

Highlights of the Chiral Unitary Approach

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- **Introduction: The Chiral Unitary Approach**
- **Non perturbative effects in Chiral EFT**
- **Formalism**
- **Meson-Baryon**
- **Nucleon-Nucleon**
- **Meson-Meson, Nuclear environment**
- **Summary**

The Chiral Unitary Approach

1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies).
 - Meson-meson processes, both scattering and (photo)production, involving $I=0,1,1/2$ S-waves, $J^{PC}=0^{++}$ (vacuum quantum numbers)
 $I=0$ $\sigma(500)$ - really low energies
Not low energies - $I=0$ $f_0(980)$, $I=1$ $a_0(980)$, $I=1/2$ $\kappa(700)$.
Related by SU(3) symmetry.
 - Processes involving $S=-1$ (strangeness) S-waves meson-baryon interactions $J^P=1/2^-$. $I=0$ $\Lambda(1405)$, $\Lambda(1670)$, $I=1$ $\Sigma(1670)$, possible $\Sigma(1400)$.
One also finds other resonances in $S=-2, 0, +1$
 - Processes involving scattering or (photo)production of, particularly, the lowest Nucleon-Nucleon partial waves like the 1S_0 , 3S_1 or P-waves.
Deuteron, Nuclear matter, Nuclei.
2. Then one can study:
 - Strongly interacting coupled channels.
 - Large unitarity loops.
 - Resonances.

4. This allows as well to use the Chiral Lagrangians for higher energies. (BONUS)
5. Since one can also use the chiral Lagrangians for higher energies it is possible to establish a connection with perturbative QCD, $\alpha_s(4 \text{ GeV}^2)/\pi \approx 0.1$. (OPE). E.g. providing phenomenological spectral functions for QCD Sum Rules (going definitively beyond the sometimes insufficient hadronic scheme of narrow resonance+resonance dominance).
6. The same scheme can be applied to productions mechanisms. Some examples:
 - Photoproduction: $\gamma\gamma \rightarrow \pi^0\pi^0, \pi^+\pi^-, K^+K^-, K^0K^0, \pi^0\eta$;
 $\gamma p \rightarrow K^+ \Lambda(1405)$; $(\gamma, \pi\pi)$; $\gamma d \rightarrow d$; $\gamma NN \rightarrow NN$; $\gamma d \rightarrow \gamma d$; ...
 - Decays: $\phi \rightarrow \gamma \pi^0\pi^0, \pi^0\eta, K^0K^0$; $J/\Psi \rightarrow \phi(\omega) \pi\pi, KK$;
 $f_0(980) \rightarrow \gamma\gamma$; branching ratios ...

Chiral Perturbation Theory

Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian

**Hilbert Space
Physical States**

u, d, s massless quarks
 $SU(3)_L \otimes SU(3)_R$

Spontaneous Chiral Symmetry Breaking



$SU(3)_V$

Goldstone Theorem

Octet of massless pseudoscalars

π, K, η

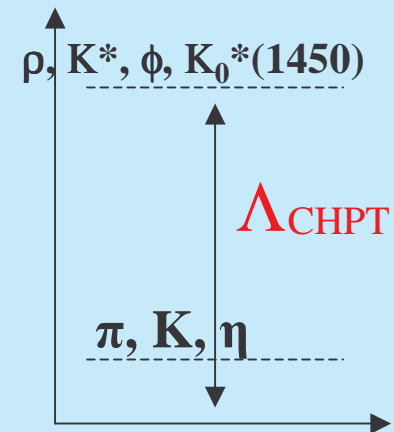
Energy gap

Non-zero masses

$m_P^2 \propto m_q$

$m_q \neq 0$. Explicit breaking
of Chiral Symmetry

Perturbative expansion in powers of
the external four-momenta of the
pseudo-Goldstone bosons over Λ_{CHPT}^2



$$L = L_2 + L_4 + \dots$$

$$\frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda_{\text{CHPT}}^2}\right)$$

$$\Lambda_{\text{CHPT}} \approx 1 \text{ GeV} \approx M_\rho$$

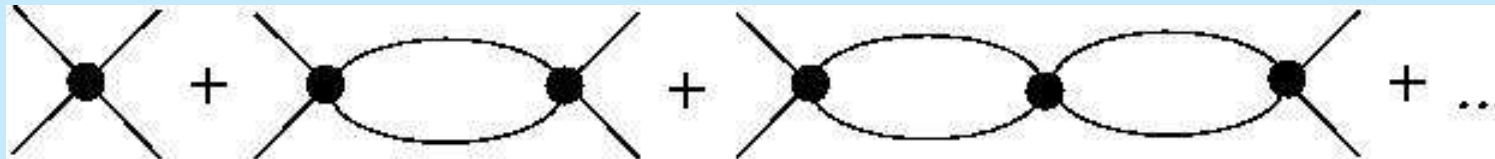
$$\approx 4\pi f_\pi \approx 1 \text{ GeV}$$

- There are good and well established reasons why the unitarity corrections are so enhanced in the previous examples giving rise to non-perturbative physics in S-wave meson-meson, meson-baryon, nucleon-nucleon.

- There are good and well established reasons why the unitarity corrections are so enhanced in the previous examples giving rise to non-perturbative physics
- **New scales or numerical enhancements can appear that makes definitively smaller the overall scale Λ_{CHPT} , e.g:**
 - Scalar Sector (S-waves) of meson-meson interactions with $I=0,1,1/2$ the unitarity loops are enhanced by numerical factors.

$$\begin{array}{ccc} \text{P-WAVE} & & \text{S-WAVE} \\ \frac{s - 4m_\pi^2}{6f^2} & \longrightarrow & \frac{s - m_\pi^2}{f^2} \end{array} \quad \text{Enhancement by a factor } 6^L$$

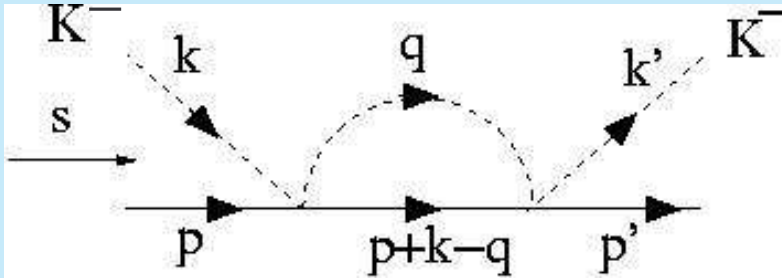
σ meson



- Presence of **large masses** compared with the typical momenta, e.g: Kaon masses in driving the appearance of the $\Lambda(1405)$ close to threshold in $\bar{K}N$. This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass.

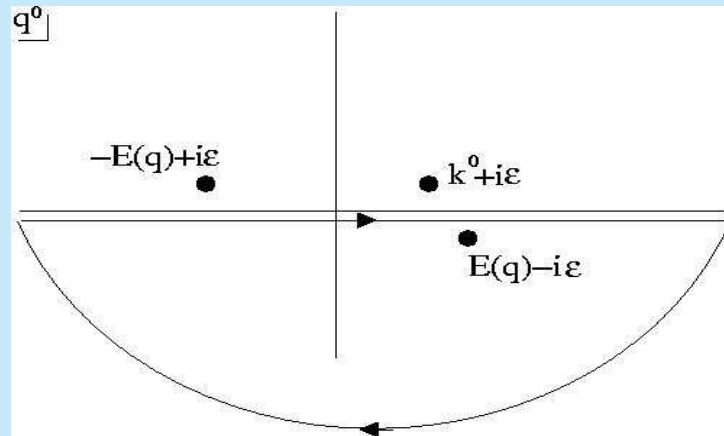
Let us keep track of the kaon mass, $M_K \approx 500 \text{ MeV}$

We follow similar arguments to those of S. Weinberg in NPB363,3 ('91) respect to NN scattering (nucleon mass).



Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\epsilon)(q^0 + E(q) - i\epsilon)(q^0 - E(q) + i\epsilon)}$$

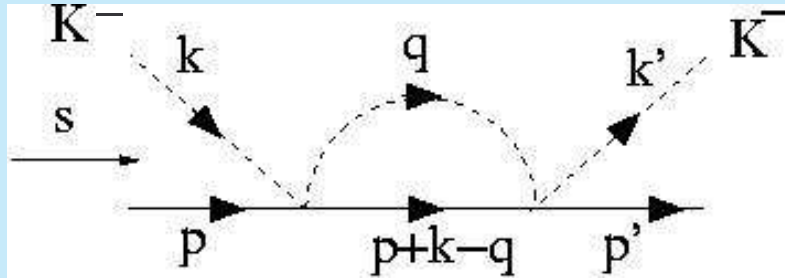


$$\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \cong \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

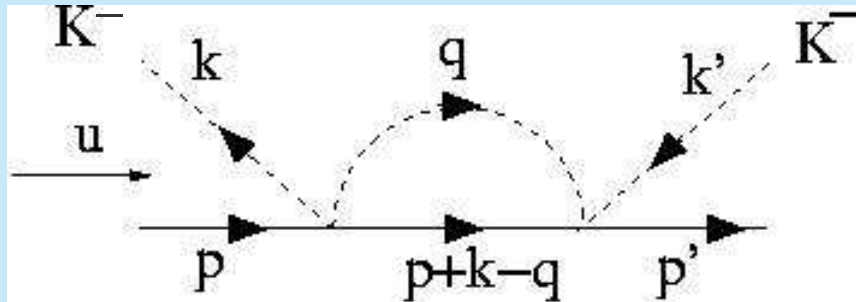
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Unitarity Diagram

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Let us take now the crossed diagram
 $k \rightarrow -k$

$$\frac{1}{k^0 + E(q)} \frac{1}{2E(q)} \cong \frac{1}{4M_K^2}$$

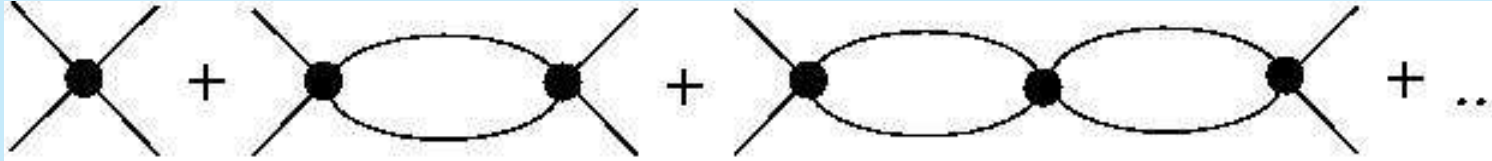
Unitarity & Crossed loop diagram:

$$\frac{4M_K^2}{k^2 - q^2}$$

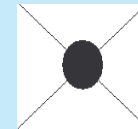
Unitarity enhancement for low three-momenta:

$$\frac{2M_K}{q}$$

In all these examples the **unitarity cut** (sum over the unitarity bubbles) is **enhanced**.



UCHPT makes an expansion of an ``Interacting Kernel''



from the appropriate EFT and then the unitarity cut is fulfilled to all orders (non-perturbatively)

- Other important non-perturbative effects arise because of the presence of nearby resonances of non-dynamical origin with a well known influence close to threshold, e.g. the $\rho(770)$ in P-wave $\pi\pi$ scattering, the $\Delta(1232)$ in πN P-waves,...

