ARTICLE TEMPLATE

Mathematical Modeling for the development of traffic based on the theory of system dynamics

Juan Manuel Sánchez[a], Adrián Valverde[a], Juan L.G. Guirao[a,‡], Huatao Chen[b]

[a] Department of Applied Mathematics and Statistics, Technical University of Cartagena, Hospital de Marina, Cartagena 30203, Spain
[b] Division of Dynamics and Control, School of Mathematics and Statistics, Shandong University of Technology, ZiBo 255000, China

ARTICLE HISTORY
Compiled July 31, 2023

ABSTRACT
This paper concerns with the mathematical modeling for the development of Shandong traffic. The system dynamics model of development of traffic in Shandong is established. In term of this model, it is shown that highway operation as well as rail transit promotes the development of traffic, while traffic accidents inhibit traffic development. Moreover, the maximum error between the output data and the statistics bureau, based on which some forecasts for the development of traffic in the further are given, is obtained, some suggestions and optimization schemes for traffic development are given. Finally, a neural network model of the development of Shandong traffic is also derived.

KEYWORDS
Traffic development, System dynamics, Neural network model.

1. Introduction

Recently, the traffic situation has been improved, which is reflected by the amount of passenger in highway of 15 billion people in 2018, and the total length of rail lines in China has reached 5761.4 kilometers, therefore, the study of the traffic system by different theoretical is crucial. For example, based on the mathematical modeling, reference [16] show that the emergency vehicle management solution can reduce the travel times of EVs without causing any performance degradation of normal vehicles. Zhu[46] proposed a bayesian updating approach based on the dirichlet model to describe the traffic system performance.By Machine Learning, Saleem [35] proposed a fusion-based intelligent traffic congestion control system to alleviate traffic congestion in smart cities.

The traffic system is a complex dynamic system whose influencing factors is complex and diverse,thus, the mathematical modeling of traffic system is difficult. In order to understand the dynamic behavior of traffic system comprehensively, the system dynamic (SD) method is one of the most powerful tools[15]. The theory of system

‡ Corresponding Author
juanmasanchezparra@gmail.com (JSP); adrian_valverde12@hotmail.com; juan.garcia@upct.es(JLGG); htchencn@sdut.edu.cn(HC)

dynamics founded by Forrester[11] can be used to simplify the multi-variable system, it is widely used to deal with the social and economic problems.As for the application of the theory of system dynamics, one can refer to [40, 41, 7, 27], too name but a few. The basic concepts of system dynamics are as follows

(1) **Level variable** represented by a rectangle in VENSIM software and only be affected by rate variables, which can be expressed by integrating the rate variables.
(2) **Rate variables** are time functions, and it determines the level variable.
(3) For ease of communication and clarity, it is often to define **auxiliary variables** which are neither stock nor flow.
(4) The **constant variable** is a constant, and it can be characterized by table functions.
(5) **Table functions** conveniently represent nonlinear relationships.
(6) **Flow graph** is a characteristic diagram in system dynamics. It simplifies the equations of models. In the sequel, the flow graph can be transformed into VENSIM equations.

The modeling steps for system dynamics in VENSIM software are shown by Fig. 1

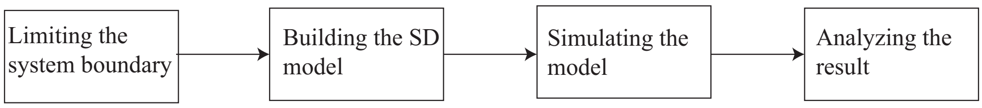


Fig. 1 Modeling steps of system dynamics.

Recently, the research based on system dynamic theory mainly focuses on urban traffic, low-carbon traffic as well as traffic policies. According to the restriction policy, Wen[42] calculated that the implementation of the tail licensing restriction policy could effectively reduce carbon dioxide emissions of 3.86%, moreover, they found that the vehicle license limit policy could effectively curb the growth of car ownership, alleviate traffic congestion. Jia[19] established a SD model of traffic congestion charge and subsidy, furthermore, it can be found that zero-subsidy and low charge reduce carbon dioxide emissions.He[15] concluded that the railway occupation rate was directly proportional to the railway length. Additionally, in order to assess the relation between numbers of available public traffic and traffic congestion in Jakarta, W. Sardjono [39] developed a SD model for traffic conditions, which can reduce traffic congestion. Based on the road accident statistics, Victor[22] proposed the methodology of evaluated long-term trends in the dynamics of traffic safety improvement. Rajput[32] present a system dynamics simulation model to reduce the traffic congestion by implementing an Intelligent Transportation System in metropolitan cities of India.Ye[45] concluded that highway investment had a certain pulling effect on economic growth based on a concrete analysis on the relationship between highway construction and economic growth.
In China, the traffic system mainly includes taxi, road vehicle, private car and rail transit[12]. The interactive communication between them forms a complex dynamic system of traffic. To the knowledge of the author, there still lacks proper mathematical model to study the traffic system including the six subsystem by the SD method.Based on the actual situation of traffic system in Shandong Province, this paper proposes a SD model to describe the relationship among the six subsystem which contains the highway vehicle subsystem, the private car subsystem, the highway mileage subsystem,

the rail transit subsystem and the traffic accident subsystem.Based on the SD model of traffic system,some forecasts for the development of traffic in the further are given,some suggestions and optimization schemes for traffic development are given.And this model is helpful to improve the convenience of traffic service and promote to construct the stable traffic.

2. Main results

2.1. SD model

In highway vehicle subsystem, the number of operating vehicles is affected by highway mileage and highway density[10]. Generally speaking,highway has positive effects on the number of vehicles. In addition, the scrap of private cars can force people to take public traffic, which requests the increasing of investment of the public traffic[4]. Thus, the above factors are defined as the influencing factors of the number of operating vehicles.

In taxi subsystem, taxi growth and taxi scrap are considered as two main factors which lead to the overall change of taxi.

