Final State Interactions in Hadronic D decays
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- Introduction
- FSI in the D$^+ \rightarrow \pi^-\pi^+\pi^+$ decay
- FSI in the D$_s^+ \rightarrow \pi^-\pi^+\pi^+$ decay
- FSI in the D$^+ \rightarrow K^-\pi^+\pi^+$ decay
- Summary
Introduction

1. Some decays of D mesons offer experiments with high statistics where the meson-meson S-waves are dominant. This is very interesting and new.

2. This has given rise to observe experimentally with large statistical significance the $f_0(600)$ or $\sigma$ E791 Collaboration PRL 86, 770 (2001) $D^+ \rightarrow \pi^- \pi^+ \pi^+$, and the $K^*_0(800)$ or $\kappa$ mesons E719 Collaboration PRL 89, 121801 (2002) $D^+ \rightarrow K^- \pi^+ \pi^+$.

3. Clear observation of the $\sigma$ resonance has been also reported by the Collaborations CLEO, Belle and BaBar.

4. There are no Adler zeroes that destroy the bumps of the $\sigma$ and $\kappa$, contrary to scattering.
However in the E791 Analyses:

- The **phases** of the Breit-Wigner‘s used for the $\sigma$ and $\kappa$ do not follow the $I=0$ $\pi\pi$ S-wave and $I=1/2$ $K\pi$ S-wave phase shifts, respectively. Despite that at low two-body energies one expects that the spectator hypothesis should work and then it should occur by Watson‘s theorem.

- Furthermore, the $f_0(980)$ resonance has non standard couplings, e.g. it couples just to pions while **having a coupling to kaons** compatible with zero.

- The width of the $K^*_0(1430)$ is a factor of 2 smaller than PDG value, typical from scattering studies.
FSI in the $D^+ \rightarrow \pi^- \pi^+ \pi^+$ decay

$D^+ \rightarrow \pi^- \pi^+ \pi^+$ E791 Col. PRL 86, 770 (2002) 1686 candidates, Signal:background 2:1 $\rightarrow$ 1124 events.

E791 Analysis follows the Isobar Model to study the Dalitz plot

It is based on two assumptions: Third pion is a spectator, and one sums over intermediate two-body resonances.
FSI in the $D^+ \to \pi^-\pi^+\pi^+$ decay

$D^+ \to \pi^-\pi^+\pi^+$  E791 Col. PRL 86, 770 (2002) 1686 candidates,
Signal:background  2:1  $\to$ 1124 events.

E791 Analysis follows the Isobar Model to study the Dalitz plot

It is based on two assumption: Third pion is an spectator, and
one sums over intermmediate two body resonances

$$
\mathcal{A} = a_0 e^{i\delta_0} N_0 + \sum_{n=1}^{N} a_n e^{i\delta_n} A_n(s_{12}, s_{13}) N_n
$$

$\pi^-(1), \pi^+(2), \pi^+(3)$

$s_{12}=(p_1+p_2)^2$,  $s_{13}=(p_1+p_3)^2$

$a_0 e^{i\delta_0}$: Non-Resonant Term  $A_n$: Breit-Wigner (BW) term corresponding to
a (12) or (13) resonance

$$
A_n[(12)3] = BW_n(s_{12}) \mathcal{M}_n^{(J)} [(12)3] F_D^{(J)} (s_{12}) F_n^{(J)} (s_{12})
$$

Then is Bose-symmetrized because of the two identical $\pi^+$
When in the sum over resonances the $\sigma\pi^+$ state was not included, then $\chi^2$/dof=1.5 $\rightarrow$ CL=10^{-5}  WHEN included $\chi^2$/dof=0.9 $\rightarrow$ CL=76 

Exchanged resonances:

$\rho^0(770), f_0(980), f_2(1270), f_0(1370), \rho^0(1450), \sigma$
$\text{BW}_\sigma(s) = (s - m_\sigma^2 + i m_\sigma \Gamma(s))^{-1}$  

Energy dependent width:

$$\Gamma(s) = \Gamma_\sigma \frac{m_\sigma p(s) F_n^{(0)}(s)^2}{\sqrt{s} p_0 F_n^{(0)}(m_\sigma^2)^2}$$

