On Coretractable Modules

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Let $R$ be a ring and let $M$ be an $R$-module. The module $M$ is called coretractable if $\text{Hom}(M/N, M)$ is nonzero for all proper submodule $N$ of $M$. Recall that a module $M$ is a Kasch module if every simple module in $\sigma[M]$ can be embedded in $M$. Amini, Ershad and Sharif proved that $R_R$ is a Kasch module if and only if $R_R$ is a coretractable module (see [2]). In this work we generalize this result as follows:

**Theorem:** Let $M_R$ be a finitely generated self-generator module. Then $M$ is coretractable if and only if it is Kasch.

Then we study rings whose all right modules are coretractable.

**Theorem:** For a ring $R$ the following are equivalent:

1. Every right $R$-module is coretractable.
2. $R$ is right perfect and every right $R$-module is small coretractable.
3. $R$ is right perfect and for every right $R$-module $M$, there exists a nonzero $f \in \text{Hom}(P, M)$ such that $P/\text{Ker} f$ is a small coretractable module, where $P$ is the projective cover of $M$.
4. $R$ is right perfect and for all right $R$-modules $M$ and $X$, $\text{Hom}(X, M) = 0$ if and only if $\text{Hom}(P, M) = 0$, where $P$ is the projective cover of $X$.
5. All torsion theories on $R$ are cohereditary.

We also prove that being coretractable is a Morita invariant property.

We will call $M$ mono-coretractable if for every submodule $N$ of $M$ there is a monomorphism from $M/N$ to $M$. We show that coretractable modules are a proper generalization of mono-coretractable modules. And we investigate some properties of mono-coretractable modules.

**References**