As for the private car subsystem, it is used in traffic frequently, but it brings a lot of exhaust pollution[18].

In highway mileage subsystem, with the increasing of all kinds of vehicles, highway construction has been accelerated. On the contrary, old roads reduce highway mileage[10]. Highway mileage is an important embodiment of traffic development. In order to indicate the situation of highway mileage, we represent it by old roads and the number of vehicles.

In rail transit subsystem, with the increasing of damage of the track, the length of rail transit decreased.Track is influenced by the using of various vehicles and track investment[14, 43].Then, the number of private cars and track damage are chosen as the influencing factors of rail transit length.

In traffic accident subsystem, the national economy can be affected by traffic accidents, a suitable traffic condition can reduce the number of occurrence of traffic accidents effectively. Public traffic can affect the number of traffic accidents directly, which changes the choice of travel modes. Hence, we can get the influencing factors for the number of operating vehicles.

In term of the fact indicated above, the following parameters are selected as Level variables: traffic accidents, number of road vehicle, highway mileage, number of taxis, number of private cars owned and length of rail transit line. For the sake of convenience, in the following context, all the elements are summarized in Table 1. In addition,the scenario diagram of the SD model is displayed in Fig. 2.

According to the scenario diagram of traffic shown in Fig. 2, each subsystem is associated with one state variable, such as T_1, T_9, T_{15}, T_{20}, T_{26}, T_{33} and so on.All the subsystems and their influence parameters can be found in Table 1. In order to evaluate the situation of traffic, SD model of traffic system in Shandong Province was established by VENSIM software, the feedback process of flow diagram is marked by blue arrow in Fig.3, furthermore, it can be obtained that the influence between the parameters is multiplex and the influences between the six subsystems are mutual.

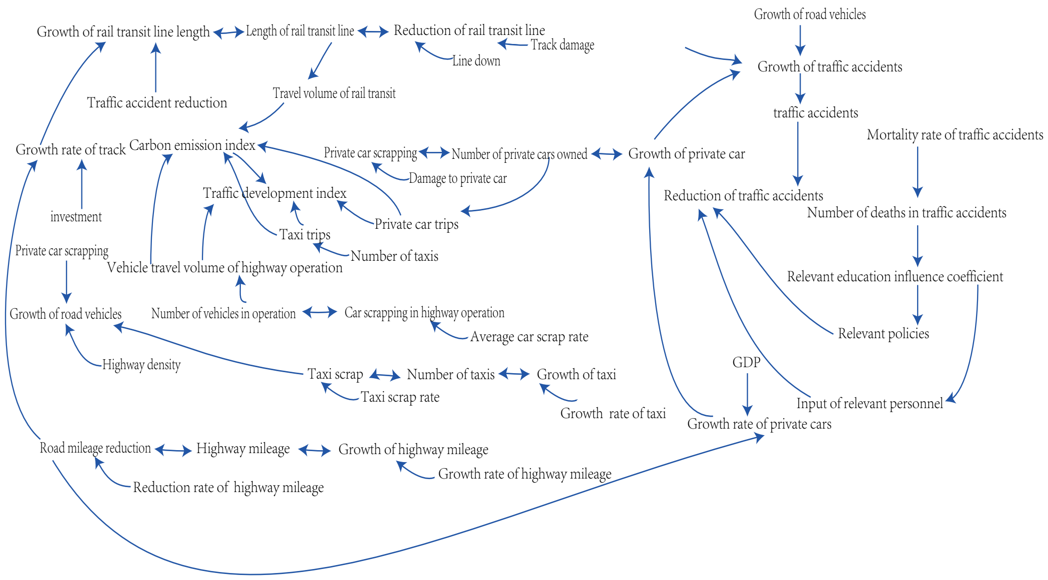


Fig. 2 Scenario diagram of SD model.

2.2. Mathematical model

In light of the flow diagram (see Fig. 3), the SD model can be established by translating the SD flow diagram into Vensim equations, thus, the level variable can be expressed as the following integral equation[40]

$$Level(t) = \int_{t_0}^{t} Rate_{in}(s) - Rate_{out}(s)ds + Level(t_0), \tag{1}$$

in which, $Level(t)$ represents level variables, $Rate_{in}$ and $Rate_{in}$ signifies inflow rate variables and outflow rate variables respectively. Thus, we have

$$\begin{cases} T_1(t) = \int_{t_0}^{t} T_2(s) - T_5(s)ds, \\ T_9(t) = \int_{t_0}^{t} T_{12}(s) - T_{13}(s)ds, \\ T_{15}(t) = \int_{t_0}^{t} T_{16}(s) - T_{18}(s)ds + 25.96, \\ T_{20}(t) = \int_{t_0}^{t} T_{22}(s) - T_{24}(s)ds + 60.119, \\ T_{26}(t) = \int_{t_0}^{t} T_{29}(s) - T_{32}(s)ds, \\ T_{33}(t) = \int_{t_0}^{t} T_{36}(s) - T_{37}(s)ds + 0.5, \end{cases} \tag{2}$$

where s is time at any time between the initial time t_o to the current time t, the unit of time assumed here is Year. On the other hand, we find

$$\frac{dLevel(t)}{dt} = Rate_{in}(t) - Rate_{out}(t),$$

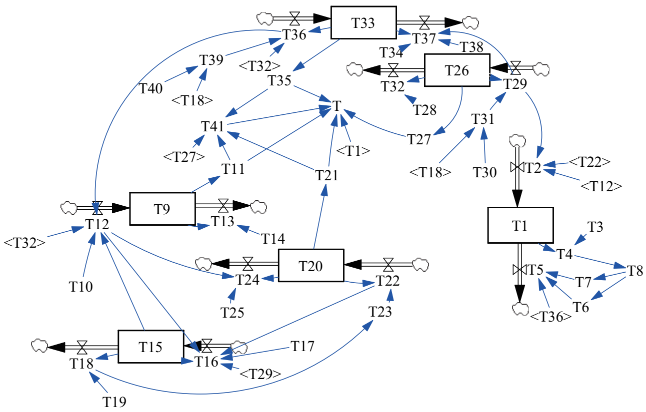


Fig. 3 Flow diagram of the SD model.