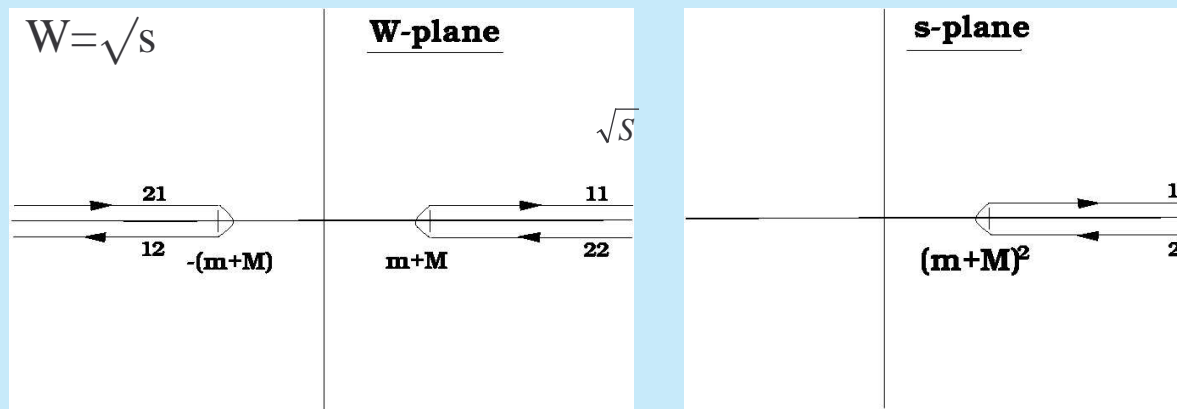
Unitarity only dresses these resonances but it is not responsible of its generation (typical $q\bar{q}$, qqq , ... states)

These resonances are included explicitly in the interacting kernel in a way consistent with chiral symmetry and then the right hand cut is fulfilled to all orders.

General Expression for a Partial Wave Amplitude

- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

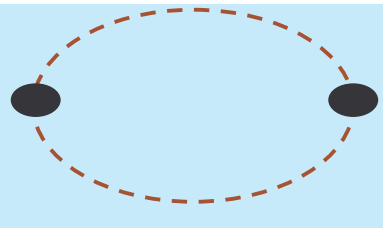
$$\text{Im } T_{ij} = \sum_k T_{ik} \rho_k T_{kj}^* \longrightarrow \text{Im } T_{ij}^{-1} = -\rho_i \delta_{ij} \quad \text{Unitarity Cut}$$



We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known)

$$T_{ij}^{-1} = \underbrace{R_{ij}^{-1}}_{\text{The rest}} + \delta_{ij} \left(g(s_0)_i - \frac{s - s_0}{\pi} \int_{s_{th;i}}^{\infty} \frac{\rho(s')_i}{(s' - s - i0^+)(s' - s_0)} ds' \right)$$

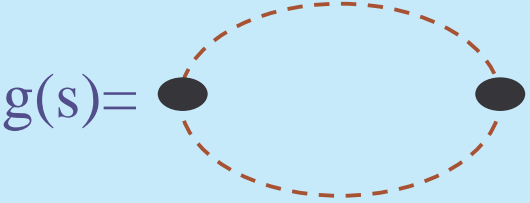
$g(s)_i$: Single unitarity bubble

$$g(s) = \text{---} \bullet \text{---} \bullet \text{---} \quad g(s) = \frac{1}{4\pi^2} \left(a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right)$$


The diagram shows two black dots representing vertices, connected by a dashed orange line forming a loop. The label 'g(s)=' is placed to the left of the first vertex.

$$T = \left[R^{-1} + g(s) \right]^{-1} = \left[I + R \cdot g \right]^{-1} \cdot R \quad \sigma(s) = \frac{2q}{\sqrt{s}}$$

1. T obeys a CHPT/alike expansion



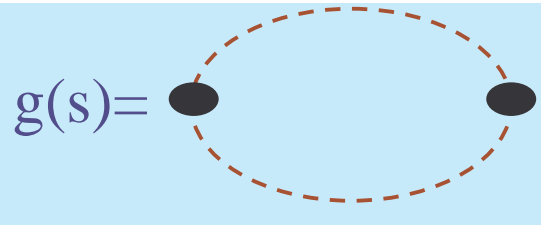
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1. T obeys a CHPT/alike expansion
2. R is fixed by **matching** algebraically with the CHPT/alike
CHPT/alike+Resonances
expressions of T , $R = R_2 + R_4 + \dots$

In doing that, one makes use of the CHPT/alike counting for $g(s)$

The counting/expressions of $R(s)$ are consequences of the known ones of $g(s)$ and $T(s)$



The diagram shows two black dots representing external vertices, connected by a dashed orange line forming a loop. To the left of the diagram is the label $g(s)=$.

$$g(s) = \frac{1}{4\pi^2} \left(a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right)$$

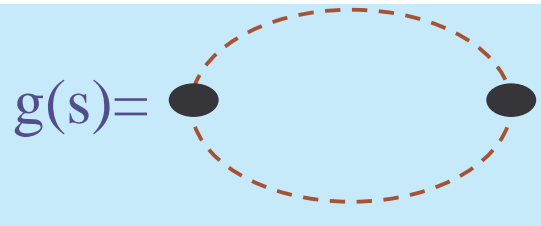
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3. The CHPT/alike expansion is done to $R(s)$. Crossed channel dynamics is included perturbatively.



$$g(s) = \text{bubble diagram} \quad g(s) = \frac{1}{4\pi^2} \left(a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right)$$

$$T = [R^{-1} + g(s)]^{-1} = [I + R \cdot g]^{-1} \cdot R \quad \sigma(s) = \frac{2q}{\sqrt{s}}$$

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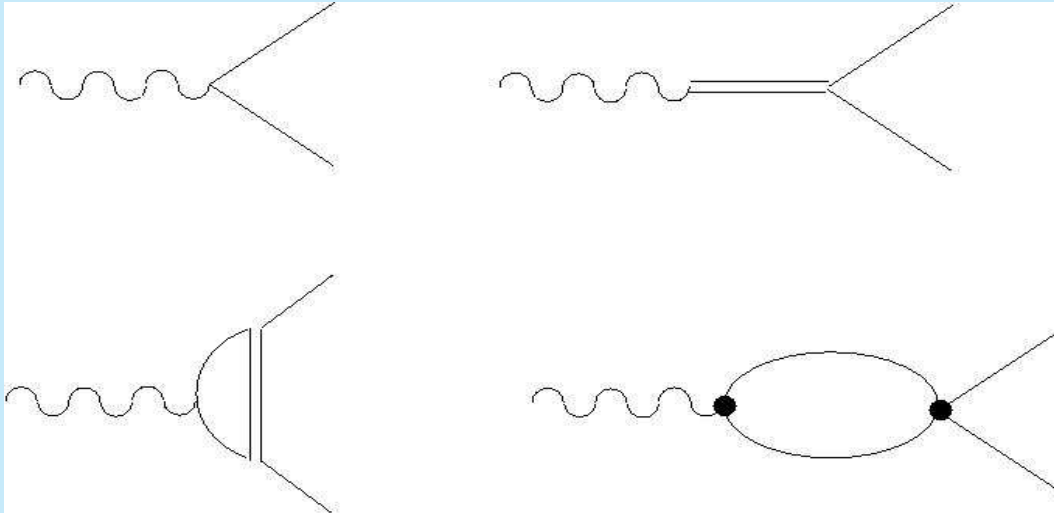
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The counting/expressions of $R(s)$ are consequences of the known ones of $g(s)$ and $T(s)$

3. The CHPT/alike expansion is done to $R(s)$. Crossed channel dynamics is included perturbatively.
4. The final expressions fulfill unitarity to all orders since R is real in the physical region (T from CHPT fulfills unitarity perturbatively as employed in the matching).

Production Processes

The re-scattering is due to the strong „final“ state interactions from some „weak“ production mechanism.



$$\text{Im} F_i = \sum_k F_k \rho_k T_{ki}^*$$

We first consider the case with only the right hand cut for the strong interacting amplitude, R^{-1} is then a sum of poles (CDD) and a constant. It can be easily shown then:

$$F = [I + R \cdot g]^{-1} \cdot \xi$$

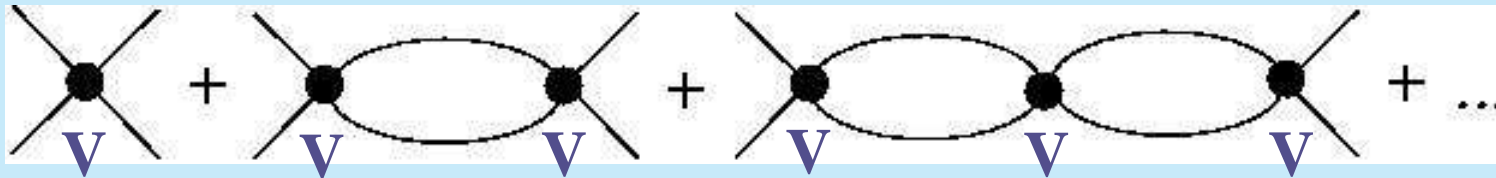
Finally, ξ is also expanded perturbatively (in the same way as R) by the **matching** process with CHPT/alike expressions for F , order by order, $\xi = \xi_2 + \xi_4 + \dots$

The crossed dynamics, as well for the production mechanism, are then included perturbatively.

Historically, the first approach to apply a Chiral expansion to an interacting KERNEL was:

S. Weinberg, PL B251(1990)288, NP B363(1991)3, PL B295 (1992)114 FOR THE NUCLEON-NUCLEON INTERACTIONS.

The Chiral expansion was applied to the set of two nucleon irreducible diagrams, THE POTENCIAL, which was then iterated through a Lippmann-Schwinger equation.



The solution to the LS equation is NUMERICAL

Further regularization is needed when solving the LS equation (cut-off dependence) so that the new divergences are not reabsorbed by the counterterms introduced in V

N. Kaiser, P.B. Siegel and W.Weise NP A594(1995)325 proceeded analogously in the S-wave strangeness= -1 meson-baryon sector

E.Oset, J.A.O. NP A620(1997)438 (E NPA652('99)407) applied it to meson-meson interactions in S-wave σ , $f_0(980)$, $a_0(980)$ resonances

§ However the approach was fully algebraic since it was demonstrated that the off-shell part of the potential (LO CHPT) when iterated in the LS equation only renormalizes the potential itself.

