UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Geometry of Submanifolds

Syllabus

Course code:	6370
Number of ECTS credits:	3
Semester:	1st (September-January)
Recommended components:	Geometry of Curves and Surfaces (1589); Global Geometry of Surfaces (1594); Riemannian Geometry (1602).
Language of instruction:	Spanish.

Course description

The theory of submanifolds as a field of differential geometry is as old as differential geometry itself, beginning with the theory of curves and surfaces. However, the theory of submanifolds given in this course is relatively new in the realm of contemporary differential geometry. The student is assumed to be somewhat familiar with the general theory of differential geometry as can be found, for example, in the book "Foundations of Differential Geometry", by S. Kobayashi and K. Nomizu.

Learning outcomes and competences

After completion of this course you will:

1. understand the basic notion of submanifold of Euclidean space.
2. know the basic elements of semi-Riemannian geometry (differentiable maps, tangent spaces, metrics, etc.).
3. know the basic properties of tensors and, particularly, vector fields and one forms.
4. know the main differences between intrinsic geometry and extrinsic geometry of a submanifold.
5. know the main results of submanifolds theory, its applications and extensions.

Course contents

1. Semi-Riemannian manifolds.

   Differentiable manifolds. Differentiable functions. Tangent space. Differentiable maps. Vector fields. One forms. Semi-Riemannian metrics. Levi-Civita connection. Parallel transport. Exponential map and geodesics.

2. Tensors and differentiable operators.

   Tensors. Tensor components. Contractions. Tensor derivations. Curvature tensor. Sectional curvature. Metric contractions. Ricci and scalar curvature. Gradient, divergence, Hessian and Laplacian operators.

3. Semi-Riemannian submanifolds I.

   Immersions. Induced metric. Tangent and normal vectors. Induced connection. Second fundamental form. Mean curvature vector field. Gauss and Weingarten formulas.

4. Semi-Riemannian submanifolds II.

   Normal bundle. Normal connection. Equations of Gauss, Codazzi and Ricci. Fundamental theorems of submanifolds: existence and rigidity.

5. Semi-Riemannian submanifolds III.

   Geodesics in submanifolds. Totally geodesic submanifolds. Totally umbilical submanifolds. Examples of Euclidean submanifolds.

6. Semi-Riemannian hypersurfaces.

   The shape operator. Principal curvatures. Hyperquadrics. Totally geodesic hypersurfaces. Totally umbilical hypersurfaces.

7. Minimal hypersurfaces I.

   Historical aspects. Equivalent definitions of minimality. The Weierstrass representation. Plateau problem. Minimal graphs. Bernstein's theorem.

8. Minimal hypersurfaces II.

   First and second variation of area. Jacobi operator. Index and stability. Stability in the Euclidean space and in the sphere.

9. Constant mean curvature hypersurfaces.

   CMC hypersurfaces. Weak and strong stability. First eigenvalue of Jacobi operator. Hypersurfaces in the sphere.

10. Submersions.

    Riemannian submersions. Vertical and horizontal vectors. Examples. Tensor T and A. Fundamental equations.

References

Main texts

1. Chen, B.Y. *Geometry of Submanifolds.* Marcel Dekker, Inc., New York, 1973.

2. Dajczer, M. *Submanifolds and isometric immersions.* Publish or Perish, Houston, Texas, 1990.

3. O'Neill, B. *Semi-Riemannian geometry: with applications to relativity.* Academic Press, New York, 1983.