hence

$$\begin{cases} \frac{dT_1(t)}{dx} = T_2(t) - T_5(t), \\ \frac{dT_9(t)}{dx} = T_{12}(t) - T_{13}(t), \\ \frac{dT_{15}(t)}{dx} = T_{16}(t) - T_{18}(t), \\ \frac{dT_{20}(t)}{dx} = T_{22}(t) - T_{24}(t), \\ \frac{dT_{26}(t)}{dx} = T_{29}(t) - T_{32}(t), \\ \frac{dT_{33}(t)}{dx} = T_{36}(t) - T_{37}(t). \end{cases} \tag{3}$$

Following $Increment = Growth\ rate \times Original\ data$, we have

$$\begin{cases} T_4(t) = T_1(t) \times T_3, \\ T_{13}(t) = T_9(t) \times T_{14}, \\ T_{18}(t) = T_{15}(t) \times T_{19}, \\ T_{22}(t) = T_{20}(t) \times T_{23}(t), \\ T_{29}(t) = T_{26}(t) \times T_{31}(t). \end{cases} \tag{4}$$

In traffic accident subsystem, the mortality rate is affected by travel modes, highway vehicles and taxis are often selected as the analysis objects[37, 20], rail transit can decrease the accident rate[24]. In light of the data on the traffic accidents, we can find that the increase number of car increase the number of traffic accidents, and the implementation of rail transit as well as positive policy can reduce the number of accidents, in addition, education on the safety awareness for people can also reduce the number of accidents. With respect to the highway subsystem, the number of cars is proportional to the number of road and the mileage of highway[17]. Investment on the traffic can increase the mileage of highway[45]. In taxi subsystem, the expenditure on taking taxi is higher than using of public traffic, but it is necessary to increase taxis reasonably, and the use of taxis is accompanied by the scrapping of taxis [31]. As for the private car subsystem, private cars are contradictory to public traffic, and they are positively related to highway mileage[4, 10]. In rail transit subsystem, the increase of the length of rail transit results in increasing of the investment of rail[14]. On the other hand, there exists the conflict between rail transit and private car[43]. The state of GDP is an important index of national development and it can reflect the government investment on traffic infrastructure construction. As indicated above, we obtain the following expressions on the rest of variables,

$$\begin{cases} T_2(t) = 0.316T_1(t) + 0.126T_{12}(t) + 0.512T_{22}(t), \\ T_5(t) = T_6(t) + 0.223T_7(t) + 0.125T_{36}(t), \\ T_7(t) = 0.656T_8(t), \\ T_8(t) = 5.89T_4(t), \\ T_{12}(t) = 0.316T_{10} + 0.233T_{15}(t) + 0.14T_{36}(t), \\ T_{16}(t) = T_{40} \times T_{15}(t), \\ T_{23}(t) = 0.205T_{25} - 0.065, \\ T_{24}(t) = T_{25} \times T_{20}(t) + 0.19T_{12}, \\ T_{31}(t) = 0.344T_{18}(t) + 0.756T_{28}, \\ T_{32}(t) = 0.625T_{22}(t) + 0.365T_{28}, \\ T_{36}(t) = 0.872T_{39} \times T_{33} + 0.128T_{32}, \\ T_{37}(t) = 0.233T_{29}(t) + 0.569T_{38}, \\ T_{39}(t) = 0.456T_{18}(t) + 0.544T_{40}. \end{cases} \tag{5}$$

2.3. Optimization of SD model

This subsection is devoted to optimize the SD model (see Fig. 4 and Fig.5). By com-

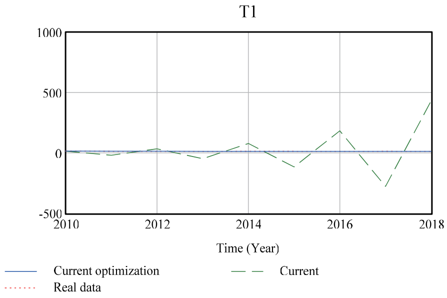
Maximum payoff found at:
T3 = 0.0966794
T10 = 80
T14 = 0.291712
T17 = 1.74955
T19 = -0.010023
T25 = -0.135105
T28 = 0.0669325
T30 = 2.49007
T34 = 0.977206
T38 = 0.233964
T4O = 41.878

Fig. 4 Optimization results of the SD model

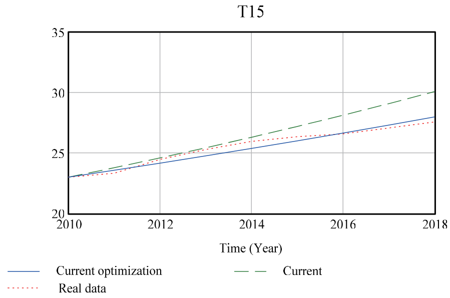
paring the output data with real data, four factors were selected in the model to judge the relative error of SD model.The data of the National Bureau of Statistics and output date are expressed by R and C respectively, O denotes the output data of optimized model, and the relative error of optimized model is signified by Oe. The following expressions to calculating Oe of four curves is defined as

$$Oe = \frac{|R - O|}{|R|}. \tag{6}$$

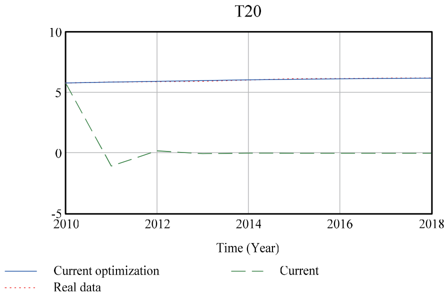
$T_3, T_{10}, T_{14}, T_{17}, T_{19}, T_{25}, T_{28}, T_{30}, T_{34}, T_{38}$ and T_{40} are selected as optimize parameters to make examination. There are four optimization models related to the eleven objective see Fig.5a, Fig.5b, Fig.5c and Fig.5d. It is shown that the optimized curves O 1:4 are close to the real curves R 1:4. It can be seen that the O curve are closer to the the R curve. Table 2 shows that the relative error of the output data is less than the limit of error of 10% in system dynamics, which means that precision of the model is enough.