Key references to the previous general and systematic version of UCHPT

E.Oset, J.R. Peláez, J.A.O. PR D59(1999)074001 (Err PR D60('99)099906)

E. Oset, J.A.O, PR D60(1999)074023

U.-G. Meißner, J.A.O., PL B500(2001)263

S-Wave, S=-1 Meson-Baryon Scattering

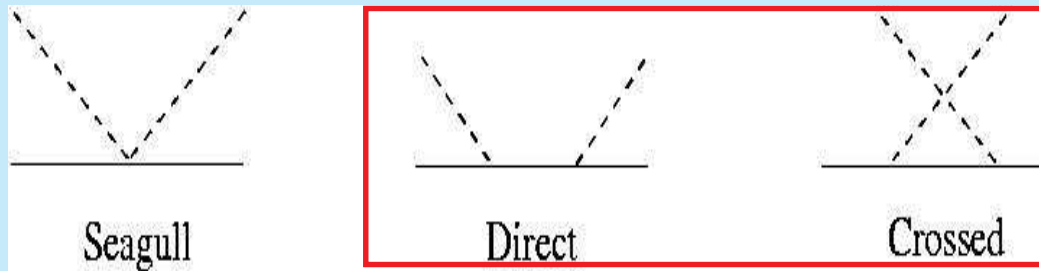
Kaiser, Weise, Siegel, NPA594(95)325; Oset, Ramos, NPA635,99 ('98). U.-G. Meißner, J.A.O, PLB500, 263 ('01), PRD64, 014006 ('01); A. Ramos, E. Oset, C. Bennhold, PRL89(02)252001, PLB527(02)99; Jido, Hosaka et al., PRC68(2003)018201, nucl-th/0305011, nucl-th/0305023, hep-ph/0309017, etc

Jido, Oset, Ramos, Meißner, J.A.O, NPA725(03)181 TWO LAMBDA S(1405)

Enhancement of the unitarity cut

$$T = [R^{-1} + g(s)]^{-1} = [I + R \cdot g(s)]^{-1} \cdot R(s)$$

$T = T_1 = R_1$ **LEADING ORDER:** $g(s)$ is order p in meson-baryon



No bare resonances

Non-negligible for energies greater than 1.3 GeV

Many channels: $K^- p, \bar{K}^0 n, \pi^0 \Sigma^0, p^+ \Sigma^-, \pi^- \Sigma^+, \pi^0 \Lambda, \eta \Lambda, \eta \Sigma^0, K^+ \Xi^-, K^0 \Xi^0$

Important isospin breaking effects due to cusp at thresholds, we work with the physical basis

In Meißner, J.A.O PLB500, 263 ('01), several poles were found.

1. All the poles were of dynamical origin, they disappear in Large N_c , because $R.g(s)$ is order $1/N_c$ and is subleading with respect to the identity I.

$$T = (I + R.g(s))^{-1} R(s) \rightarrow R(s) \quad N_c \rightarrow \infty$$

The subtraction constant corresponds to evaluate the unitarity loop with a cut-off Λ of natural size (scale) around the mass of the ρ .

$$a_{SL} = -2 \text{Log} \left(1 + \sqrt{1 + \frac{M_N^2}{\Lambda^2}} \right) \cong -2 \quad M_N \rightarrow N_c$$

2. Two $I=1$ poles, one at 1.4 GeV and another one at around 1.5 GeV.
3. The presence of **two resonances (poles)** around the nominal mass of the $\Lambda(1405)$.

These points were further studied in: Jido, Oset, Ramos, Meißner, J.A.O, NP A725 (03)181, taking into account as well another study of Oset, Ramos, Bennhold PLB527,99 (02).

SU(3) decomposition $8 \otimes 8 \rightarrow 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus 27$

Isolating the different SU(3) invariant amplitudes one observes the presence of poles for the Singlet (1), **Symmetric Octet (8_s), Antisymmetric Octet (8_a)**.

DEGENERATE

Table 3: Pole positions and couplings to $I = 0$ physical states from the model of Ref. [3]

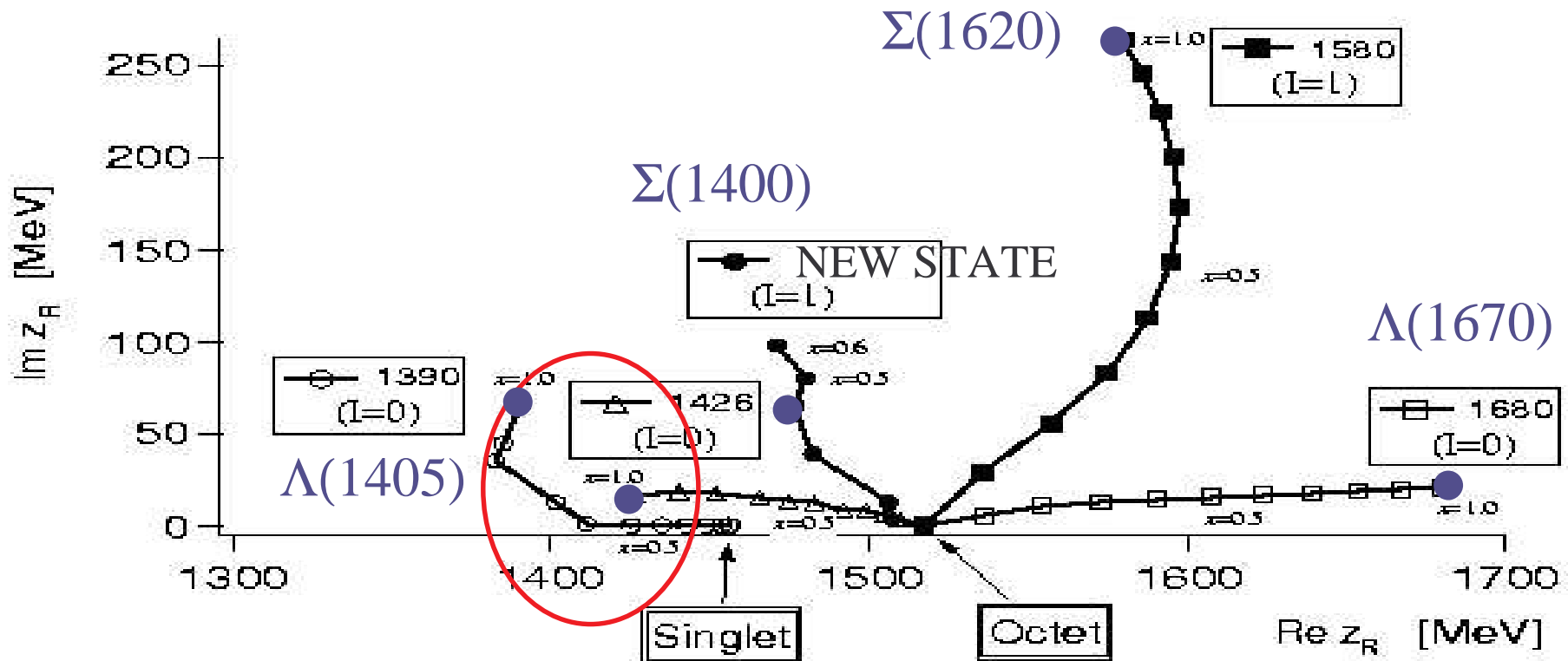
z_R ($I = 0$)	1379 + 27i		1434 + 11i		1692 + 14i	
	g_i	$ g_i $	g_i	$ g_i $	g_i	$ g_i $
$\pi\Sigma$	-1.76 - 0.62i	1.87	-0.56 - 1.02i	1.16	-0.08 - 0.32i	0.33
$\bar{K}N$	0.86 + 0.70i	1.11	-1.74 + 0.63i	1.85	0.32 + 0.41i	0.52
$\eta\Lambda$	0.19 + 0.33i	0.38	-1.20 + 0.23i	1.23	-0.83 - 0.19i	0.85
KE	-0.52 - 0.19i	0.55	-0.20 - 0.30i	0.36	3.87 + 0.05i	3.87

a) $\Lambda(1405)$ b) $\Lambda(1670)$

a) is more than twice wider than b)
(Quite Different Shape)

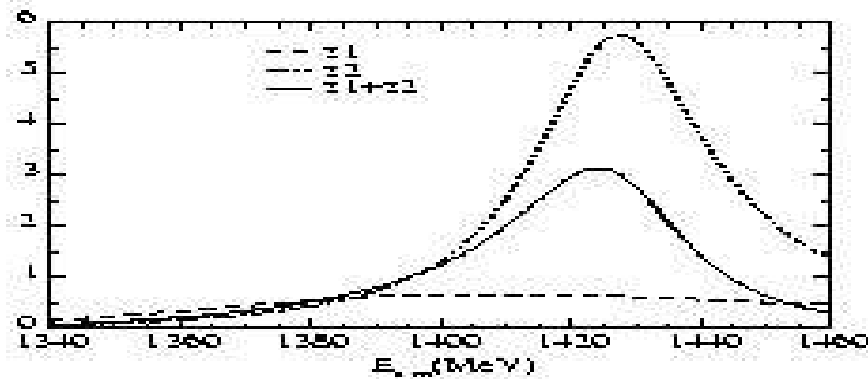
b) Couples stronger to $\bar{K}N$ than to $\pi\Sigma$
contrarily to a)

It depends to which resonance the
production mechanism couples
stronger that the shape will move
from one to the other resonance

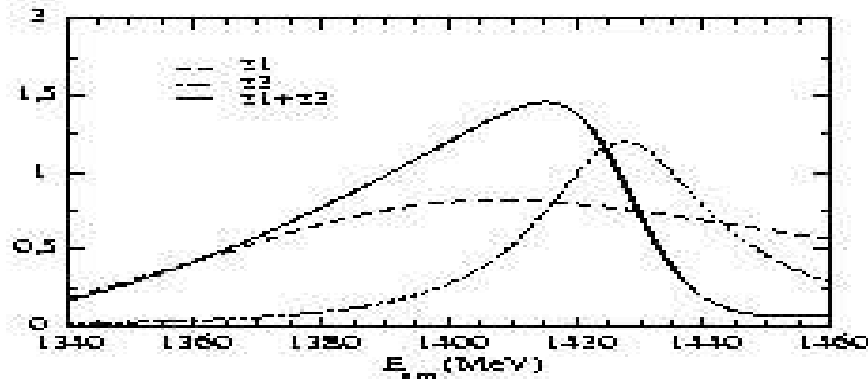


Simple parametrization of our own results with BW like expressions

$$g_{R_1}^2 \frac{1}{W - M_{R_1} + i\Gamma_{R_1}/2} g_{R_1}^2 + g_{R_2}^2 \frac{1}{W - M_{R_2} + i\Gamma_{R_2}/2} g_{R_2}^2$$



$$g_{R_1}^2 \frac{1}{W - M_{R_1} + i\Gamma_{R_1}/2} g_{R_1}^2 + g_{R_2}^2 \frac{1}{W - M_{R_2} + i\Gamma_{R_2}/2} g_{R_2}^2$$



SU(3) Decomposition of the Physical Resonances

Pole (MeV)	C_1	C_{8_A}/C_1	C_{8_S}/C_1	$ C_1 ^2$	$ C_{8_A} ^2$	$ C_{8_S} ^2$
1379+ i 27	0.96	0.15+ i 0.11	0.15- i 0.19	0.92	0.03	0.05
1434+ i 11	0.49	0.64+ i 0.77	0.71+ i 1.28	0.24	0.24	0.52
1692+ i 14	0.48	1.58+ i 0.37	0.78+ i 0.16	0.23	0.63	0.14

I=0

Pole (MeV)	C_{8_A}	C_{8_S}/C_{8_A}	$ C_{8_A} ^2$	$ C_{8_S} ^2$
1401+ i 40	0.81	0.72+ i 0.07	0.66	0.34
1488+ i 114	0.59	1.37- i 0.06	0.35	0.65