BW does not follow the experimental S-wave $I=0$ $\pi\pi$ phase shifts

$M_\sigma = 478 \pm 24 \text{ MeV}$

$\Gamma_\sigma = 324 \pm 42 \text{ MeV}$
Laurent series around the $\sigma$ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\sigma} + \gamma_1 + \gamma_2(s - s_\sigma) + \ldots$$

From the T-matrix of Oset, J.A.O., NPA620 (1997) 435

$$s_\sigma = (0.47 - i 0.22)^2 \text{GeV}^2$$

$$\gamma_0^2 = 5.3 + i 7.7 \text{GeV}^2$$

$$\gamma_1 = -8.1 + i 36.9$$

$$\gamma_2 = 1.1 + i 0.1 \text{GeV}^{-2}$$
Laurent series around the $\sigma$ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\sigma} + \gamma_1 + \gamma_2(s - s_\sigma) + \ldots$$

We substitute the $\sigma$ BW by

$$a_1 e^{i \delta_1}$$

The movement of the phase of the pole follows the experimental phase shifts.

Adler zero: the reason why the background is so large in order to cancel the $\sigma$ pole for low energies.
At the same time the phase motion of the $\sigma$ contribution follows the experimental phase shifts.
Full Final State Interactions (FSI) from the T-matrix of Oset, J.A.O., NPA620 (1997) 435

**SCATTERING:**

Primary Vertex: \( N_{\ell k} \)  

Primary Vertex: \( \xi_{\ell D} \)

\[
T_{\ell k} = \sum_j [I + N \cdot g]_{\ell j}^{-1} N_{jk} \quad A_{\ell k} = \sum_j [I + N \cdot g]_{\ell j}^{-1} \xi_{jD}
\]

In **Unitarized Chiral Perturbation Theory** the matrix of scattering amplitudes \( N \) is fixed by matching at a given chiral order with the full amplitude \( T \) calculated in CHPT.

\( g \) is the scalar loop function or unitarity bubble.
\[
D = [I + N \cdot g]
\]

\[
\left[ D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13}) \right] a_{\pi\pi} e^{i\delta_{\pi\pi}} + \left[ D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13}) \right] a_{K\bar{K}} e^{i\delta_{K\bar{K}}}
\]

We now have meson-meson intermediate states

The \(\sigma\) and \(f_0(980)\) resonances appear as poles in the D matrix (they are dynamically generated)

Thus, the \(\sigma\) and \(f_0(980)\) BW’s are removed and substituted by the previous expression.

\(\chi^2/\text{dof}=2/152\), solid line
Dashed line, $\sigma$ pole instead of $\sigma$ BW $\chi^2/$dof$=3/152$

Solid line, full results. The BW’s of the $\sigma$ and $f_0(980)$ are removed, $\chi^2/$dof$=2/152$

FSI are driven by the fixed scattering amplitudes from UCHPT in agreement with scattering experimental data

No background, in the Laurent expansion of $D_{11}$ the background accompanying the $\sigma$ pole is negligible (No Adler zero)
FSI in the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ decay

E791 Collaboration PRL86,765 (2001) 625 events

Isobar Model: $f_0(980)$, $\rho^0(770)$, $f_2(1270)$, $f_0(1370)$ with a $f_0(980)$ dominant contribution.

$$D_s^+ \rightarrow \begin{array}{c} \text{non-resonant} \\ \rho^0(770)\pi^+ \\ f_0(980)\pi^+ \\ f_2(1270)\pi^+ \\ f_0(1370)\pi^+ \\ \rho^0(1450)\pi^+ \end{array} \rightarrow \pi^-\pi^+\pi^+$$

