



Characterization, classification and ordering of distributions

Syllabus

Course code:	6377
Number of ECTS credits:	3
Semester:	1st (September-January)
Recommended components:	Real functions of a single variable I (1568), Real functions of a single variable II (1573), Functions of several real variables I (1578), Functions of several real variables II (1579), Probability theory (1590), Advanced probability and stochastic processes (1595).
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

In this course the student will study the role of several functions, like the quantile, hazard rate and mean residual lifetime, in the characterization, classification and ordering of distributions. These functions are of special interest in the context of reliability and risk theory. Additionally the notion of copula and several measures of concordance and their main properties are also studied.

Learning outcomes and competences

After completion of this course you will:

1. understand the notions of hazard rate and mean residual life and their role in the context of reliability and risk theory.
2. know how to characterize a distribution function in terms of the hazard rate and mean residual life functions.
3. know how to use the hazard rate and mean residual life functions to provide non parametric classifications of distributions. The provided non parametric classes include IFR (DFR), NBU (NWU), DMRL (IMRL) and NBUE (NWUE) ageing classes and bathtub distributions.
4. know how to use the survival, quantile, hazard rate and mean residual life functions to provide stochastic orderings of distributions. The provided stochastic orders include usual stochastic, hazard rate, mean residual life, likelihood ratio and dispersive orders.
5. know how to model the dependence of multivariate distributions through the notion of copula.

6. know how to measure the concordance of two random variables.
7. know how to characterize a distribution function in terms of the entropy of truncated random variables.

Course contents

THEORY

I. Introduction

Introduction: distribution, quantile and survival functions. Properties and examples.

II. Truncated random variables. Reliability measures

Truncated random variables. Hazard rate function. Mean residual life function. Properties and examples.

III. Classification of distributions.

IFR (DFR), NBU (NWU), DMRL (IMRL) and NBUE (NWUE) ageing classes. Bathtub distributions. Properties and relationships. Glasser's theorem.

IV. Stochastic orders

Stochastic orders. Characterizations, sufficient conditions and relationships.

V. Continuous multivariate distributions and copulas

Multivariate normal distribution. t -Student multivariate distribution. Copulas. Sklar's theorem. Archimedean copulas.

VI. Concordance measures.

Kendall's tau and Spearman's rho measures. Properties.

VII. Characterization of distributions

Characterization of distribution functions through truncated means, the hazard rate and mean residual life functions and the entropy of residual life.

References

Main texts

1. Belzunce, F., Martínez-Riquelme, C. & Mulero, J. *An introduction to stochastic orders*; Elsevier-Academic Press, 2015.
2. Galambos, J. & Kotz, S. *Characterization of probability distributions*, Springer, 1978.
3. Joe, H. *Dependence modelling with copulas*, Chapman & Hall, 2015.
4. Nelsen, R.E. *An introduction to copulas*; Springer, 2006.
5. Lai, C.D. & Xie, M. *Stochastic ageing and dependence for reliability*, Springer, 2006.
6. Shaked, M. & Shanthikumar, J.G. *Stochastic orders*, Springer, 2007.

Supplementary references

1. Barlow, R. & Proschan, F. *Statistical theory of reliability and life testing*, Holt, Rinehart and Winston, 1976.
2. Belzunce, F., Navarro, J., Ruiz, J.M. & del Aguila, Y. *Some results on residual entropy functions*, *Metrika*, **59**, 147-161, 2004.
3. Kayid, M. & Izadkhah, S. *Characterization of the exponential distribution by the concept of residual life at random time*, *Statistics and Probability Letters*, **107**, 164-169, 2015.
4. Kotz, S., Navarro, J. & Ruiz, J.M. *Characterization of Arnold and Strauss and related bivariate exponential models*, *Journal of Multivariate Analysis*, **98**, 1494-1507, 2007.
5. Kotz, S. & Shanbhag, D.N. *Some new approaches to probability distributions*, *Advances in Applied Probability*, **12**, 497-499, 1980.
6. Kundu, C., Nanda, A.K. & Maiti, S.S. *Some distributional results through past entropy*, *Journal of Statistical Planning and Inference*, **140**, 1280-1291, 2010.
7. Zoroa, P., Ruiz, J.M. & Marín, J. *A characterization based on conditional expectations*, *Communications in Statistics - Theory and Methods*, **19**, 3127-3135, 1990.