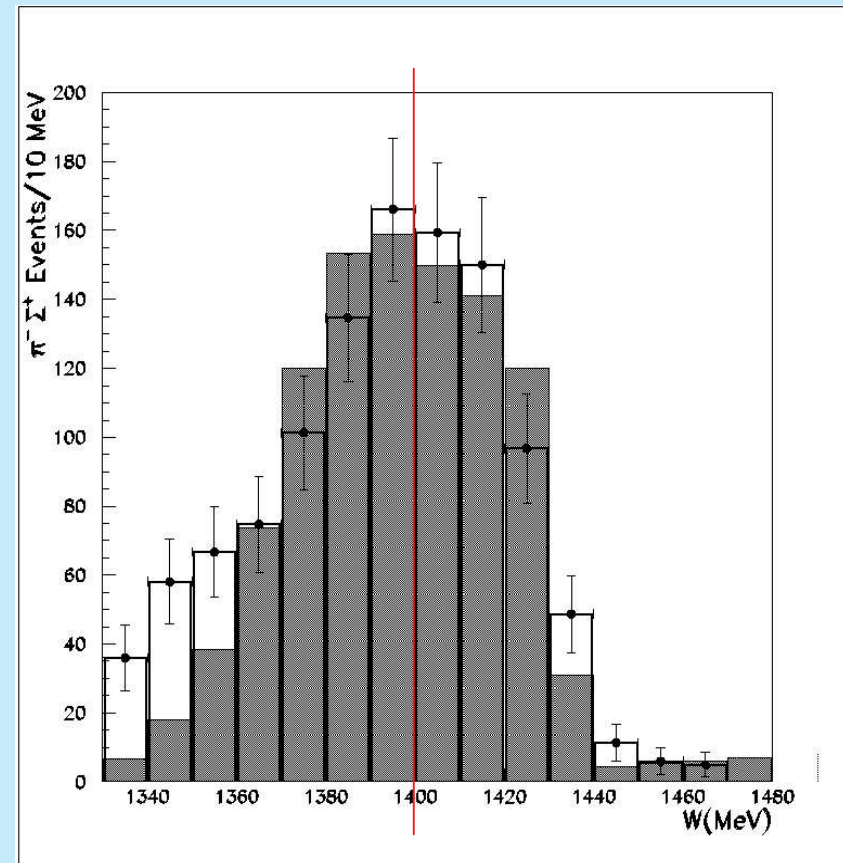
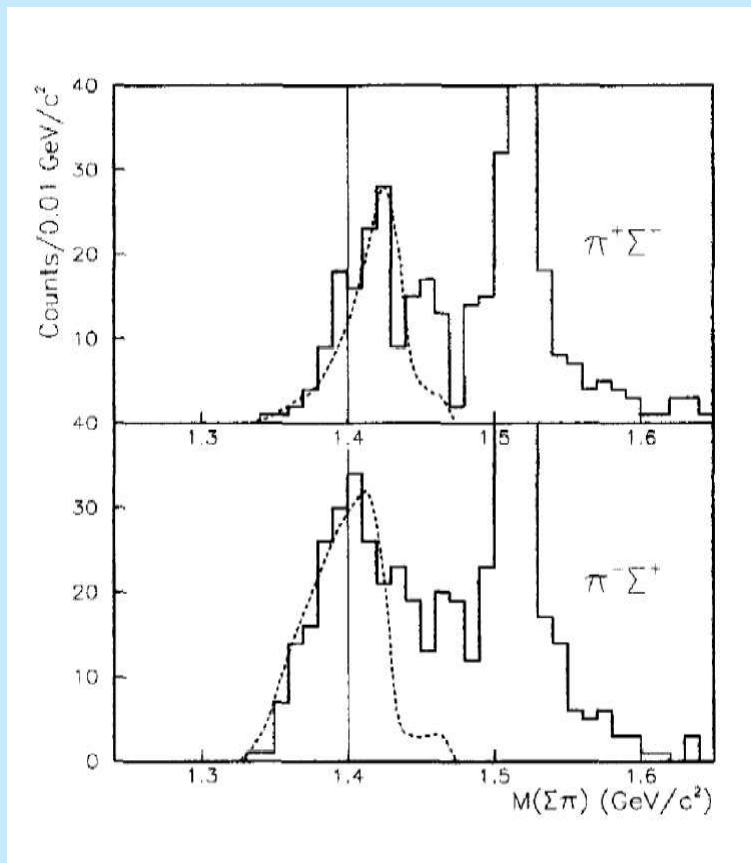
I=1

ACCURATE AGREEMENT between **UCHPT PREDICTIONS** ON THE POSITION OF THE $\Lambda(1405)$ bump AND EXPERIMENT:

$\gamma p \rightarrow K^+ \Lambda(1405) \rightarrow K^+ \pi^+ \Sigma^-, \pi^- \Sigma^+$

Nacher, Oset, Toki, Ramos PL B455 ('99)55

Recently measured experimentally in Spring8 J.K. Ahn, NP A721 ('03) 715c



We are now working in order to include as well the higher orders, $\mathcal{O}(q^2)$ and $\mathcal{O}(q^3)$
This means that theoretical uncertainties are expected to be of the order

$$(M_K/4\pi f_\pi)^3 \lesssim 10 \%$$

As for the leading order (Meißner, J.A.O, PLB500 ('01)263, PRD64 ('01)014006) the approach is relativistic (important since $M_K/M_N \simeq 0.5$). We will also consider higher partial waves. Data on scattering, photoproduction and baryon masses will be considered simultaneously, together with πN low energy data, for fixing counterterms.

Kaiser, Weise, Siegel, NPA594(95)325; Caro Ramón, Kaiser, Wetzel, Weise Nucl.Phys. A672('00)249 performed calculations at $\mathcal{O}(q^2)$ in SU(3) HBCHPT (Non-Relativistic) without the non-negligible $\eta\Lambda$ channel as well.

APPLICATION TO DEAR (Dafne Exotic Atom Research) EXPERIMENT

Calculation of $T_{\bar{K}p}^{th}$ at threshold in the isospin limit (QCD, $m_u=m_d$).

They have measured the $2p \rightarrow 1s$ kaonic-hydrogen line (“very preliminary” results in EPJ A19,s01 (2004)185) “strong” shift:

$$\epsilon - i\frac{\Gamma}{2} = (150 \pm 45) + i(125 \pm 45) \text{ eV}$$

$$\epsilon - i\frac{\Gamma}{2} = \underbrace{-2\alpha^3 \mu_r^2 T_{\bar{K}p}^{th}}_{\text{Deser's formula}} (1 + X)$$

Deser's formula

Isospin violating corrections
Meißner,Raha,Rusetsky

30% error they expect to improve precision in the near future

$$X = 0 \rightarrow T_{\bar{K}p}^{th} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10) \text{ fm}$$

$$\epsilon_{12} = -2\alpha^3 \mu_r^2 T_{\bar{K}p}^{th} (1 + X)$$

Isospin violating corrections
Meißner, Raha, Rusetsky

DEAR $X = 0 \rightarrow T_{\bar{K}p}^{th} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10)$ fm

M.Iwasaki et al. PRL78(1997)3067 $\epsilon_{12}^{-i\frac{\Gamma}{2}} = (323 \pm 63 \pm 11) - i(200 \pm 100 \pm 50)$ eV

$X = 0 \rightarrow T_{\bar{K}p}^{th} = (-0.78 \pm 0.15) + i(0.50 \pm 0.30)$ fm

Scattering experiment B.R. Martin NP B94 (1975)413

$$T_{\bar{K}p}^{th} = (-0.67 \pm 0.10) + i(0.64 \pm 0.10)$$
 fm

Oset, Ramos: -0.85+i 1.24 // Kaiser et al.: -0.97+i1.1 //
Meißner, J.A.O.: -0.51+i0.9 LO (relativistic) UCHPT

Rather controversial (not very precise) situation:

Experimentally: more precision is needed in kaonic atoms experiments (hopefully DEAR)

Theoretically: 1) Higher orders are necessary to be considered and one must check the convergence of the UCHPT expansion to calculate $T_{\bar{K}p}^{th}$

2) To compute X

Nucleon-Nucleon Interaction

- Ideal system to apply the UCHPT
- At (very) low energies one finds already non-perturbative physics.
- Bound state (deuteron) and antibound state just below threshold (new and non-natural scale).
- Large nucleon masses that enhances the unitarity cut.

Nucleon-Nucleon Interaction

- Ideal system to apply the UCHPT
- At (very) low energies one finds already non-perturbative physics.
- Bound state (deuteron) and antibound state just below threshold (new and non-natural scale).
- Large nucleon masses that enhances the unitarity cut.
- **Weinberg scheme:** The chiral counting is applied for calculating the NN potential that then is iterated in a Lippmann-Schwinger equation.

S. Weinberg, PL B251(1990)288, NP B363(1991)3, PL B295 (1992)114 ;
C. Ordoñez, L. Ray, U. Van Kolck, PRC53(1996)2086

E. Epelbaum, W. Glöckle, U.-G. Meißner NP A671(2000)295, **etc.**

- **Kaplan-Savage-Wise EFT:** like in CHPT one works out directly the scattering amplitude following a chiral like counting called the KSW counting. Problems with the convergence of the series.

D.B. Kaplan, M.J. Savage, M.B. Wise, NP A637(1998)107; NP B534(1998)329.

S. Fleming, T. Mehen, I. Stewart, NP A677(2000)313, PR C61(2000)044005, **etc.**

WE FIX R MATCHING WITH KSW:

Since $T = \frac{4\pi}{M} \frac{1}{R^{-1} + g}$ is easier to fix R by matching with the inverses of the KSW amplitudes

$${}^1S_0, \quad \begin{array}{c} g = -v - ip \\ \text{O}(p^0) \text{O}(p^1) \end{array} \quad R = \begin{array}{cccc} R_0 & + & R_1 & + & R_2 & + & R_3 & + & \text{O}(p^4) \\ \text{O}(p^0) & & \text{O}(p) & & \text{O}(p^2) & & \text{O}(p^3) & & \end{array}$$

$$\frac{1}{A_{KSW}} = \frac{1}{A_{-1}} - \frac{A_0}{A_{-1}^2} + \frac{A_0^2 - A_1 A_{-1}}{A_{-1}^3} + \text{O}(p^4)$$

$$\text{O}(p) \quad \text{O}(p^2) \quad \text{O}(p^3)$$

$$\frac{1}{R} + g = \left(\frac{1}{R_0} - v \right) + \left(\frac{R_1}{R_0^2} + ip \right) - \left(\frac{R_0 R_2 - R_1^2}{R_0^3} \right) + \left(\frac{R_1^3 - 2R_0 R_1 R_2 + R_0^2 R_3}{R_0^4} \right) + \text{O}(p^4)$$

$$R_0 = \frac{1}{v} \quad R_1 = \frac{1}{a_s v^2} \quad R_2 = \frac{\frac{2v p^2}{\pi} + \gamma^2 M + \frac{v 4\pi A_0}{A_{-1}^2}}{M v^3}$$

