NORM EQUALITIES FOR OPERATORS

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Abstract. We investigate norm equalities for operators of the form

\[(1) \quad \|g(T)\| = f(\|T\|)\]

and

\[(2) \quad \|Id + g(T)\| = f(\|g(T)\|),\]

where \(g\) is analytic and \(f\) is continuous. The most famous equation of this form is the Daugavet equation

\[(DE) \quad \|Id + T\| = 1 + \|T\|,\]

which holds for all weakly compact operators on many spaces as \(C[0,1]\), \(L_1[0,1]\) or \(A(\mathbb{D})\). The fact that all rank-one operators on a Banach space satisfy (DE), called the Daugavet property, has many geometrical and topological implications on the space.

Our aim is to investigate the properties one could define by requiring that all rank-one operators on a Banach space satisfy equation (1) or equation (2) for fixed \(g\) and \(f\). For the first case, we show that the only non-trivial property of this kind is the Daugavet property. For the second case, a property different from the Daugavet property appears; however, only these two properties may happen in the complex case.

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