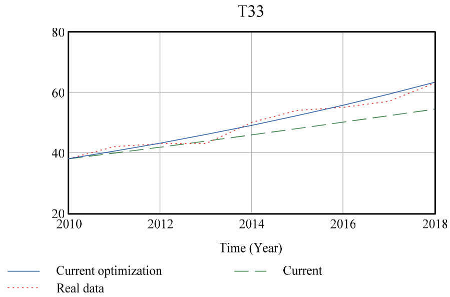
(a) Optimization of T_1.



(b) Optimization of T_{15}.



(c) Optimization of T_{20}.



(d) Optimization of T_{33}.

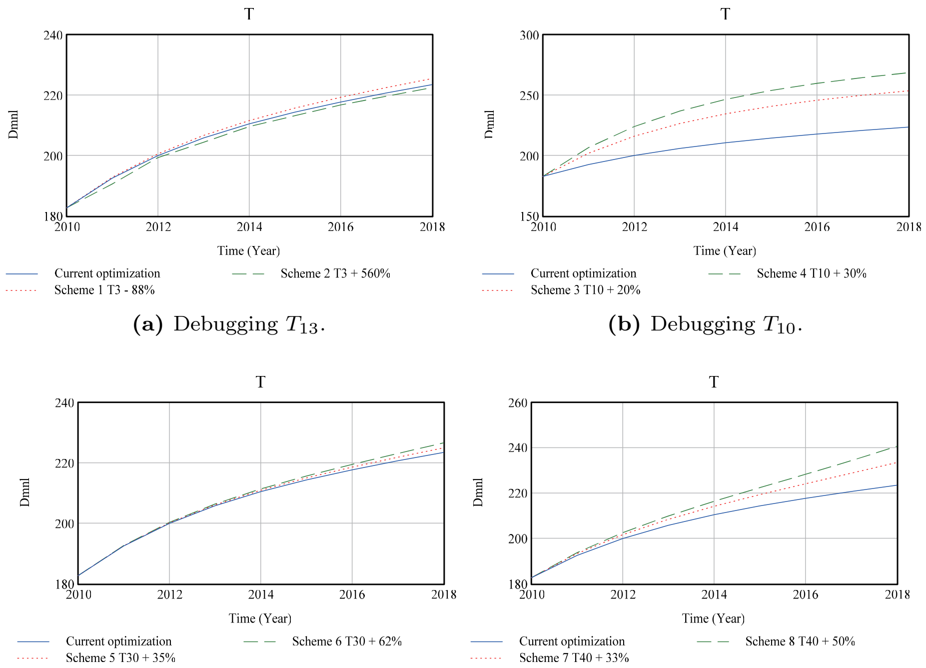
Fig. 5 Optimization of SD model.

2.4. Parameter analysis

Based on the SD model, ten debugging models are established, in which T is taken as the objective function, and T_3, T_{10}, T_{30} and T_{40} are taken as the design variables. By using these debugging models, the optimal design of traffic can be obtained to provide better traffic control strategy. Based on the results of debugging, the effects of different debugging models on the traffic are evaluated.

From the Fig. 6 which shows the current optimization curve of T, we have

(1) The variation of T associated with the traffic accident subsystem is the maximal one in the scheme 1 (see Fig. 6a), which means that the government should take measures to reduce the number of traffic accidents.
(2) Although there exist relationships between T and all parameters of the global system, the road density is more important than others, therefore, by adjusting road density, we can control the process of development of traffic effectively. As length of highway can increase T, the government should improve the quality and length of highway, moreover, high utilization factor of highway must be kept.
(3) Fig.6 shows comparison between current optimization curves and the schemes proposed. By Fig.6c and Table 3, we assert that T_{30} exerts dominant influence on the increasing of T, the best way to promote development of traffic is to improve the quality and length of highway and rail transit[8].
(4) Fig. 7 indicates that the relationship between debuging two parameters and debuging one parameter is not linear, furthermore, the effect of two parameters is better than one parameter.



(a) Debugging T_{13}.

(b) Debugging T_{10}.

(c) Debugging T_{30}.

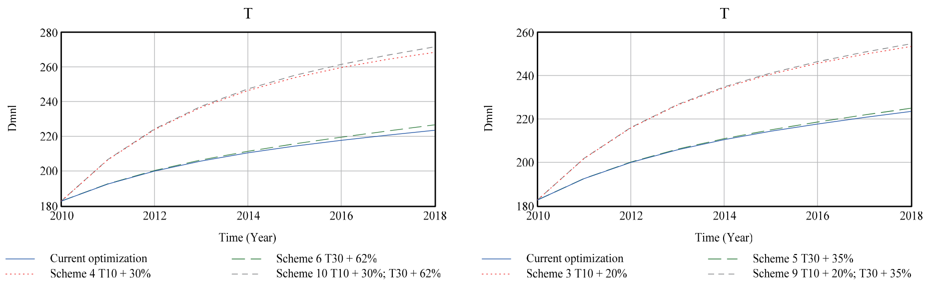
(d) Debugging T_{40}.

Fig. 6 Debugging results of one parameter.