$$\frac{R_1}{R_0} = \frac{1}{a_s v} \rightarrow 0$$

etc

PHENOMENOLOGY

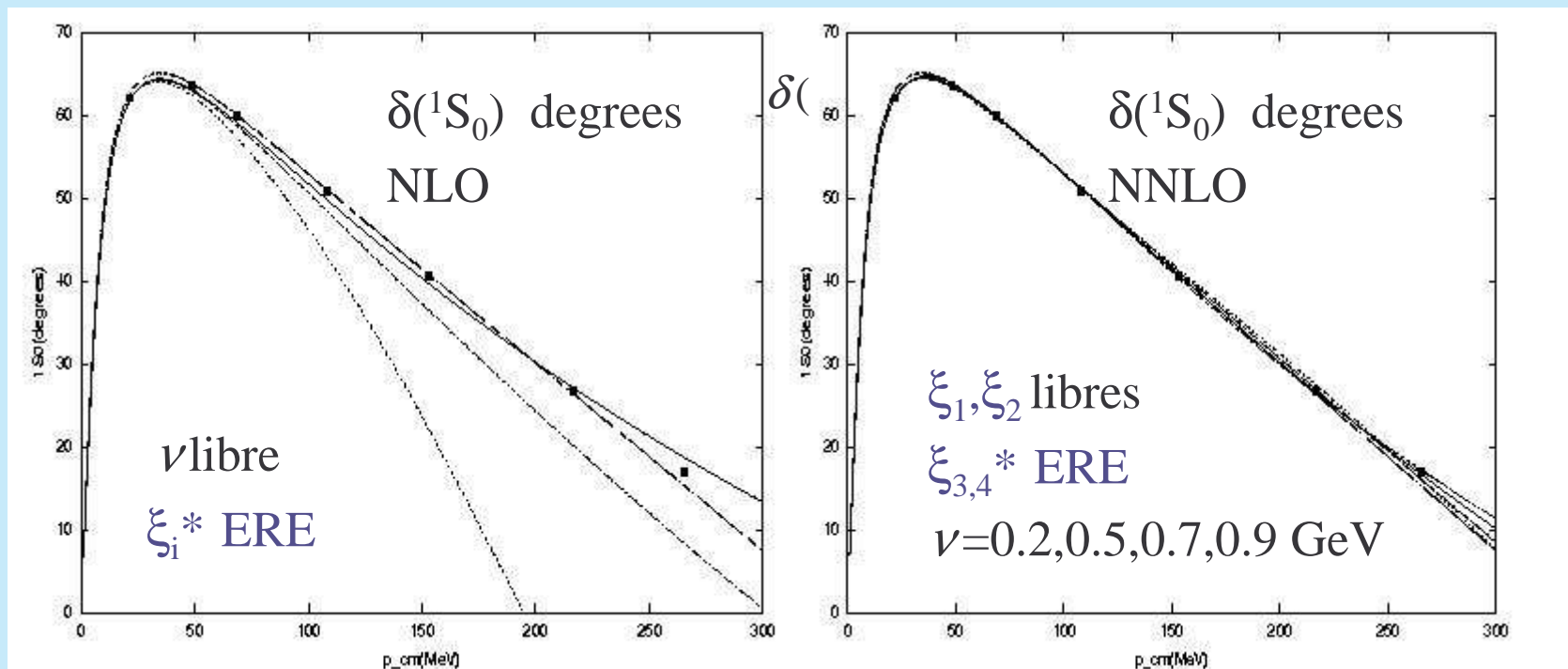
1S_0

Counterterms: NLO: ξ_1, ξ_2 ; NNLO: ξ_3, ξ_4

At every order in the expansion of R , two counterterms are fixed in terms of a_s, r_0 and ν :

NLO: $\xi_1(a_s, r_0, \nu), \xi_2(a_s, r_0, \nu)$

NNLO: $\xi_3(\xi_1, \xi_2, a_s, r_0, \nu), \xi_4(\xi_1, \xi_2, a_s, r_0, \nu)$



3S_1

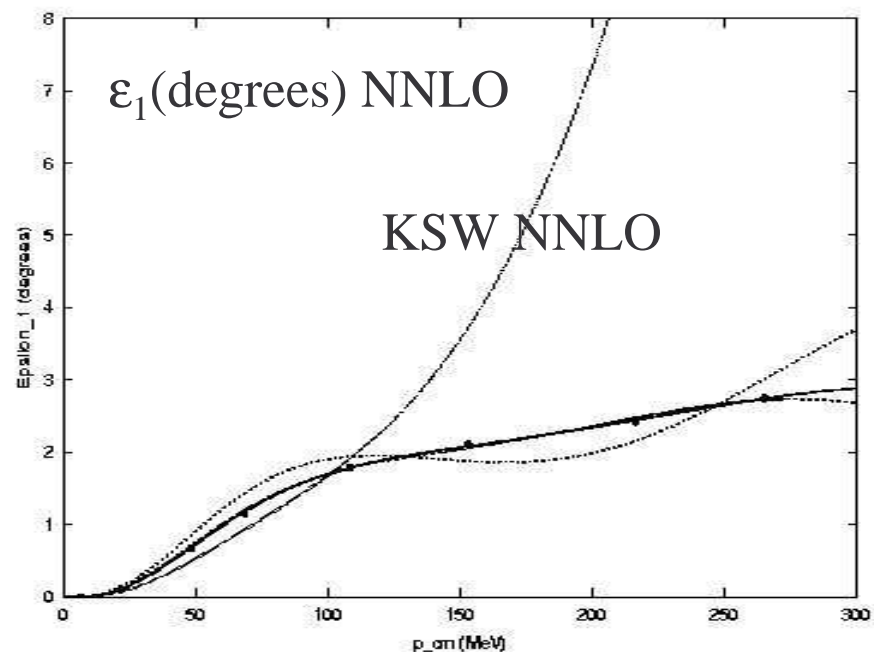
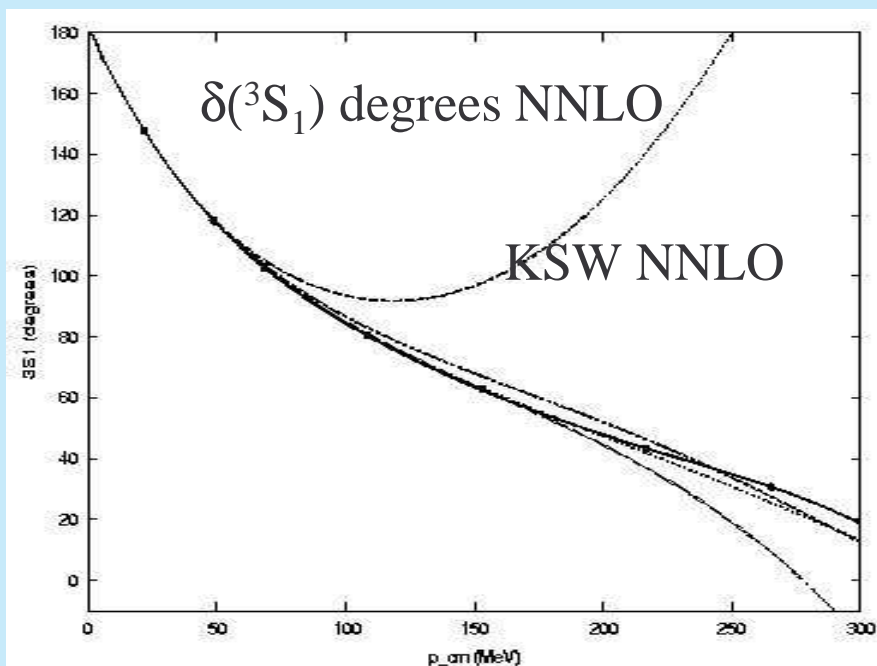
Counterterms: NLO: ξ_1, ξ_2 ; NNLO: $\xi_3, \xi_4, \xi_5, \xi_6$

At every order in the expansion of R , two counterterms are fixed in terms of a_s, r_0 and v :

J.A.O. NPA725('03)85

NLO: $\xi_1(a_s, r_0, v), \xi_2(a_s, r_0, v)$

NNLO: $\xi_3(\xi_1, \xi_2, a_s, r_0, v), \xi_4(\xi_1, \xi_2, a_s, r_0, v)$



$v=500 \text{ MeV}, \gamma=0.37 \text{ fm}^{-1}, \xi_5 = 0.44,$
 $\xi_6 = 0.58$

In **PDS** one includes a new scale μ when removing poles in $D=3$ dimensions, guarantying natural size for counterterms.

The subtraction constant ν accounts for such scale as well.

$$T = (R^{-1} + g(s))^{-1} \quad T = \frac{4\pi}{M} \frac{1}{p \cot\delta - ip} \quad g = -\nu - ip$$

$$\frac{1}{R} = p \cot\delta + \nu = \nu - \frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_{n=0} r_n \left(\frac{p^2}{\Lambda^2} \right)^{n+1}$$

We take ν as a quantity of Λ_χ and since we always the combination $\nu - 1/a$ *The expansion for R is well behaved !!*

Thus we shall **abandon** the more intricate PDS power counting and apply **the plain Weinberg's** counting for calculating the **whole amplitude** in order to obtain, by matching, the kernel R .

Our aim is to perform an $\mathcal{O}(q^4)$ calculation (as already done within standard Weinberg's approach by Entem, Machleid PRC68('03)041001; Epelbaum, Glöckle, Meißner nucl-th/0405048) **but ...**

- 1) In a way free of regularization schemes as required in an EFT.**
- 2) Algebraic results, as in standard CHPT**

Within UCHPT one can also study the properties of bound states such as the **deuteron** by making use of well known results in S-matrix theory within quantum field theory

E.g: $\gamma \mathbf{d} \rightarrow \mathbf{d}$ (EM form factors of the deuteron): we can study it by considering first $\gamma \mathbf{N} \mathbf{N} \rightarrow \mathbf{N} \mathbf{N}$ and then isolating the deuteron pole contribution:

$$\begin{aligned} \langle NN\gamma | S | NN \rangle &= (2\pi)^4 \delta^{(4)}(P_f + k - P_i) \frac{\pi}{E_d} \frac{\pi}{E'_d} \delta(P_f^0 - E'_d) \delta(P_i^0 - E_d) \\ &\times \langle NN | S | d, P_f \rangle \langle d, P_f; \gamma k | S | d, P_i \rangle \langle d, P_i | S | NN \rangle \end{aligned}$$

This remembers the interpolating field for the deuteron employed in **D.B.Kaplan, M.J.Savage and M.B.Wise, PRC59(1999)617**

Again, one avoids in this way further cut-off dependences that occurs when convoluting with deuteron Wave Functions taken from phenomenological Lagrangias or from W-CHPT in present nuclear EFT a la Weinberg.

In the same way we can study other processes of interest like the Compton Scattering on the deuteron ($\gamma d \rightarrow \gamma d$) where new data are available [Beane, Malheiro, McGovern, Phillips and U. van Kolck nucl-th/0403088](#)),

$n d \leftrightarrow d \gamma, p p \rightarrow d e^+ \nu, \nu d \rightarrow p p e^+, \nu d \rightarrow \nu d, \text{ etc}$

Meson-Meson Sector: Scattering and production

Important contributions have been achieved employing the UCHPT, some highlights:

Ø Scalar meson-meson spectroscopy: Clarification of the nature of the lightest scalar mesons $\sigma, f_0(980), a_0(980), \kappa$ giving rise to the lightest scalar nonet of dynamically generated resonances versus preexisting nonet around 1.4 GeV

J.A.O.
NPA727('03)353
Oset, J.A.O.
PRD62('99)074023;
NPA620('97)438 (E
A652,407)

Ø Reproduction of **all meson-meson scattering** data up to 1.2 GeV in S and P-waves in terms of 7 free parameters ($\mathcal{O}(q^4)$ CHPT counterterms, reproducing all the S and P-waves resonances, $\rho, K^*, \phi(1020), \sigma, f_0(980), a_0(980), \kappa$

Oset, Peláez, J.A.O
PRL80('98)3452
PRD59('99)074001
(E D60,099906)

Ø Production processes (clarification of the role of the scalar dynamics played in FSI):

Ø $\gamma\gamma$ **meson-meson** (more than one of magnitude of effects by FSI due to the $f_0(980)$)

Oset, J.A.O
NPA629('98)739

Ø $\phi(1020)$ decays: $f_0(980), a_0(980)$

J.A.O NPLB426('98)7 ; J.Palomar et al,
NPA729('03)743 ;

Ø Role of the $a_0(980)$ in $\eta \rightarrow \pi^0 \gamma\gamma$ decay

J.Oset, Peláez, Roca
NPRD67('03)073013 ;

Ø Exotic $\pi_1(1400)$, $\pi_1(1600)$ resonances, as dynamically generated resonances from **coupled channel dynamics**
 $\eta\pi$, $\eta'\pi$ in P-waves

Szczepaniak, Swat,
Dzierba, Teige
PRL91('03)092002

Ø Application to QCD sum rules: Most reliable determination of m_s mass by scalar **QCD sum rules** thanks to apply UCHPT to determine the $K\pi$ S-wave $I=1/2$ partial wave

Jamin, Pich, J.A.O,
NPB587('00)331;
NPB622('02)279;EP
JC24('02)237

Ø Improvement of the knowledge of V_{us} from $K \rightarrow \pi \ell^+ \nu_\ell$ fixing the scale in which is applicable a former calculation performed by **Leutwyler and Ross** *Z Phys* **C25('84)91**, performing as well a consistency check of the given value.