$M_{f_0(980)}=(977 \pm 4)$ MeV, $g_K = 0.02 \pm 0.05$, $g_\pi=0.09 \pm 0.01$

g_K \gg g_\pi and g_K compatible with zero !! In constrast with its proved affinity to couple with strangeness sources, SU(3) analysis, etc
We employ our formalism to take into account FSI from UCHPT

\[
\begin{align*}
&D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13}) \quad a_{\pi\pi} \text{e}^{i\delta_{\pi\pi}} + \\
&D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13}) \quad a_{K\bar{K}} \text{e}^{i\delta_{K\bar{K}}}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Resonance</th>
<th>$a_n$ Fraction</th>
<th>$\delta_n$ (radians)</th>
<th>$a_n$ Fraction(%)</th>
<th>$\delta_n$ (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>0.40</td>
<td>0.16</td>
<td>0.40</td>
<td>-0.24</td>
</tr>
<tr>
<td>($\pi\pi$)$\pi^+$</td>
<td>13%</td>
<td>2.36</td>
<td>14%</td>
<td>2.23</td>
</tr>
<tr>
<td>($K\bar{K}$)$\pi^+$</td>
<td>1(fixed)</td>
<td>0(fixed)</td>
<td>5%</td>
<td>0(fixed)</td>
</tr>
<tr>
<td>$\rho^0(770)\pi^+$</td>
<td>78%</td>
<td>0.14</td>
<td>84%</td>
<td>4%</td>
</tr>
<tr>
<td>$f_0(1370)\pi^+$</td>
<td>4%</td>
<td>0.60</td>
<td>4%</td>
<td>1.68</td>
</tr>
<tr>
<td>$f_2(1270)\pi^+$</td>
<td>28%</td>
<td>20%</td>
<td>28%</td>
<td>0.39</td>
</tr>
<tr>
<td>$\rho^0(1450)\pi^+$</td>
<td>20%</td>
<td>0.50</td>
<td>19%</td>
<td>0.67</td>
</tr>
<tr>
<td>$\chi^2/\nu$</td>
<td>11/142</td>
<td></td>
<td>8.5/140</td>
<td></td>
</tr>
</tbody>
</table>
Notice that the $f_0(980)$ resonant pole position is already fixed from scattering from UCHPT, as given in the $D$ matrix.

The $\sigma$ and $f_0(980)$ poles were fixed in Oset, JAO, NPA620, 435 (1997) in terms of just one free parameters plus CHPT at leading order.
FSI in the $D^+ \rightarrow K^-\pi^+\pi^-$ decay

E791 Collaboration PRL 89,121801 (2002) 28400 events only 6% background

Similar situation to the $D^+ \rightarrow \pi^-\pi^+\pi^+$ case $\sigma \leftrightarrow \kappa$

Isobar Model: non-resonant

\[ D^+ \rightarrow K^*_{0}(1430)\pi^+ \]
\[ \rightarrow K^*_{2}(1430)\pi^+ \]
\[ \rightarrow K^*_{2}(1680)\pi^+ \]
\[ \rightarrow [\kappa\pi^+] \]

Without $\kappa\pi^+$: $\chi^2$/dof=2.7 $\rightarrow$ CL=10^{-11}

With $\kappa\pi^+$: $\chi^2$/dof=0.73 $\rightarrow$ CL=95%
$\text{BW}_\kappa(s) = (s - m_\kappa^2 + i m_\kappa \Gamma(s))^{-1}$ energy dependent width

$\Gamma(s) = \Gamma_\kappa \frac{m_\sigma p(s) F_n^{(0)}(s)^2}{\sqrt{s} p_0 F_n^{(0)}(m_\kappa^2)^2}$

$\text{BW}$ does not follow the experimental $S$-wave $I=1/2$ $K\pi$ phase shifts

$M_\kappa = 797 \pm 47$ MeV

$\Gamma_\kappa = 410 \pm 97$ MeV
Laurent series around the $\sigma$ pole position at the second Riemann sheet:

$$t^{II}_{11}(s) = \frac{\gamma_0^2}{s - s_\kappa} + \gamma_1 + \gamma_2(s - s_\kappa) + \ldots$$

$s_\kappa = (0.71 - i0.31)^2\text{GeV}^2$ \hspace{1cm} $\gamma_0^2 = 19.0 + i5.9\text{GeV}^2$

$\gamma_1 = 12.5 + i43.1$ \hspace{1cm} $\gamma_2 = 0.8 + i7.3\text{GeV}^{-2}$


UCHPT matching with U(3) CHPT +Resonances+large $N_c$ constraints (vanishing of scalar form factors for $s \to \infty$)

$K\pi, K\eta, K\eta'$ channels are included
Laurent series around the $\kappa$ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \gamma_0^2 \frac{\gamma_1 + \gamma_2(s - s_\kappa)}{s - s_\kappa} + \ldots$$

The movement of the phase of the pole follows the experimental phase shifts.