2.5. Data estimation

Table 3 shows that relative error of T_1, T_9, T_{15}, T_{20} and T_{33} are under 6.83%, thus, the model has high accuracy. In light of this SD model, Table 4 displays the next ten years developments of traffic in Shandong Province, it is found that the following trends of traffic in the future,

(1) The number of traffic accidents is increasing every year, which may be related to the growth of private cars and taxis. The growth of private cars and taxis is inevitable, therefore, the government must take active measures to reduce the number of traffic accidents.
(2) Due to the needs of the citizen and environmental protection, the number of operating vehicles will always keep rising. The increase of operating vehicles means that the number of buses on remote roads and the diversification of routes are improved, which is convenient for citizens.
(3) The larger number of all kinds of vehicles leads to the increase of loading of road, which will brings more traffic accidents. Obviously, the length of highway mileage should be enlarged.
(4) Although the number of taxis will continue to rise in the next ten years, its growth is slower than other means of traffic. Taxis and buses are convenient for people to take, but the taxi results in more serious pollution than bus, furthermore, unlike the bus, the taxi is not safety. Therefore, the number of taxis will decrease in the future.
(5) The number of private cars will grow rapidly in the future. The private cars not only bring convenience to people, but also bring great burden to traffic. The citizens should appropriately reduce the use of private cars and take public



(a) Debugging T_{10} and T_{35}. **(b)** Debugging T_{10} and T_{35}.

Fig. 7 Debugging results of two parameters.

traffic.

(6) The rail transit's length will maintain rapid growth in the next decades, which means that government pay more attention to the development of rail transit, the rail transit can greatly reduce traffic accidents and environmental pollution.

2.6. The neural network model

Based on the SD model, the following neural network model of traffic problem considered can be got

$$\begin{cases} \frac{d(T_{33}(t))}{dt} = a_1T_{33}(t) + b_{11}T_{33}(t) + b_{13}T_{15}(t)T_{33}(t) + b_4T_{26}^2(t) + c_1, \\ \frac{T_{15}(t)}{dt} = -a_2T_{15}(t) + b_{21}T_{15}(t) + b_{23}T_{26}(t) + b_{24}T_{15}(t) + b_{25}T_{33}(t)T_{15}(t) + \\ \qquad b_{26}T_{26}(t)T_{15}(t) + b_{27}T_{20}T_{15}(t) + c_2, \\ \frac{d(T_{26}(t))}{dt} = -a_3T_{26}(t) + b_{31}T_{26}(t) + b_{32}T_{15}(t)T_{26}(t), \\ \frac{d(T_{20}(t))}{dt} = -a_4T_{20}(t) + b_{41}T_{33}(t) + b_{42}T_{33}(t) + b_{43}T_{26}(t) + b_{44}T_{33}(t)T_{15}(t) + \\ \qquad b_{45}T_{20}(t)T_{15}(t)) + c_4, \\ \frac{d(T_9(t))}{dt} = -a_5T_9(t) + b_{51}T_{33}(t) + b_{52}T_{15}(t) + b_{53}T_{26}(t) + b_{54}T_{33}(t)T_{15}(t) + c_5, \\ \frac{d(T_1(t))}{dt} = -a_6T1_(t) + b_{62}T_{15}(t) + b_{63}T_{26}(t) + b_{64}T_{20}(t) + b_{65}T_{33}(t)T_{15}(t) + \\ \qquad b_{66}T_{26}(t)T_{15}(t) + b_{67}x_4(t)T_{15}(t) + c_6, \\ T(t) = a_{11}T_{33}(t) + a_{12}T_{15}(t) + a_{13}T_{26}(t) + a_{14}T_{20}(t) + a_{15}T_9(t) + a_{16}T_1(t), \end{cases} \tag{7}$$

where the meaning of coefficient of (7) can be found in App (10). The five neurons are represented by T_3, T_{14}, T_{19}, T_{25} and T_{28}.The capacitance of each neuron is fixed to be 1.$\gamma_i = \frac{1}{T_i}$($i = 3, 14, 19, 25, 28$) are the resistances of per neuron[1, 3, 36]. T_0 is the initial value, I signifies the expected value, S_i are the years to achieve I of each neuron, and T_s is T_0 after s years, then the neural network model is

$$T_s = T_0 + (I - T_0) \times (1 - \exp\frac{-S_i}{\gamma_i}),$$

further

$$S_i = \gamma_i \times \ln\frac{I - T_0}{I - T_s}, \tag{8}$$

which provides an algorithm to calculate I(see Example 2.1)

Example 2.1. T in 2015 and 2018 are noted as $T_0 = 214.74$ and $T_s = 224.73$ respectively, I is set to be 300, then

$$S_i = \gamma_i \times \ln \frac{I - T_0}{I - T_s} = \gamma_i \times \ln \frac{85.27}{75.29}, \tag{9}$$

where

$$\gamma_3 = 10.34,\ \gamma_{14} = 3.43,\ \gamma_{19} = 99.80,\ \gamma_{25} = 7.40,\ \gamma_{28} = 14.94,$$

so we can get

$$S_3 = 1.29,\ S_{14} = 0.43,\ S_{19} = 12.42,\ S_{25} = 0.92,\ S_{28} = 1.86.$$

Model 9 shows that the length of time to achieve expected value of each subsystem are $1.29, 0.43, 12.42, 0.17, 0.92$, and 1.86 years respectively, obviously, the maximum is 12.42 years which is the length of time to arrive the expected value. The SD and neural network models are advantageous to improve the speed and efficiency of the development of traffic system.

3. Conclusion

This paper consider the development of traffic system of Shandong Province, based on the system dynamics method, the SD model of traffic system related the six subsystem which contains the highway vehicle subsystem, the private car subsystem, the highway mileage subsystem,the rail transit subsystem and the traffic accident subsystem, in light of which prediction on the further situation of traffic system of Shandong Province, is derived. Furthermore, the neural network model of the problem considered in paper is also given. In term of SD model and neural network model, we can give some policies to force the traffic systems to develop to the situation we want. For example, the government need to developing rail transit and take some measures to reduce the number of traffic accidents.The model is helpful to improve the convenience of traffic service and promote to construct the stable traffic. Furthermore, based on the neural network model, we can investigate the dynamics of the traffic system, we will pursue this line in the further.