Jamin, Pich, J.A.O,
JHEP 0402('04)047

Future Plans in meson-meson:

Ø We are studying the role of the σ and $f_0(980)$ in $D \rightarrow 3\pi$, where a clear sigma for the σ resonance is seen but the present analysis E791 Coll. PRL86('01)770 is in disagreement with the Watson theorem for FSI.

Ø Scalar form factors with $I=0,1$ and $\frac{1}{2}$ to deliver a direct calculation of the masses of the lightest quarks, u, d, s from QCD sum rules, check of quark mass ratios from CHPT.

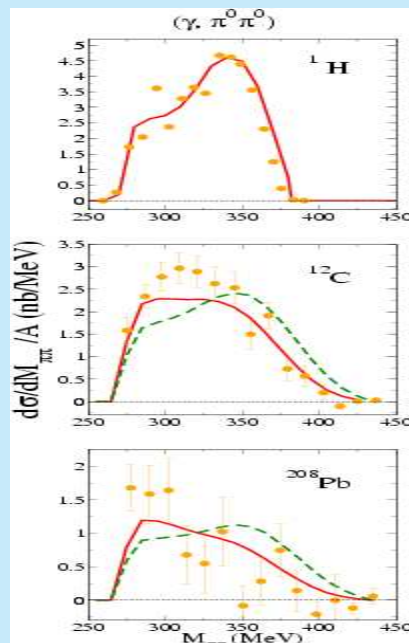
Ø Sizes of the scalar resonances (since we conclude that they are of dynamical origin some of them can be much larger than the typical hadronic size). We also determine the dependence on the quark masses of the masses of the lightest scalar resonances (sigma terms).

Ø Study the vacuum quantum numbers region around 1.4 GeV, focusing on the $f_0(980)$, $f_0(1500)$, $f_0(1370)$ to discriminate possible glueball. For that it is essential to properly handle $\sigma \sigma$ scattering.

UCHPT in the Nuclear Medium

It is still not systematic since there is no counting to apply to the nuclear corrections.

In terms of including one or the other or both mechanisms Oset et al. have performed a sound work e.g. on kaonic atoms (kaon self energy in the nuclear medium) Ramos, Oset NPA671('00)481; Hirenzaki, Okumura, Toki, Oset, Ramos PRC61('00)055205 or on the change of the properties of resonances in the medium Roca, Oset, Vicente Vacas PLB541('02)77 ($\gamma, \pi\pi$) in nuclei (σ). Their prediction was confirmed rather accurately by experiment at Mainz PRL 89('02)222302:



However, see also U. Mosel's talk on Wednesday after lunch session, [nucl-th/0401042](#)

UCHPT in the Nuclear Medium

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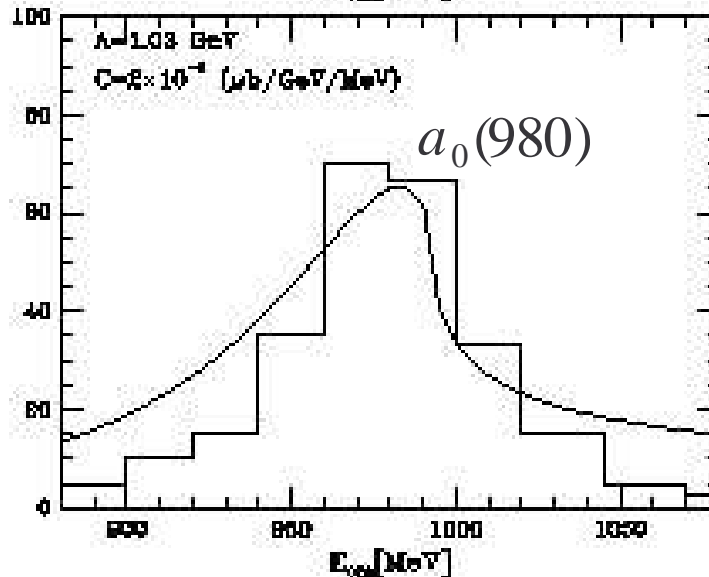
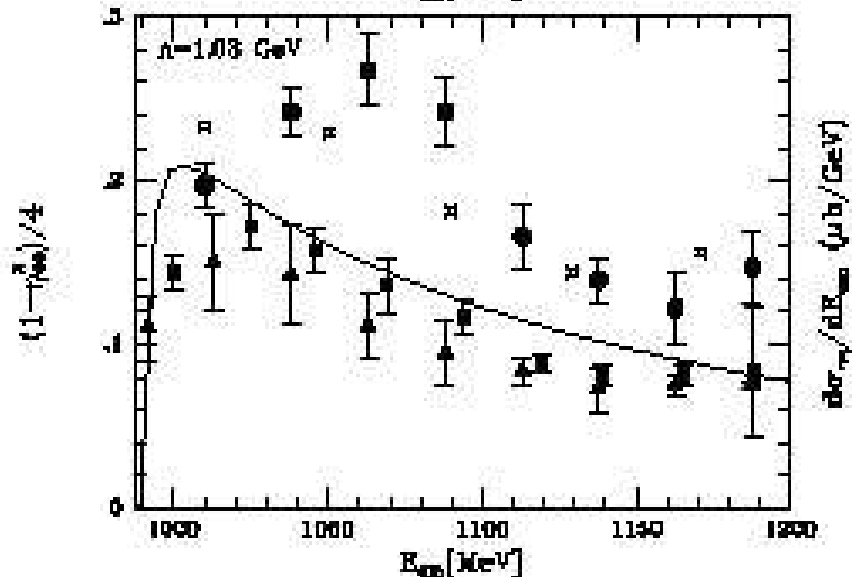
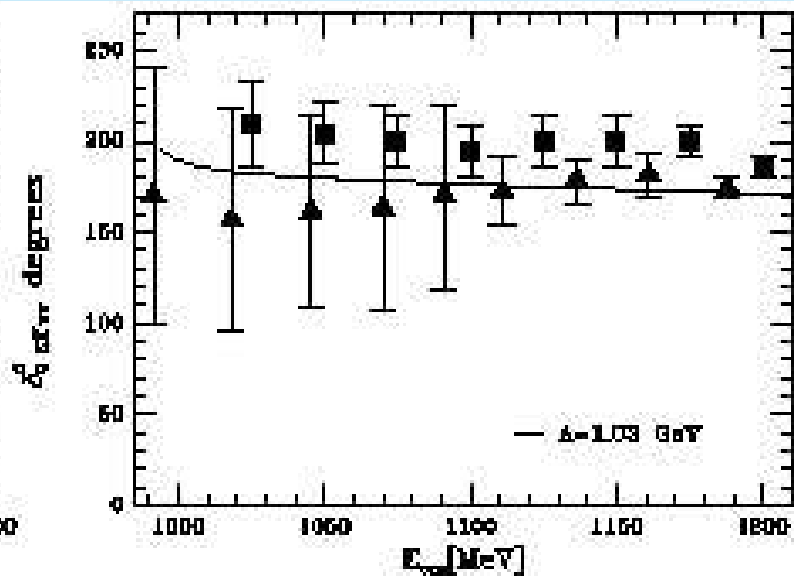
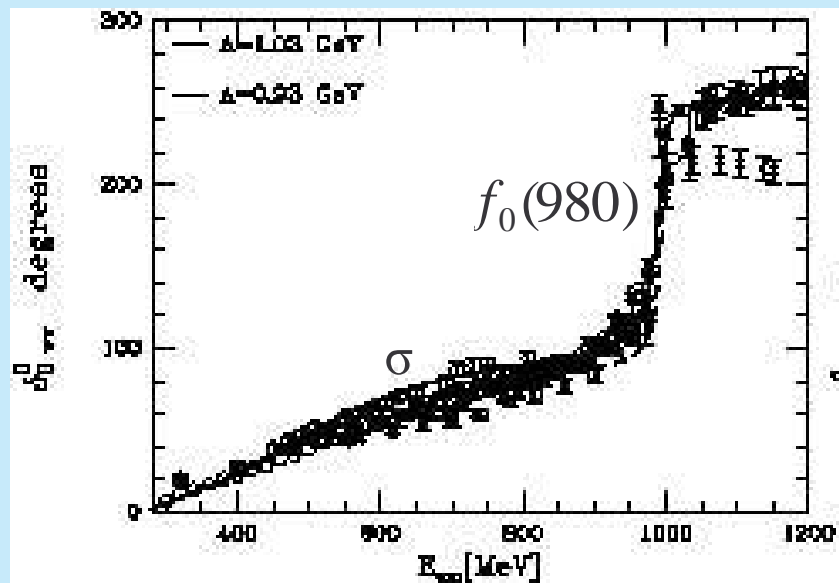
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Work is being developed to include in the medium NN interactions in vacuum from UCHPT and establish a chiral power counting in nuclear matter including then 'short range' interactions (NN contact interactions) and extending the results of J.A.O. PRC65('02)025204 ; Meißner, Wirzba, J.A.O. Ann.Phys.297('02)27 where a chiral counting was established but only with long range interactions (mediated by pions)

Summary

- **Chiral Unitary Approach:**

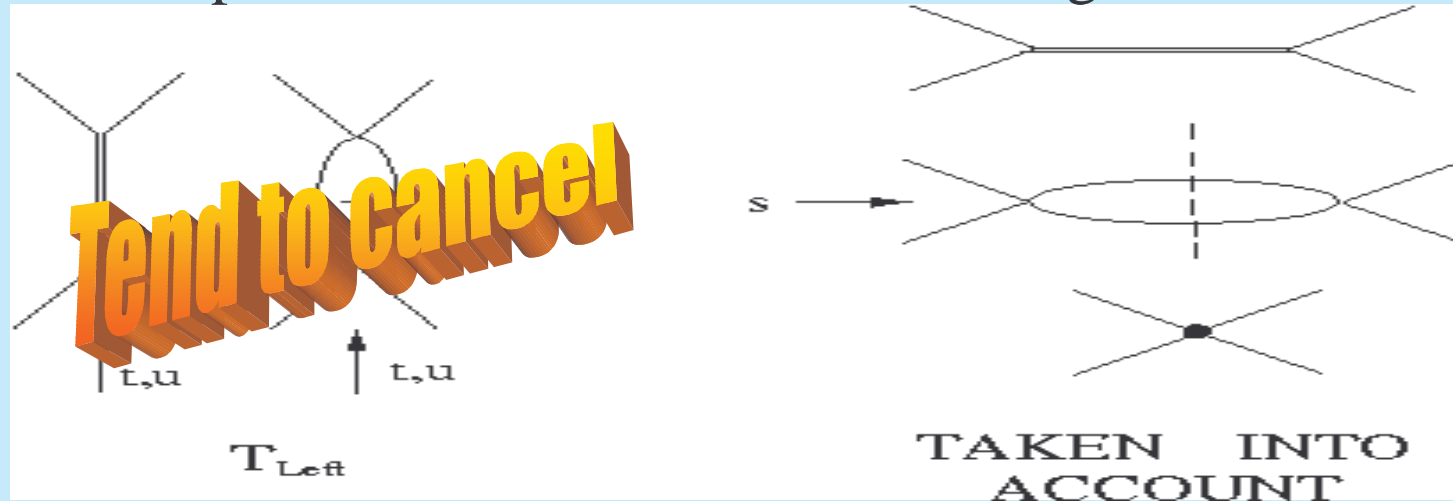
- Systematic and versatile scheme to treat self-strongly interacting channels (Meson-Meson Scattering, Meson-Baryon Scattering and Nucleon-Nucleon scattering), through the chiral (or other appropriate EFT) expansion of an interaction kernel R .
- Based on **Effective Field Theory, Analyticity and Unitarity**.
- The same **scheme is amenable** to correct Production Processes from FSI
- It treats both resonant (**preexisting/dynamically generated**) and background contributions.
- Amenable to be applied in a nuclear environment.
- It can also be extended to higher energies to fit data in terms of Chiral Lagrangians and, e.g., to provide phenomenological spectral function for QCD sum rules.



