Bode optimal loop shaping with CRONE compensators

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PROBLEM:
- for operational bandwidth $0 \leq \omega \leq \omega_0$
- where desired $|L(j\omega)| = M_0 \gg 1$
- given crossover frequency $\omega_c$
- compute $L(j\omega)$ which maximizes $\omega_0$

SOLUTION: decrease $|L(j\omega)|$ as fast as possible...
**Problem Statement**

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Problem Statement

- \(|L(j\omega)|_{dB}\)
- \(M_0\)
- \(\omega_0\)
- \(\omega_c\)
- \(\arg(L(j\omega))\)
- \(-180^\circ\)
- \(-180^\circ\)
- \(\omega_0\)
- \(\omega_c\)
Ideal Bode Characteristic

For ideal optimal structure

\[
\begin{align*}
|L(j\omega)| &= M_0, \quad 0 < \omega < \omega_0 \\
\angle L(j\omega) &= -\alpha \pi, \quad \omega > \omega_0
\end{align*}
\]

solution is equivalent to maximize phase lag,
or minimizing stability margin,
so it is necessary to trade-off between $\omega_0$ and stability margin...
Introduction: Bode Optimal Loops
Loop Shaping with CRONE Compensators
Design Examples
Conclusions

Four Parameters Bode Optimal Loop
Eight Parameters Bode Optimal Loop

Goal

Ideal Bode Characteristic

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\[
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solution is equivalent to maximize phase lag, or minimizing stability margin, so it is necessary to trade-off between \(\omega_0\) and stability margin...
In general, the problem is well defined as a function of parameters:

- $M_0$
- $\alpha$
- $\omega_0$
- $\omega_c$

Not all of them independent.
Practical considerations about high frequency

- In practice, four parameters Bode Optimal Loop has to be modified:
  - To cope with sensor noise amplification
  - Because it is not realistic to assume good control of $|L(j\omega)|$ for high frequency.
Seven Parameters Bode Optimal Loop
Seven Parameters Bode Optimal Loop

\[ |L(j\omega)|_{dB} \]

\[ \arg(L(j\omega)) \]

\( \omega_0 \)

\( \omega_c \)

\( \omega_1 \)

\( \omega_2 \)

\( M_0 \)

\(-\alpha \ 180^\circ\)

\(-180^\circ\)

\(-90^\circ\)
In order to add integrators to the loop, for a good steady state response ...
Eight Parameters Bode Optimal Loop

- $M_0$
- $|L(j\omega)|_{dB}$
- $\omega_0$
- $\omega_c$
- $\omega_1$
- $\omega_2$
- $e 90^\circ$
- $-180^\circ$
- $-\pi 180^\circ$

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Eight Parameters Bode Optimal Loop

Parameters:
- $M_0$
- $M_1$
- $\alpha$
- $\omega_0$
- $\omega_c$
- $\omega_1$
- $n$
- $e$

Not all of them are independent.
Establish **relations between** these **eight parameters** and the **parameters of** a proposed **CRONE structure**, so that a first approach to Bode optimal loop can be obtained in an easy and fast way.
CRONE Features for Bode Optimal Loop Shaping

- Easy to tune
- Few parameters

For the 2/3 CRONE generation band defined compensator

\[ D_r = \left( C_0 \frac{1}{1+s/\omega_l} \right)^a \cos \left[ -b \log \left( C_0 \frac{1}{1+s/\omega_h} \right) \right], \]

- Phase and gain slope only depend on \( a \) (real differentiation order)
- Gain and phase slope only depend on \( b \) (complex differentiation order)

Idea: for a Bode optimal loop shape, with constant phase at \((\omega_l, \omega_h)\) and constant gain at \((\omega'_l, \omega'_h)\), use real differentiator at \((\omega_l, \omega_h)\) \((b=0)\) and complex differentiator \((a=0)\) at \((\omega'_l, \omega'_h)\).
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CRONE Features for Bode Optimal Loop Shaping

- Easy to tune
- Few parameters
- For the 2/3 CRONE generation band defined compensator
  \[ D_r = \left( C_0 \frac{1+\omega}{1+\omega_a} \right) \]
  - Phase and gain slope only depend on \( a \) (real differentiation order)
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- Idea: for a Bode optimal loop shape, with constant phase at \((\omega_l, \omega_h)\) and constant gain at \((\omega'_l, \omega'_h)\), use real differentiator at \((\omega_l, \omega_h)\) (\(b=0\)) and complex differentiator \((a=0)\) at \((\omega'_l, \omega'_h)\).
Additionally, two terms to shape low and high frequencies

Final structure:

\[
L(s) = k \left( \frac{\omega_l}{s} + 1 \right)^{n_l} \left( C_0 \frac{1 + \frac{s}{\omega_l}}{1 + \frac{s}{\omega_h}} \right)^a \\
\cos \left[ -b \log \left( C_0' \frac{1 + \frac{s}{\omega_l'}}{1 + \frac{s}{\omega_h'}} \right) \right] \frac{1}{\left( \frac{s}{\omega_h} + 1 \right)^{n_h}}
\]
Real Differentiator Term

\[ L_2(s) = \left( C_0 \frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}} \right)^a \]

Design relations:
- \[ a \left( \frac{\pi}{2} - 2\theta_l(\omega_u) \right) = (1 - \alpha)\pi \]
- \[ \left( \frac{\omega_h}{2\omega_l} \right)^{-a} \approx M_0 M_1 \]
Real Differentiator Term

$L_2(s) = \left( C_0 \right.$

Design relations:

\[ a \left( \frac{\pi}{2} - 2\theta \right) \left( \frac{\omega_h}{2\omega_l} \right)^{-a} \]

\[ \approx M_0 M_1 \]