Acknowledgement

This work is supported by Natural Science Foundation of Shandong Province (No. ZR2020MA054) and Research Foundation for Talented Scholars of SDUT (No.4041/419023)

References

[1] Syed Ahmed Aamir, Paul Müller, Andreas Hartel, Johannes Schemmel and Karlheinz Meier, A highly tunable 65-nm CMOS LIF neuron for a large scale neuromorphic system, in: *ESSCIRC Conference 2016: 42nd European Solid-State Circuits Conference*, IEEE, pp. 71–74, 2016.

[2] Shanthi Ameratunga, Martha Hijar and Robyn Norton, Road-traffic injuries: confronting disparities to address a global-health problem, *The Lancet* **367** (2006), 1533–1540.
[3] David Bau, Jun-Yan Zhu, Hendrik Strobelt, Agata Lapedriza, Bolei Zhou and Antonio Torralba, Understanding the role of individual units in a deep neural network, *Proceedings of the National Academy of Sciences* (2020).
[4] Robert Cervero and Yuhsin Tsai, City CarShare in San Francisco, California: second-year travel demand and car ownership impacts, *Transportation Research Record* **1887** (2004), 117–127.
[5] Huseyin Ceylan, Ozgur Baskan and Cenk Ozan, Modeling and forecasting car ownership based on socio-economic and demographic indicators in Turkey, *TeMA Journal of Land Use, Mobility and Environment* (2018), 47–66.
[6] Liu Chengliang, Yan Qi and Luo Jing, System dynamics simulation on the coupling of economy resources environment system in Wuhan Metropolitan Region, **32** (2013), 857–869.
[7] Robert G Coyle, System dynamics modelling: a practical approach, *Journal of the Operational Research Society* **48** (1997), 544–544.
[8] Tadeusz Dyr, Przemysław Misiurski and Karolina Ziółkowska, Costs and benefits of using buses fuelled by natural gas in public transport, *Journal of Cleaner Production* **225** (2019), 1134–1146.
[9] Randall W Eberts et al., Public infrastructure and regional economic development, *Economic Review* **26** (1990), 15–27.
[10] Lu Fan and Aihu Wang, CO 2 emissions and technical efficiency of logistics sector: An empirical research from China, in: *Proceedings of 2013 IEEE International Conference on Service Operations and Logistics, and Informatics*, IEEE, pp. 89–94, 2013.
[11] Jay Wright Forrester, Industrial dynamics, *Journal of the Operational Research Society* **48** (1997), 1037–1041.
[12] Juan Martín García, *Theory and Practical Exercises of System Dynamics: Modeling and Simulation with Vensim PLE. Preface John Sterman*, Juan Martin Garcia, 2020.
[13] Yang Miang Goh and Peter ED Love, Methodological application of system dynamics for evaluating traffic safety policy, *Safety science* **50** (2012), 1594–1605.
[14] Qian-Wen Guo, Shumin Chen, Paul Schonfeld and Zhongfei Li, How time-inconsistent preferences affect investment timing for rail transit, *Transportation Research Part B: Methodological* **118** (2018), 172–192.
[15] Shou-kui He and Jing Li, A Study of Urban City Traffic Congestion Governance Effectiveness Based on System Dynamics Simulation, *Int. Ref. J. Eng. Sci.* **8** (2019), 37–47.
[16] Mamoona Humayun, Maram Fahhad Almufareh and Noor Zaman Jhanjhi, Autonomous traffic system for emergency vehicles, *Electronics* **11** (2022), 510.
[17] Gregory K Ingram and Zhi Liu, *Determinants of motorization and road provision*, The World Bank, 1999.
[18] Shuwei Jia, Lizhu Bi, Wanminghao Zhu and Tianhui Fang, System dynamics modeling for improving the policy effect of traffic energy consumption and CO2 emissions, *Sustainable Cities and Society* **90** (2023), 104398.
[19] Shuwei Jia, Guangle Yan, Aizhong Shen and Jun Zheng, A system dynamics model for determining the traffic congestion charges and subsidies, *Arabian Journal for Science and Engineering* **42** (2017), 5291–5304.
[20] Annisa Jusuf, Ignatius Pulung Nurprasetio and Arya Prihutama, Macro data analysis of traffic accidents in Indonesia, *Journal of Engineering and Technological Sciences* **49** (2017), 132–143.
[21] Dean C Karnopp, Donald L Margolis and Ronald C Rosenberg, System dynamics, *Modeling and simulation of mechatronic systems* **4** (1990).
[22] Victor Kolesov and Artur Petrov, System dynamics of process organization in the sphere of traffic safety assurance, *Transportation research procedia* **36** (2018), 286–294.
[23] Reinhart Kühne, Electric buses–An energy efficient urban transportation means, *Energy* **35** (2010), 4510–4513.
[24] Wenxing Li and Shuai Yin, Analysis on cost of urban rail transit, *Journal of Transportation Systems Engineering and Information Technology* **12** (2012).
[25] Yanmei Li and Shuangshuang Lu, Study on the optimization of urban passenger traffic structure based on multi-objective linear programminga case study of Beijing, *Environmental Science and Pollution Research* (2020), 1–15.

[26] Lin Lin, Xiaohua Liu, Tao Zhang and Xiaochen Liu, Energy consumption index and evaluation method of public traffic buildings in China, *Sustainable Cities and Society* (2020), 102132.

[27] Armin Monirabbasi, Ali RamezaniKhansari and Leila Majidi, Simulation of Delay Factors in Sewage Projects with the Dynamic System Approach, *Industrial Engineering and Strategic Management* **1**, 15–30.

[28] Katsuhiko Ogata, *System dynamics*, 13, Pearson/Prentice Hall Upper Saddle River, NJ, 2004.

[29] Xiuqin Pan, Wangsheng Zhou, Yong Lu and Na Sun, Prediction of network traffic of smart cities based on DE-BP neural network, *IEEE Access* **7** (2019), 55807–55816.

[30] Huanhuan Qin, Amy Cha-tien Sun, Jie Liu and Chunmiao Zheng, System dynamics analysis of water supply and demand in the North China Plain, *Water Policy* **14** (2012), 214–231.