In Oset, J.A.O PRD60,074023(99) we studied the $I=0, 1, 1/2$ S-waves. The input included leading order CHPT plus Resonances:

1. **Cancellation** between the crossed channel loops and crossed channel resonance exchanges. (**Large N_c violation**).

The loops were taken from next to leading CHPT for the estimation.



2. Dynamically generated resonances ($M \sim N_c^{1/2}$). σ , $f_0(980)$, $a_0(980)$, $\kappa(700)$

The tree level or preexisting resonances move higher in energy (octet around 1.4 GeV). Pole positions were very stable under the improvement of the kernel R (convergence).

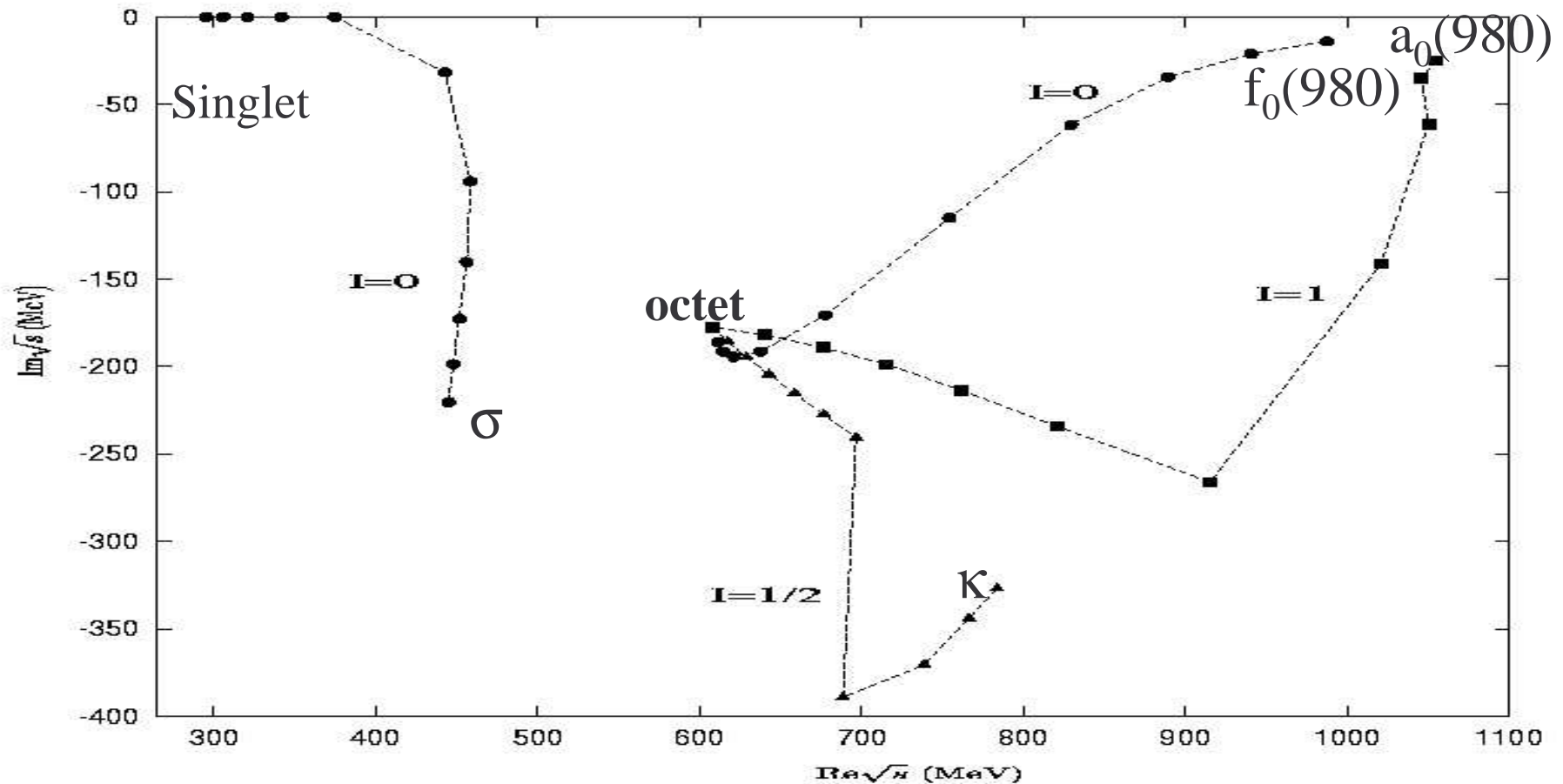
3. In the **SU(3) limit** we have a degenerate octet plus a singlet of dynamically generated resonances

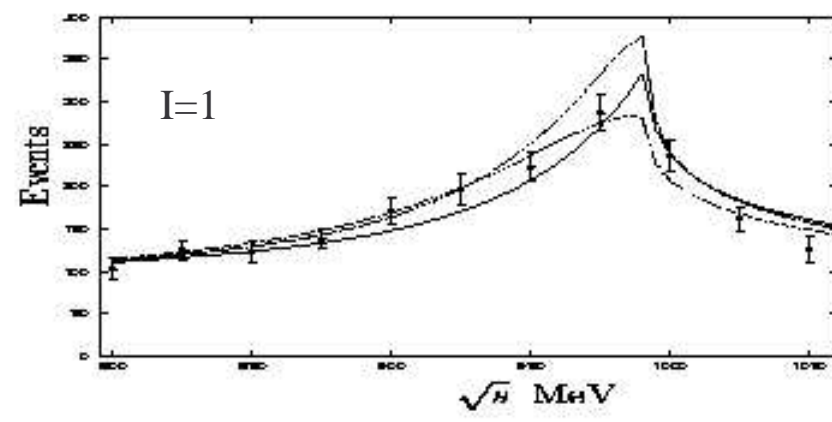
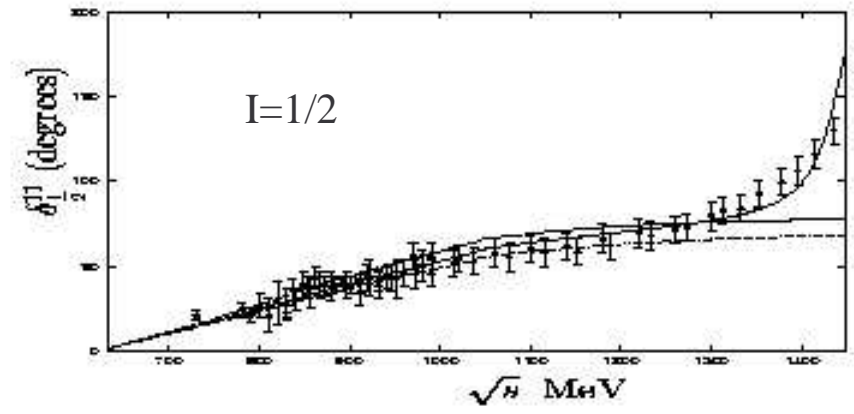
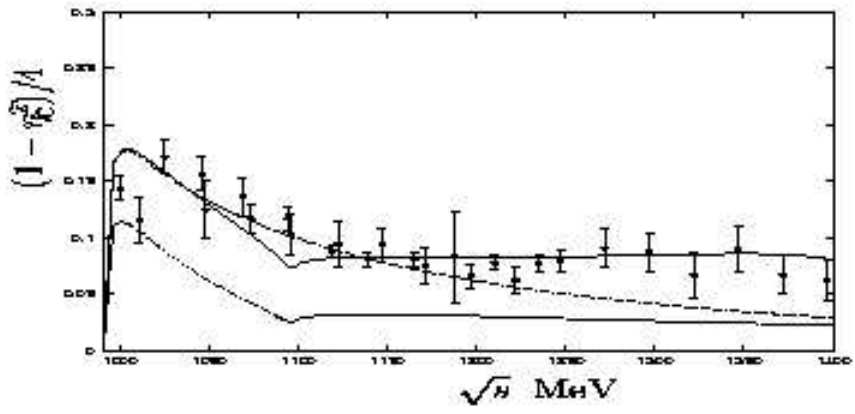
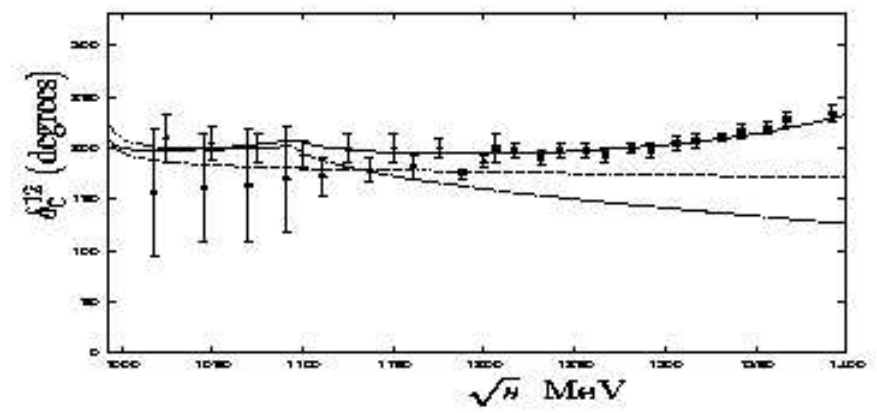
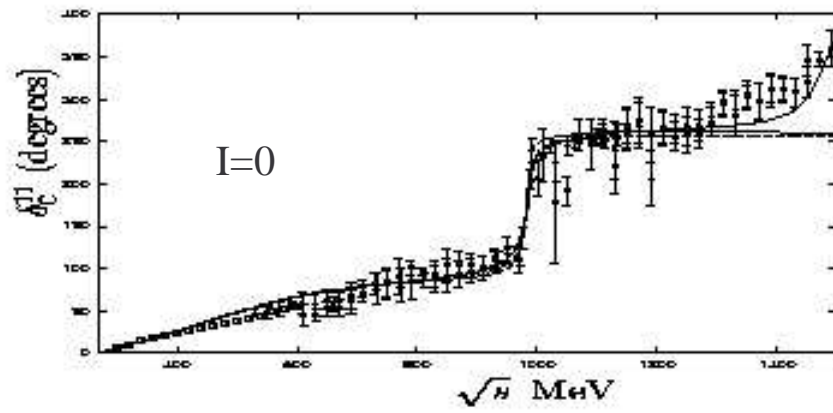
In [J.A.O. hep-ph/0306031](#) (to be published in NPA) a SU(3) analysis of the couplings constants of the $f_0(980)$, $a_0(980)$, $\kappa(900)$, $f_0(600)$ and κ was done.

