UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Applied Mathematical Analysis I

Syllabus

Course code:	6364
Number of ECTS credits:	3
Semester:	1st (September-January)
Recommended components:	Natural prerequisites are real analysis courses from the degree in mathematics, such as Partial Differential Equations and Fourier series (1593) or Functional Analysis (1599), as well as basic knowledge in measure and integral theory in $\mathbb{R}^n$ (1579,1583). It is strongly recommended to take as well the course Classical Mathematical Analysis (6365) from this master.
Language of instruction:	Spanish (questions, homework and exams are allowed in English)

Course description

This course intends to introduce various mathematical analysis tools which are useful in the study of partial differential equations, as well as to develop physical models of PDEs and applications of these. The course will cover some of the following topics (depending on time and students' background)

- Distributions and Sobolev spaces: weak derivatives, Sobolev embedding theorem.
- Functional analysis techniques: fixed point theorems.
- Harmonic analysis techniques: Fourier transform.
- Elliptic equations: boundary value problems, eigenvalues, variational methods.
- Evolution equations: heat equation, Schrödinger equation, semigroups.
- Wave equation: finite propagation, regularity.

Learning outcomes and competences

After completion of this course, an average student is expected to

1. know the most basic tools in real analysis which are relevant in the study of PDEs
2. know the main models of linear PDEs (elliptic, parabolic and hyperbolic), their physical formulation and their most relevant properties
3. be familiar with a few basic texts from Real Analysis and PDEs, which can be used for reference and further research in the field

Course contents

I. The Fourier transform

The Fourier transform in L^2, the Schwartz class, formal resolution of PDEs.

II. Distributions and Sobolev spaces

The concept of distribution. Weak derivatives, convolution, Fourier transform. Fundamental solutions of PDEs. Introduction to Sobolev spaces, density, inclusions, Sobolev embedding theorem.

III. Laplace equation

Physical models. Fundamental solutions, maximum principle, regularity estimates. Dirichlet and Neumann conditions, variational methods.

IV. Heat equation

Physical model. Fundamental solution. Semigroup property. Convergence to initial data. Introduction to the Schrodinger equation.

V. Wave equation

Physical model. Fundamental solution, finite propagation, regularity.

References

Main texts

1. L. Evans, *Partial Differential Equations.* Amer Math Soc, 1997.
2. E. Stein, R. Shakarchi, *Fourier Analysis, an introduction.* Princeton Univ Press, 2003.

Supplementary references

1. H. Brezis, *Análisis Funcional*, Alianza Ed, 1984.
2. G. Folland, *Lectures on Partial Differential Equations*, Springer-Tata Institute, 1983.
   http://www.math.tifr.res.in/∼publ/ln/tifr70.pdf
3. D. Gilbarg, N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer 2001 .
4. E. Lieb, M. Loss, *Analysis*, 2nd Ed, Amer Math Soc, 2001.
5. I. Peral, *Primer curso de Ecuaciones en Derivadas Parciales*, Addison-Wesley-UAM, 1994.
   https://www.uam.es/personal_pdi/ciencias/ireneo/libro.pdf
6. R. Strichartz. *A guide to Distribution Theory and Fourier Transforms.* CRC Press 1994.