We substitute the $\kappa$ BW by $a_1 e^{i \delta_1}$.

Adler zero: the reason why the background is so large in order to cancel the $\kappa$ pole for low energies.
Dashed line, \( \kappa \) pole instead of \( \kappa \) BW \( \chi^2/\text{dof}=6.5/132 \) \( s_\kappa \) is fixed

At the same time the phase motion of the \( \kappa \) pole contribution follows the experimental phase shifts.

$$
\begin{align*}
\left[ D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13}) \right] a_{K\pi} e^{i\delta_{K\pi}} + \\
\left[ D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13}) \right] a_{K\eta} e^{i\delta_{K\eta}} + \\
\left[ D_{13}^{-1}(s_{12}) + D_{13}^{-1}(s_{13}) \right] a_{K\eta'} e^{i\delta_{K\eta'}}
\end{align*}
$$

We now have meson-meson intermediate states.

The $\kappa$ and $K^*_0(1430)$ resonances appear as poles in the D matrix.

Thus, the $\kappa$ and $K^*_0(1430)$ BW’s are removed and substituted by the previous expression.

$\chi^2/dof=127/128$, solid and dashed lines
Points from E791 fit

Solid line, full results. The BW’s of the $\kappa$ and $K^*_0(1430)$ are removed.

Dashed line: the $K\eta$ channel is removed. Stability.

FSI are driven by the fixed scattering amplitudes from UCHPT in agreement with scattering experimental data.

No background, in the Laurent expansion of $D_{11}$ the background accompanying the $\kappa$ pole is negligible (No Adler zero).
<table>
<thead>
<tr>
<th>Resonance</th>
<th>$\alpha_n$</th>
<th>$\delta_n$</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>1.60</td>
<td>0.10</td>
<td>29.6%</td>
</tr>
<tr>
<td>$(K\pi)_0\pi^+$</td>
<td>1.66</td>
<td>4.10</td>
<td>31.8%</td>
</tr>
<tr>
<td>$(K\eta)_0\pi^+$</td>
<td>0.86</td>
<td>2.63</td>
<td>2.0%</td>
</tr>
<tr>
<td>$(K\eta')_0\pi^+$</td>
<td>2.33</td>
<td>−1.54</td>
<td>9.8%</td>
</tr>
<tr>
<td>$K_1^*(892)\pi^+$</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
<td>11.6%</td>
</tr>
<tr>
<td>$K_2^*(1430)\pi^+$</td>
<td>0.11</td>
<td>−0.62</td>
<td>0.2%</td>
</tr>
<tr>
<td>$K_3^*(1680)\pi^+$</td>
<td>0.72</td>
<td>0.80</td>
<td>5.9%</td>
</tr>
<tr>
<td>$\chi^2/\nu$</td>
<td>127/128</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$K^*_0(1430)$ E791... $M_{K^*_0(1430)} = 1459 \pm 9$ MeV

$\Gamma_{K^*_0(1430)} = 175 \pm 17$ MeV

PDG... $M_{K^*_0(1430)} = 1412 \pm 6$ MeV

$\Gamma_{K^*_0(1430)} = 294 \pm 23$ MeV

Jamin,Pich,JAO (1430-1450,140-160) MeV$\sim$ (M,$\Gamma/2$)

In their fit (6.10) $(1450,142)$ MeV

employed here
We have considered simultaneously the FSI driven by the S-waves in the decays: \( D^+ \rightarrow \pi^-\pi^+\pi^+ \), \( D_{s}^+ \rightarrow \pi^-\pi^+\pi^+ \), \( D^+ \rightarrow K^-\pi^+\pi^+ \).