[31] Huijie Qu and Zhi Zhao, The Application of Lagrange Relaxation on Taxi Dispatchments During Evening Rush Hours, in: *Journal of Physics: Conference Series*, 1650, IOP Publishing, p. 032019, 2020.

[32] Aditi Rajput and Madhuri Jain, System Dynamics Simulation Model to Reduce the Traffic Congestion of Metropolitan Cities of India by Implementing Intelligent Transportation System, in: *International Conference on Mathematical Sciences and Statistics 2022 (ICMSS 2022)*, Atlantis Press, pp. 440–455, 2022.

[33] Gerald W Recktenwald, An Introduction to Numerical Methods with MATLAB, (2000).

[34] Homayoun Sadeghi-Bazargani, Sakineh Sharifian, Davoud Khorasani-Zavareh, Raana Zakeri, Mehdi Sadigh, Mina Golestani, Mousa Amiri, Reza Masoudifar, Farzad Rahmani, Nasser Mikaeeli et al., Road safety data collection systems in Iran: a comparison based on relevant organizations, *Chinese journal of traumatology* **23** (2020), 265–270.

[35] Muhammad Saleem, Sagheer Abbas, Taher M Ghazal, Muhammad Adnan Khan, Nizar Sahawneh and Munir Ahmad, Smart cities: Fusion-based intelligent traffic congestion control system for vehicular networks using machine learning techniques, *Egyptian Informatics Journal* **23** (2022), 417–426.

[36] Subhrajit Samanta, Sundaram Suresh, J Senthilnath and N Sundararajan, A new neuro-fuzzy inference system with dynamic neurons (nfis-dn) for system identification and time series forecasting, *Applied Soft Computing* **82** (2019), 105567.

[37] Sigit Puji Santosa, Andi Isra Mahyuddin and Febrianto Guntur Sunoto, Anatomy of injury severity and fatality in Indonesian traffic accidents, *Journal of Engineering and Technological Sciences* **49** (2017), 412–422.

[38] Hasimah Sapiri, Jafri Zulkepli Hew, Norazura Ahmad, Norhaslinda Zainal Abidin and Nurul Nazihah Hawari, *Introduction to system dynamic modelling and vensim software*, UUM Press, 2017.

[39] W Sardjono, E Selviyanti and WG Perdana, Modeling the relationship between public transportation and traffic conditions in urban areas: a system dynamics approach, *JPhCS* **1465** (2020), 012023.

[40] John Sterman, System Dynamics: systems thinking and modeling for a complex world, (2002).

[41] Janez Usenik and T Turnšek, Modeling conflict dynamics with fuzzy logic inference, *Journal of US-China Public Administration* **10** (2013), 457–474.

[42] Lei Wen and Lu Bai, System dynamics modeling and policy simulation for urban traffic: a case study in Beijing, *Environmental Modeling & Assessment* **22** (2017), 363–378.

[43] Na Wu, Shengchuan Zhao and Qi Zhang, A study on the determinants of private car ownership in China: Findings from the panel data, *Transportation Research Part A: Policy and Practice* **85** (2016), 186–195.

[44] Wenyi Xia and Anming Zhang, Air and high-speed rail transport integration on profits and welfare: Effects of air-rail connecting time, *Journal of Air Transport Management* **65** (2017), 181–190.

[45] NJ Ye, WJ Li, Y Li and YF Bai, Spatial Econometric Research on the Relationship between Highway Construction and Regional Economic Growth in China: Evidence from the Nationwide Panel Data, in: *IOP Conference Series: Earth and Environmental Science*, 100, IOP Publishing, p. 012138, 2017.

[46] Zheng Zhu, Shanjiang Zhu, Zhengfei Zheng and Hai Yang, A generalized Bayesian traffic model, *Transportation research part C: emerging technologies* **108** (2019), 182–206.

4. Appendix

$$
\begin{aligned}
&a_1 = 0.125T_{34},\\
&a_2 = T_{19},\\
&a_3 = 0.135T_{30},\\
&a_4 = 3.1550e - 05T_{30} + T_{25} - 0.245,\\
&a_5 = T_{14},\\
&a_6 = 0.223a_2 + 4.031T_3,\\
&b_{11} = 0.0047T_{40},\\
&b_{12} = 0.0040T_{19},\\
&b_{13} = -0.00801T_{19},\\
&b_{14} = -0.00561T_{30},\\
&b_{21} = 7.0176e - 06T_{40},\\
&b_{22} = 0.010.751T_{17} + 0.233,\\
&b_{23} = 9.1428e - 06T_{19} \times T_{30} + 9.3700e - 05T_{30},\\
&b_{24} = -9.6500e - 05,\\
&b_{25} = 0.000129,\\
&b_{26} = 7.3600e - 05T_{19},\\
&b_{27} = 3.0500e - 04a_2,\\
&b_{31} = 0.075T_{30}, b_{32} = 0.0344T_{19},\\
&b_{41} = -1.3375e - 05T_{40},\\
&b_{42} = -4.4720e - 04,\\
&b_{43} = -0.1801,\\
&b_{44} = -1.1172e - 05T_{19},\\
&b_{45} = 0.205a_2,\\
&b_{51} = 7.0176e - 04T_{40},\\
&b_{52} = 0.233,\ b_{52} = 0.0233,\\
&b_{53} = 0.0166T_{30},\\
&b_{54} = 5.9098e - 04T_{19},\\
&b_{25} = 0.205a_2,\\
&b_{51} = 0.0070T_{40},\\
&b_{53} = 0.016644T_{30},\\
&b_{54} = 5.8824e - 04T_{19},\\
&b_{61} = -5.2550e - 04T_{40},\\
&b_{62} = 0.02935,\ b_{64} = -0.0331,\\
&b_{63} = -0.0259T_{30},\\
&b_{65} = -4.4050e - 04T_{19},\\
&b_{66} = 0.0108a_2, b_{67} = 0.104a_2,\\
&a_{11} = 0.0703,\ a_{12} = 0.298,\\
&a_{13} = 1.765,\ a_{14} = 0.226,\\
&a_{15} = 0.096,\ a_{16} = 0.011,\\
&c_1 = 4.6720e - 04T_{28} - 0.569T_{38},\\
&c_2 = 0.316T_{10} + 1.3128e - 06T_{28}\\
&c_4 = -6.0040e - 04T_{10} + 1.3128e - 06T_{28}\\
&c_5 = 0.316T_{10} + 6.9146e - 05T_{28},\\
&c_6 = -5.8400e - 05T_{28} + 0.0398T_{10} + 8.6899e - 06T_{28},\\
&a = 0.456T_{19},\ \bar{a} = T_{19},\ b = 0.544T_{40}, d = 0.04672T_{28},\ \bar{d} = T_{30},\\
&f = 0.751T_{17},\ k = 5.8T_3, j = 0.569T_{38},\ h = 0.316T_{10},\ r = T_{25},\ q = T_{14}.
\end{aligned}
\tag{10}
$$

