Main Lectures

Richard M. Aron
Some problems related to algebras of holomorphic functions in finite and infinite dimensions

Frédéric Bayart
Coordinatewise summability, inclusion theorems and p-Sidon sets

Andreas Defant
Bohr’s phenomenon for functions on the Boolean cube

Anna Kamińska
Abstract Lorentz spaces and Köthe duality

Mieczysław Mastyło
Analytic families of multilinear operators

Vassili Nestoridis
A project on generic non-extendability of functions belonging to some new spaces and their properties, in one, several or infinitely many complex variables

Eve Oja
Lifting of nest approximation properties and related principles of local reflexivity

Thomas Schlumprecht
On the coarse embeddability of Hilbert space

Kristian Seip
Hardy spaces of Dirichlet series and the Riemann zeta function
Schedule

Tuesday 17.10.2017
09:30-10:00 Opening
10:00-11:00 A. Defant
11:00-11:30 Coffee
11:30-12:30 V. Nestoridis

Wednesday 18.10.2017
09:30-10:30 M. Mastyło
10:30-11:00 Coffee
11:00-12:00 E. Oja
12:00-13:00 T. Schlumprecht

Thursday 19.10.2017
10:00-11:00 A. Kamińska
11:00-11:30 Coffee
11:30-12:30 F. Bayart

Friday 20.10.2017
10:00-11:00 K. Seip
11:00-11:30 Coffee
11:30-12:30 R.M. Aron
Abstracts

Richard M. Aron
Kent State University

Some problems related to algebras of holomorphic functions in finite and infinite dimensions

Let $X$ be a complex Banach space with open unit ball $B$. As usual, let $\mathcal{H}_b(X)$ denote the Fréchet algebra of entire functions $f : X \to \mathbb{C}$ such that $f$ is bounded on $nB$ for all $n \in \mathbb{N}$. Also, let $\mathcal{H}_\infty(B) = \{ f : B \to \mathbb{C} \mid f$ is bounded and holomorphic on $B \}$, and call $\mathcal{A}_u(B)$ the subalgebra of those functions in $\mathcal{H}_\infty(B)$ that are uniformly continuous. We will review some old, and perhaps a few new, open problems related to homomorphisms on these algebras.

Frédéric Bayart
Université Blaise Pascal

Coordinatewise summability, inclusion theorems and $p$-Sidon sets

Let $m \geq 1$, $X_1, \ldots, X_m$, $Y$ Banach spaces and $T : X_1 \times \cdots \times X_m \to Y$ $m$-linear. For $r \geq 1$ and $\mathbf{p} = (p_1, \ldots, p_m) \in [1, +\infty)^m$, we say that $T$ is multiple $(r, \mathbf{p})$-summing if there exists a constant $C > 0$ such that for all sequences $x(f) \subset X_j^N$, $1 \leq j \leq m$,

$$\left( \sum_{k \in \mathbb{N}^m} \| T(x_k) \|^r \right)^{\frac{1}{r}} \leq C w_{p_1}(x(1)) \cdots w_{p_m}(x(m))$$

In this talk, we discuss the multiple summability of $T$ when we have information on the summability of the maps it induces on each coordinate, extending important results of Defant, Popa and Schwartz for $(r, 1)$-multiple summability. Our methods have applications to inclusion theorems for multiple summing multilinear mappings and to the product of $p$-Sidon sets.

Andreas Defant
Universität Oldenburg

Bohr’s phenomenon for functions on the Boolean cube

We study the asymptotic decay of the Fourier spectrum of real functions on the Boolean cube $\{-1, 1\}^N$ in the spirit of Bohr’s phenomenon from complex analysis. Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S \subseteq \{1, \ldots, N\}} \hat{f}(S) x^S$, where $x^S = \prod_{k \in S} x_k$. Given a class $\mathcal{F}$ of functions on the Boolean cube $\{-1, 1\}^N$, the Boolean radius of $\mathcal{F}$ is defined to be the largest $\rho \geq 0$ such that
\[ \sum S |\hat{f}(S)| \rho |S| \leq \|f\|_{\infty} \text{ for every } f \in \mathcal{F}. \]

We indicate the precise asymptotic behaviour of the Boolean radius of several natural subclasses, as e.g. the class of all real functions on \([-1,1]^N\), the subclass made of all homogeneous functions or certain threshold functions. Compared with the classical complex situation subtle differences as well as striking parallels occur. An interesting side aspect of our work is the following Bohnenblust-Hille type inequality for real functions on \([-1,1]^N\): There is an absolute constant \(C > 0\) such that the \(\ell_{2d/(d+1)}\)-sum of the Fourier coefficients of every function \(f : [-1,1]^N \to [-1,1]\) of degree \(d\) is bounded by \(C \sqrt{d \log d}\). It was recently proved that a similar result holds for complex-valued polynomials on the \(n\)-dimensional torus \(\mathbb{T}^n\), but that in contrast to this a replacement of the \(n\)-dimensional torus \(\mathbb{T}^n\) by the \(n\)-dimensional cube \([-1,1]^n\) leads to a substantially weaker estimate. Joint work with Mieczysław Mastyło and Antonio Pérez Hernández.

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**Gilles Godefroy**
Université Paris 6

*Recent results of Lipschitz-free spaces*

The purpose of this talk is to survey some recent results on the so-called Lipschitz-free spaces (that is, isometric preduals of spaces of Lipschitz functions on metric spaces) due to several authors, and to recall some open questions on these spaces.