$\lambda = 0$ physical limit

$m_0 = 300 \text{ MeV}$ $\lambda = 1$ SU(3) Symmetric point **SOFT EVOLUTION**

$$m_\pi(\lambda) = m_\pi + \lambda(m_0 - m_\pi); m_K(\lambda) = m_K + \lambda(m_0 - m_K); m_\eta(\lambda) = m_\eta + \lambda(m_0 - m_\eta)$$





Spectroscopy: Dynamically generated resonances.

σ	$0.445 - i 0.220$ $ g_{\pi\pi} = 3.01$ $ g_{K\bar{K}} = 1.09$ $ g_{\eta_8\eta_8} = 0.09$	$0.443 - i 0.213$ $ g_{\pi\pi} = 2.94$ $ g_{K\bar{K}} = 1.30$ $ g_{\eta_8\eta_8} = 0.04$	$0.442 - i 0.214$ $ g_{\pi\pi} = 2.95$ $ g_{K\bar{K}} = 1.34$
$f_0(980)$	$0.988 - i 0.014$ $ g_{\pi\pi} = 1.33$ $ g_{K\bar{K}} = 3.63$ $ g_{\eta_8\eta_8} = 2.85$	$0.983 - i 0.007$ $ g_{\pi\pi} = 0.89$ $ g_{K\bar{K}} = 3.59$ $ g_{\eta_8\eta_8} = 2.61$	$0.987 - i 0.011$ $ g_{\pi\pi} = 1.18$ $ g_{K\bar{K}} = 3.83$
$a_0(980)$	$1.055 - i 0.025$ $ g_{\pi\eta_8} = 3.88$ $ g_{K\bar{K}} = 5.50$	$1.032 - i 0.042$ $ g_{\pi\eta_8} = 3.67$ $ g_{K\bar{K}} = 5.39$	$1.030 - i 0.086$ $ g_{\pi\eta_8} = 4.08$ $ g_{K\bar{K}} = 5.60$
κ	$0.784 - i 0.327$ $ g_{K\pi} = 5.02$ $ g_{K\eta_8} = 3.10$	$0.804 - i 0.285$ $ g_{K\pi} = 4.93$ $ g_{K\eta_8} = 2.96$	$0.774 - i 0.338$ $ g_{K\pi} = 4.89$ $ g_{K\eta_8} = 3.00$

PLUS the values of the $\pi\pi$ AND $K\bar{K}$ scalar form factors in the $f_0(980)$ peak

Weighted Averages of the first and second SU(3)
Analysis (Final results):

$$\cos^2 \theta = 0.925 \pm 0.013$$

$$\theta = 15.9^\circ \pm 1.4^\circ$$

$$|g_8| = 8.6 \pm 0.5 \text{ GeV}$$

$$|g_1| = 3.7 \pm 0.5 \text{ GeV}$$

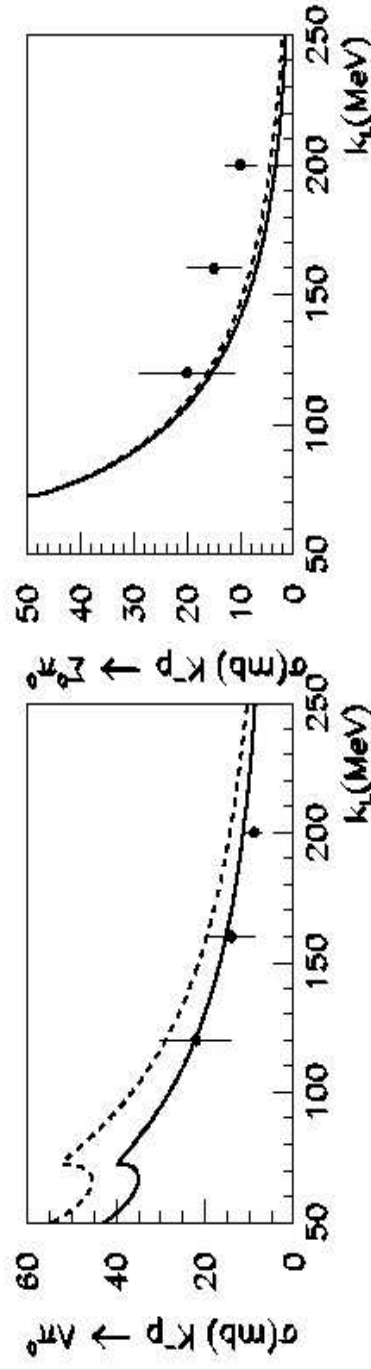
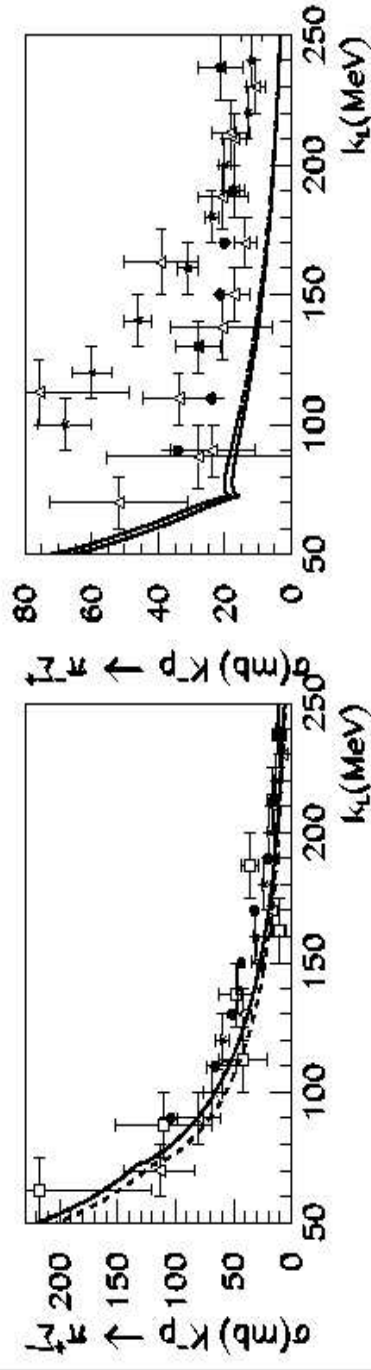
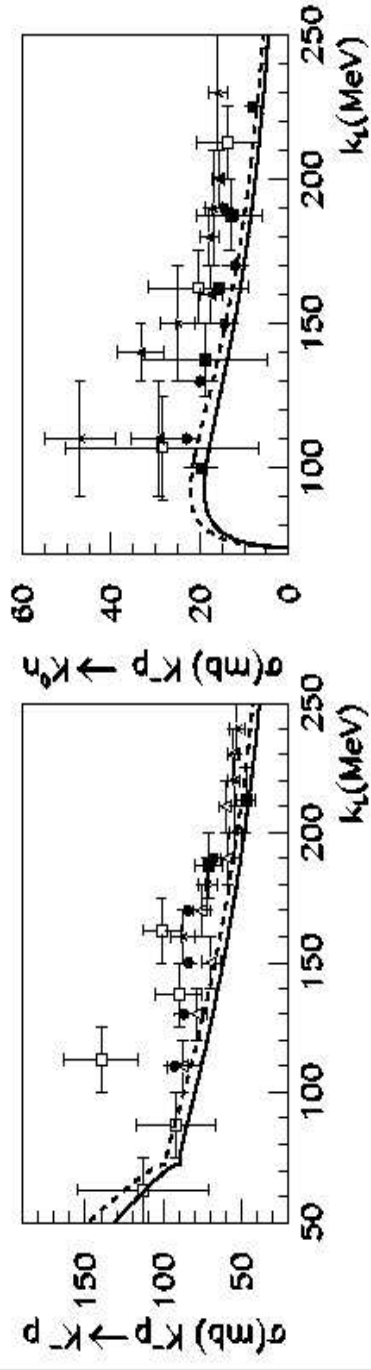
$$\sigma = \cos \theta S_1 + \sin \theta S_8$$

$$f_0 = -\sin \theta S_1 + \cos \theta S_8$$

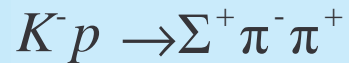
1. The σ is mainly the singlet state. The $f_0(980)$ is mainly the I=0 octet state. The $\kappa(700)$ the I=1/2 octet member and the $a_0(980)$ the isovector one.

2. Very similar to the mixing in the pseudoscalar nonet but **inverted**.

η Octet \longrightarrow σ Singlet ; η' Singlet \longrightarrow $f_0(980)$ Octet. (Anomaly)



πΣ Mass Distribution



Typically one takes: $\frac{dN_{\pi^- \Sigma^+}}{dE} = C |T_{\pi\Sigma \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$ As if the process were elastic

E.g: Dalitz, Deloff, JPG 17,289 ('91); Müller, Holinde, Speth NPA513,557('90), Kaiser, Siegel, Weise NPB594,325 ('95); Oset, Ramos NPA635, 99 ('89)

But the $\bar{K}N$ threshold is only 100 MeV above the $\pi\Sigma$ one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. The prescription is ambiguous, why not?

$$\frac{dN_{\pi^- \Sigma^+}}{dE} = C |T_{\bar{K}N \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$$

We follow the Production Process scheme previously shown:

$$F = (I + Rg)^{-1} \xi \quad \xi^T = (r_1, r_1, r_2, r_2, r_2, 0, 0, 0, 0, 0) \quad \text{I=0 Source}$$

$$\frac{r_1}{r_2} = 1.42 \quad r_1 = 0 \text{ (common approach)}$$

Our Results

$$\gamma = \frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)}{\Gamma(K^- p \rightarrow \pi^- \Sigma^+)} = 2.36 \pm 0.04$$

2.33

$$R_c = \frac{\Gamma(K^- p \rightarrow \text{Charged})}{\Gamma(K^- p \rightarrow \text{All})} = 0.664 \pm 0.011$$

0.645

$$R_n = \frac{\Gamma(K^- p \rightarrow \pi^0 \Lambda)}{\Gamma(K^- p \rightarrow \text{Neutral})} = 0.189 \pm 0.015$$

0.227

- **Scattering Lengths:** $a_0 = -0.58 + i1.19 \text{ fm}$
 $a_0 = -0.53 + i0.95 \text{ fm}$ Isospin Limit

Data:

Kaonic Hydrogen:

$$a_0 = (-0.78 \pm 0.15 \pm 0.03) + i(0.49 \pm 0.25 \pm 0.12) \text{ fm}$$

Isospin Scattering Lengths:

$$a_0 = (-0.68 \pm 0.10) + i(0.64 \pm 0.10) \text{ fm}$$