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Real Differentiator Term

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Real Differentiator Term

\[ L_2(s) = C_0: \]

Design relation:
- \( a \left( \frac{\pi}{2} - 2\theta \right) \)
- \( \left( \frac{\omega_h}{2\omega_l} \right)^{-a} \)

Why a CRONE compensator?
- Real Differentiator Term
- Low and High Frequency Terms
- Complex Differentiator Term
- Maximizing Loop Phase Lag

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**Low and High Frequency Terms**

\[ L_{CRONE2}(s) = k \left( \frac{\omega_l}{s} + 1 \right)^{n_l} \left( C_0 \frac{1 + \frac{s}{\omega_l}}{1 + \frac{s}{\omega_h}} \right)^a \frac{1}{\left( \frac{s}{\omega_h} + 1 \right)^{n_h}} \]

**Design relations:**
- \( n_l \geq n \)
- \( n_h \geq e_p \geq n \)
- \( |L_{CRONE2}(j\omega_c)| = 1 \)
- \( |L_{CRONE2}(j\omega_u)| = \frac{M_{0, dB} + M_{1, dB}}{2} \)
Low and High Frequency Terms

\[ L_{CRONE2}(s) = k \left( \frac{\omega_l}{s} + 1 \right)^{n_l} \left( C_0 \frac{1 + \frac{s}{\omega_l}}{1 + \frac{s}{\omega_h}} \right)^a \frac{1}{\left( \frac{s}{\omega_h} + 1 \right)^{n_h}} \]

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Low and High Frequency Terms

\[ L_{CRONE2}(s) = \]

Design relations:
- \( n_l \geq n \)
- \( n_h \geq e_p \)
- \( |L_{CRONE2}(j \omega)| \)
- \( |L_{CRONE2}(j \omega)| \)

\[ \omega_l = 0.1 \text{ rad/s} \]
\[ \omega_h = 10 \text{ rad/s} \]
\[ \omega_h = 100 \text{ rad/s} \]
\[ \omega_h = 1000 \text{ rad/s} \]

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**Complex Differentiator Term**

- \( L_3(s) = \cos \left[ -b \log \left( C'_0 \frac{1 + \frac{s}{\omega'_l}}{1 + \frac{s}{\omega'_h}} \right) \right] \)

- Complements \( L_2(s) \), to increase phase lag at \([\omega'_l, \omega'_h]\)

- To avoid non minimum phase:

  \[
  b \log \left( \frac{\omega'_h}{\omega'_l} \right) < \pi
  \]

  or, equivalently

  \[
  b < b_{\text{max}} = \frac{\pi}{\log(\omega'_h/\omega'_l)}
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\]
Maximizing Loop Phase Lag

- Maximized by $b = b_{max}$, but...
- Design relations:
  - $\omega_u' = \omega_h \approx \omega_1$
  - $b \approx b_{max}$
Maximizing Loop Phase Lag

- Maximized by $\gamma$
- Design relations:
  - $\omega'_u = \omega_h \approx \omega_1$
  - $b \approx b_{max}$

\[ b = b_{max} = 0.68 \]
\[ b = 0.75 \]
\[ b = 0.67 \]
\[ b = 0.60 \]
\[ b = 0.40 \]
\[ b = 0.10 \]
Maximizing Loop Phase Lag

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Design relations:

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$$b \approx b_{\text{max}}$$
8 Parameters Bode Optimal Specifications:

- $M_0, dB = M_1, dB = 30$ dB
- $\omega_0 = 0.4$ rad/s, $\omega_c = 0.4$ rad/s
- $\alpha = 0.22$ (40° phase margin)
- $e = 3$, $n = 2$
- $\omega_1 = 40$ rad/s

Loop obtained:

$$L_{ex}(s) = 0.87 \left( \frac{0.34}{s} + 1 \right)^2 \left( C_0 \frac{1 + \frac{s}{0.34}}{1 + \frac{s}{93.5}} \right)^{-1.45} \cos \left[ -1.8374 \log \left( C'_0 \frac{1 + \frac{s}{97.5}}{1 + \frac{s}{250}} \right) \right] \frac{1}{\left( \frac{s}{93.5} + 1 \right)^3}$$
**Desig Example**

- **8 Parameters Bode Optimal Specifications:**
  - \( M_0, dB = M_1, dB = 30 \text{ dB} \)
  - \( \omega_0 = 0.4 \text{ rad/s} \), \( \omega_c = 0.4 \text{ rad/s} \)
  - \( \alpha = 0.22 \) (40° phase margin)
  - \( e = 3 \), \( n = 2 \)
  - \( \omega_1 = 40 \text{ rad/s} \)

- **Loop obtained:**

\[
L_{ex}(s) = 0.87 \left( \frac{0.34}{s} + 1 \right)^2 \left( C_0 \frac{1 + \frac{s}{0.34}}{1 + \frac{s}{93.5}} \right)^{-1.45} \cos \left[ -1.8374 \log \left( C'_0 \frac{1 + \frac{s}{97.5}}{1 + \frac{s}{250}} \right) \right] \frac{1}{\left( \frac{s}{93.5} + 1 \right)^3}
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Design Example

8 Parameters
- $M_0, dB = M_1$
- $\omega_0 = 0.4 \text{ rad/s}$
- $\alpha = 0.22$ (40° phase margin)
- $e = 3$, $n = 2$
- $\omega_1 = 40 \text{ rad/s}$

Loop obtained:

$L_{ex}(s) = 0.$
Conclusions

- A special CRONE compensator has been proposed to efficiently approximate Bode optimal loop.
- Bode optimal loop has been defined based on a number of parameters, and simple design rules have been obtained for tuning the proposed compensator.
- These rules yield a first solution of a rather hard problem.
- A finest tuning may require the use of some automatic loop shaping technique.
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