We have reproduced the E791 Collaboration signal distribution functions in terms of new parameterizations.

In E791 analyses the disagreement between the phase motions of the \( \sigma \) and \( \kappa \) and the elastic S-wave I=0,1/2 phase shifts is due to the employment of BW‘s.

Once these BW‘s are substituted by the pole contributions of these resonances the agreement is restored.

These poles are fixed from T-matrices already determined from CHPT, unitarity, analiticity plus fitting scattering data.
We have also reproduced the results of E791 making use of the full results of I=0,1/2 S-wave T-matrices in agreement with scattering and from Unitarized CHPT.

The reason why the $\sigma$ and $\kappa$ pole contributions are not distorted in contrast with scattering is the absence of Adler zeroes.

These poles dominate the D matrix in the low energy region. No significative background.

The $f_0(980)$ from D decays turns out also with standard properties regarding its coupling to kaon.

The width of the $K^*_0(1430)$ from D decays is then in agreement with that from scattering data and reported in the PDG. That from E791 analysis was a factor 2 smaller.
General Expression for a Partial Wave Amplitude

- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$\text{Im} T_{ij} = \sum_k T_{ik} \rho_k T_{kj}^* \quad \rightarrow \quad \text{Im} T_{ij}^{-1} = -\rho_i \delta_{ij} \quad \text{Unitarity Cut}$$

$W = \sqrt{s}$

We perform a dispersion relation for the inverse of the partial wave (the unitarity cut is known)

$$T_{ij}^{-1} = R_{ij}^{-1} + \delta_{ij} \left( g(s_0) \frac{s - s_0}{\pi} \int \frac{\rho(s')ds'}{(s' - s - i0^+)(s' - s_0)} \right)$$

The rest

$g(s)$: Single unitarity bubble
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \left( \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \right) \]

\[ T = \left( R^{-1} + g(s) \right)^{-1} \]

\( T \) obeys a CHPT/alike expansion

\( R \) is fixed by matching algebraically with the CHPT/alike CHPT/alike+Resonances expressions of \( T \)

In doing that, one makes use of the CHPT/alike counting for \( g(s) \)

The counting/expressions of \( R(s) \) are consequences of the known ones of \( g(s) \) and \( T(s) \)

The CHPT/alike expansion is done to \( R(s) \). Crossed channel dynamics is included perturbatively.

The final expressions fulfill unitarity to all orders since \( R \) is real in the physical region (\( T \) from CHPT fulfills unitarity pertubatively as employed in the matching).
Production Processes

The re-scattering is due to the strong "final" state interactions from some "weak" production mechanism.

\[ Im F_i = \sum_k F_k \rho_k T_{ki}^* \]

We first consider the case with only the right hand cut for the strong interacting amplitude, \( R^{-1} \) is then a sum of poles (CDD) and a constant. It can be easily shown then:

\[ F = (I + R g(s))^{-1} \xi \]
Finally, $\xi$ is also expanded pertubatively (in the same way as $R$) by the matching process with CHPT/alike expressions for $F$, order by order. The crossed dynamics, as well for the production mechanism, are then included pertubatively.
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LET US SEE SOME APPLICATIONS
Meson-Meson Scalar Sector

1) The mesonic scalar sector has the vacuum quantum numbers $0^{++}$. Essential for the study of Chiral Symmetry Breaking: Spontaneous and Explicit $m_u, m_d, m_s$.

2) In this sector the mesons really interact strongly.
   1) Large unitarity loops.
   2) Channels coupled very strongly, e.g. $\pi\pi$, $\bar{K}K$, $\pi\eta$, $K, \bar{K}$...
   3) Dynamically generated resonances, Breit-Wigner formulae, VMD, ...

3) OZI rule has large corrections.
   1) No ideal mixing multiplets.
   2) Simple quark model.

Points 2) and 3) imply large deviations with respect to Large $N_c$ QCD.
4) A precise knowledge of the scalar interactions of the lightest hadronic thresholds, $\pi \pi$ and so on, is often required.