table 1 All parameters in the SD model

Subsystem	Parameter	Initial value	Description
	T		Traffic development index
	T_1	14.56 thousand times	Traffic accident
	T_2		Growth of traffic accident
	T_3		Mortality rate of traffic accidents
Traffic accident	T_4		Number of deaths in traffic accident
	T_5		Reduction of traffic accident
	T_6		Input of relevant personnel
	T_7		Relevant policy
	T_8		Education influence coefficient
	T_9	913.8 thousand units	Number of vehicles in operation
	T_{10}		Road density
Highway vehicle	T_{11}		Travel volume of highway operation
	T_{12}		Growth of road vehicles
	T_{13}		Car scrapping in highway operation
	T_{14}		Average car scrap rate
	T_{15}	229.9 thousand km	Highway mileage
	T_{16}		Growth of highway mileage
Highway mileage	T_{17}		Growth rate of highway mileage
	T_{18}		Reduction of highway mileage
	T_{19}		Reduction rate of highway mileage
	T_{20}	57.7 thousand units	Number of taxi
	T_{21}		Taxi trips
Taxis	T_{22}		Growth of taxi
	T_{23}		Growth rate of taxi
	T_{24}		Taxi scrap
	T_{25}		Taxi scrap rate
	T_{26}	5.77 million units	Number of private car
	T_{27}		Private car trip
	T_{28}		Damage of private car
Private car	T_{29}		Growth of private car
	T_{30}		GDP
	T_{31}		Growth rate of private cars
	T_{32}		Private car scrapping
	T_{33}	38 million km	Length of rail transit
	T_{34}		Line down
	T_{35}		Travel volume of rail transit
Rail transit	T_{36}		Growth of rail transit line length
	T_{37}		Reduction of rail transit line
	T_{38}		Track damage
	T_{39}		Growth rate of track
	T_{40}		Investment

table 2 The relative error of SD model.

Year	2013	2014	2015	2016	2017	2018
C T_1	-48.05	77.94	-115.93	182.81	-277.03	431.36
C T_{15}	25.42	26.29	27.18	28.11	29.07	30.06
C T_{20}	-0.08	-0.04	-0.04	-0.04	-0.04	-0.05
C T_{33}	43.82	45.88	47.98	50.10	52.25	54.39
R T_1	12.88	13.57	13.38	13.16	13.40	13.23
R T_{15}	25.28	25.95	26.34	26.57	27.06	27.56
R T_{20}	5.91	6.01	6.12	6.13	6.17	6.17
R T_{33}	43	50	54	55	57	63
O T_1	13.39	13.21	13.12	13.10	13.15	13.27
O T_{15}	24.75	25.36	25.99	26.64	27.30	27.98
O T_{20}	5.96	6.024	6.071	6.11	6.14	6.16
O T_{33}	45.98	48.88	51.98	55.24	58.66	62.23
Oe T_1	3.97%	2.62%	1.89%	0.46%	1.87%	0.36%
Oe T_{15}	2.09%	2.25%	1.3%	0.28%	0.91%	1.55%
Oe T_{20}	1.04%	0.02%	0.84%	0.32%	0.39%	0.07%
Oe T_{33}	6.83%	2.23%	3.73%	0.45%	2.92%	1.22%

* The date of this table comes from China's National Bureau of Statistics.

table 3 Growth Rate of Debugged Models

	Index of 2018	Growth rate
Scheme 1 $T_3 - 88\%$	226.36	0.73%
Scheme 2 $T_3 + 560\%$	223.67	-0.46%
Scheme 3 $T_{10} + 20\%$	254.68	13.34%
Scheme 4 $T_{10} + 30\%$	269.67	20.01%
Scheme 5 $T_{30} + 35\%$	243.78	8.49%
Scheme 6 $T_{30} + 62\%$	229.28	2.03%
Scheme 7 $T_{40} + 33\%$	234.68	4.44%
Scheme 8 $T_{40} + 50\%$	241.82	7.61%
Scheme 9 $T_{10} + 20\%, T_{30} + 35\%$	258.25	14.91%
Scheme 10 $T_{10} + 30\%, T_{30} + 62\%$	276.45	23.02%

table 4 Data estimation based on SD model

Year	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
T_1	13.46	13.74	14.11	14.59	15.18	15.92	16.83	17.93	19.25	20.85
T_9	1126	1136	1147	1157	1167	1178	1188	119.9	1210	1221
T_{15}	61.8	61.8	61.8	61.7	61.4	61.1	60.68	60.1	59.4	58.6
T_{20}	28.68	29.39	30.12	30.88	31.65	32.44	33.25	34.09	34.95	35.83
T_{26}	237.9	278.2	325.2	380.1	444.2	518.9	606.1	707.8	826.4	964.5
T_{33}	65.94	69.79	73.76	77.84	81.99	86.19	90.40	94.57	98.64	102.52