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**Anna Kamińska**
The University of Memphis

*Abstract Lorentz spaces and Köthe duality*

Motivated by the theory and applications of Lorentz spaces we define abstract Lorentz spaces which comprise of classical, weighted and Orlicz-Lorentz spaces. By our procedure we also obtain a whole array of new examples of Lorentz type spaces. We study their properties and in particular their Köthe dual spaces. Given a symmetric space \(E\) and a decreasing positive weight \(w\), the space \(E_w\) is an abstract weighted space and its symmetrization \(\Lambda_{E,w}\) is a generalized Lorentz space. The class of functions \(M_{E,w}\) equipped with the gauge \(\|\cdot\|_{M_{E,w}}\) is defined in the spirit of Marcinkiewicz spaces. This class does not need to be even a linear space, however \(\|f^{\theta}\|_{M_{E,w}}\) is a norm if it is finite, where \(f^{\theta}\) is a level function. We further identify symmetric spaces \(Q_{E,w}\) and \(P_{E,w}\) and we show that they are Köthe dual spaces of Lorentz spaces. Consequently, we prove that the Köthe dual space of \(\Lambda_{E,w}\) is equipped with three different formulas of the norm expressed in terms of Hardy-Littlewood inequalities and/or in terms of level functions. This not only allows us to recover the existing formulas of dual norms in classical and Orlicz-Lorentz spaces, but also to obtain a new one. Substituting for \(E\) various symmetric spaces we obtain a number of examples of generalized Lorentz spaces and their dual spaces. Joint work with Yves Raynaud.
Mieczysław Mastyło
Adam Mickiewicz University in Poznań

18.10.2017
09:30-10:30

Analytic families of multilinear operators

We will discuss interpolation of analytic families of multilinear operators defined on spaces generated by the Calderón method applied to couples of quasi-Banach spaces. We prove a multilinear version of Stein's classical interpolation theorem and we show applications for analytic families of operators taking values in Lorentz and Hardy spaces. These results are applied to prove that the bilinear Bochner-Riesz transform is bounded from $L^p(\mathbb{R}^n) \times L^p(\mathbb{R}^n)$ to $L^{p/2}(\mathbb{R}^n)$ for all $1 < p < 2$. This is a joint work with Loukas Grafakos.

Vassili Nestoridis
National and Kapodistrian University of Athens

17.10.2017
11:30-12:30

A project on generic non-extendability of functions belonging to some new spaces and their properties, in one, several or infinitely many complex variables

First, we remind some generic non-extendability results valid for most of the function spaces in one or several complex variables, as well as, some generic results on totally unbounded functions. These results can be extended to be valid for some new function spaces that we introduce in one, several or infinitely denumerably many variables. These new spaces are defined by several properties of holomorphic functions that we require to hold when we approach several parts of the boundary of their common domain of definition. They are Frechet spaces when they are endowed with their natural topologies. An open issue is to investigate the properties of the functions belonging to these spaces.

Eve Oja
University of Tartu

18.10.2017
11:00-12:00

Lifting of nest approximation properties and related principles of local reflexivity

Recall that a Banach space $X$ has the approximation property if there exists a net of bounded linear finite-rank operators on $X$ converging uniformly on compact subsets of $X$ to the identity operator. Nest approximation properties are defined by the requirement that the finite-rank approximating operators leave invariant all subspaces in a given nest of closed subspaces of $X$.

Nest approximation properties were launched by T. Figiel and W. B. Johnson in [1]. These properties refine the concept of approximation properties for pairs due to
T. Figiel, W. B. Johnson, and A. Pełczyński [2]. The latter concept requires the finite-rank approximating operators to leave invariant a given closed subspace of $X$ and it is, in turn, a refinement of the classical approximation properties. In this lecture, we review some old and new results and open problems related to the lifting of various approximation properties between the dual space $X^*$ and $X$. An emphasis is given to nest approximation properties, where lifting theorems rely on some new forms of the principle of local reflexivity which respect given nests of subspaces [3,4].

References


Kristian Seip
Norwegian University of Science and Technology

*Hardy spaces of Dirichlet series and the Riemann zeta function*

We have in recent years seen a notable growth of interest in certain functional analytic aspects of the theory of ordinary Dirichlet series

$$\sum_{n=1}^{\infty} a_n n^{-s}.$$  

Inspired by the classical theory of Hardy spaces and the operators acting on such spaces, this topic is also intertwined with analytic number theory and function theory on the infinite dimensional torus. Of particular interest are problems that involve an interplay between the additive and multiplicative structure of the integers, in this context embodied respectively by function theory in half-planes and the so-called Bohr lift that transforms Dirichlet series into functions of infinitely many complex variables.

In this survey talk, I will present some recent highlights from the function theory of Hardy spaces of Dirichlet series, outline some aspects of the operator theory that has been developed for these spaces, and present some applications to the Riemann zeta function $\zeta(s)$, including pseudomoments and lower bounds for the growth of $\zeta(1/2 + it)$. 
In this joint work with Florent Baudier and Gilles Lancien a new concentration inequality is proven for Lipschitz maps on the infinite Hamming graphs taking values into Tsirelson's original space. This concentration inequality is then used to disprove the conjecture, originating in the context of the Coarse Novikov Conjecture, that the separable infinite dimensional Hilbert space coarsely embeds into every infinite dimensional Banach space. Some positive embeddability results are proven for the infinite Hamming graphs and the countably branching trees using the theory of spreading models. A purely metric characterization of finite dimensionality is also obtained, as well as a rigidity result pertaining to the spreading model set for Banach spaces coarsely embeddable into Tsirelson's original space.