- Final State Interactions (FSI) in $\varepsilon'/\varepsilon$, Pich, Palante, Scimemi, Buras, Martinelli,...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- Fluctuations in order parameters of $\mathcal{N}$SB.
4) A precise knowledge of the scalar interactions of the lightest hadronic thresholds, $\pi \pi$ and so on, is often required.

- Final State Interactions (FSI) in $\varepsilon'/\varepsilon$, Pich, Palante, Scimemi, Buras, Martinelli, ...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- Fluctuations in order parameters of SUSY.

Let us apply the chiral unitary approach

- LEADING ORDER:

$$T = (R^{-1} + g(s))^{-1}$$

$T = T_2 = R_2 - R_2 g R_2 + \ldots$  \quad $R = R_2 = T_2$

$g$ is order 1 in CHPT

Oset, Oller, NPA620,438(97) \quad $a_{SL} \approx -0.5$ only free parameter, equivalently a three-momentum cut-off $\Lambda \approx 0.9$ GeV
All these resonances were dynamically generated from the lowest order CHPT amplitudes due to the enhancement of the unitarity loops.

<table>
<thead>
<tr>
<th>$f_0 (980)$ (GeV)</th>
<th>$a_0 (980)$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.993 $-i$ 0.012</td>
<td>1.009 $-i$ 0.056</td>
</tr>
<tr>
<td>$</td>
<td>g^f_{\pi\pi}</td>
</tr>
<tr>
<td>$</td>
<td>g^f_{KK}</td>
</tr>
</tbody>
</table>

$\text{Br}(f_0(980) \rightarrow \pi\pi) = 0.70 \quad \text{Br}(a_0(980) \rightarrow \pi\eta) = 0.63$
In Oset, Oller PRD60,074023(99) we studied the I=0,1,1/2 S-waves. The input included next-to-leading order CHPT plus resonances:

1. **Cancellation** between the crossed channel loops and crossed channel resonance exchanges. (Large $N_c$ violation).

2. **Dynamically generated resonances.** The tree level or preexisting resonances move higher in energy (octet around 1.4 GeV). Pole positions were very stable under the improvement of the kernel $R$ (convergence).

3. In the **SU(3) limit** we have a degenerate octet plus a singlet of dynamically generated resonances $\sigma, f_0(980), a_0(980), \kappa(700)$.
Using these T-matrices we also corrected by Final State Interactions the processes $\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0, \pi\eta, K^+K^-, K^0\bar{K}^0$
Where the input comes from CHPT at one loop, plus resonances. There were some couplings and counterterms but were taken from the literature. No fit parameters.
Oset, Oller NPA629,739(98).
CHPT+Resonances

Ecker, Gasser, Pich and de Rafael, NPB321, 311 (’98)
Resonances give rise to a resummation of the chiral series at the
tree level (local counterterms beyond $O(p^4)$).

$$\frac{1}{M^2 - q^2} = \frac{1}{q^2} + \frac{q^2}{M^4} + \frac{q^4}{M^6} + ...$$

$q^2 < M^2$

The counting used to perform the matching is a simultaneous one in the
number of loops calculated at a given order in CHPT (that increases order by
order). E.g:
- Meissner, J.A.O, NPA673, 311 (’00) the $\pi N$ scattering was
  studied up to one loop calculated at $O(p^3)$ in HBCHPT+Resonances.
- Jamin, Pich, J.A.O, NPB587, 331 (’00), \( K\pi, K\eta, K\eta' \) scattering.

- The inclusion of the resonances require the knowlodge of their bare masses and couplings, that were fitted to experiment. A theoretical input for their values would be very welcome:
  - The CHUA would reduce its freedom and would increase its predictive power.
  - For the microscopic models, one can then include the so important final state interactions that appear in some channels, particularly in the scalar ones. Also it would be possible to identify the final physical poles originated by such bare resonances and to work simultaneously with those resonances dynamically generated.

\[
L_{S\pi\pi} = g_s \Psi \Psi S \\
L_{s\pi\pi} = S(\bar{c}_m \text{Tr}(X^+) + \bar{c}_d \text{Tr}(u_\mu u^\mu))
\]