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Radiative ϕ Decays and Scalar Dynamics

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- **Introduction**
- **Chiral Unitary Approach**
- **Scalar Sector**
- **Φ Radiative Decays. FSI.**

1. Introduction

1) The mesonic scalar sector has the vacuum quantum numbers 0^{++}. Essencial for the study of Chiral Symmetry Breaking: Spontaneous and Explicit m_u, m_d, m_s .

2) In this sector the hadrons really interact strongly.

   1) Large unitarity loops.

   2) Channels coupled very strongly, e.g. $\pi\pi$- $K\bar{K}$, $\pi\eta$- $K, \bar{K}$ ...

   3) Dynamically generated resonances, ~~Breit-Wigner formulae~~, ~~VMD~~, ...

3) OZI rule has large corrections.

   1) No ideal mixing multiplets.

   2) ~~Simple quark model.~~

Points 2) and 3) imply large deviations with respect to Large Nc QCD.

4) A **precise knowledge** of the scalar interactions of the lightest hadronic thresholds, $\pi\pi$ and so on, is often required.

- Final State Interactions (FSI) in ε'/ε, Pich, Palante, Scimemi, Buras, Martinelli,...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)

5) The effective field theory of QCD at low energies is Chiral Perturbation Theory (CHPT).

This allows a systematic treatment of pion physics.

Nevertheless, the energy range of convergence is too small ($\sqrt{s} < 0.4$-0.5 GeV) to deal with resonance physics (intermmediate energies, $\sqrt{s} > 0.5$ GeV) and large rescattering effects like in 0^{++}.

Chiral Perturbation Theory

Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian — **Hilbert Space**
Physical States

u, d, s massless quarks — **Spontaneous Chiral Symmetry Breaking**

$\mathbf{SU(3)_L \otimes SU(3)_R \longrightarrow SU(3)_V}$

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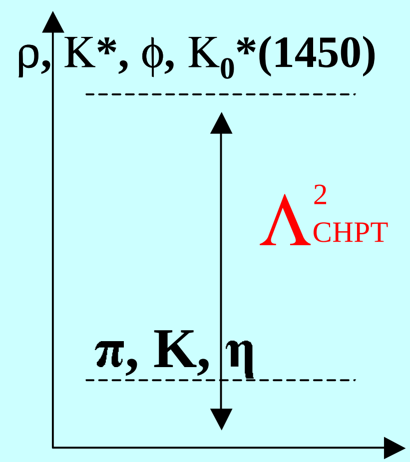
Goldstone Theorem

Octet of massles pseudoscalars
π, K, η
Energy gap



$m_q \neq 0$. Explicit breaking of Chiral Symmetry — **Non-zero masses**
$m_P^2 \propto m_q$

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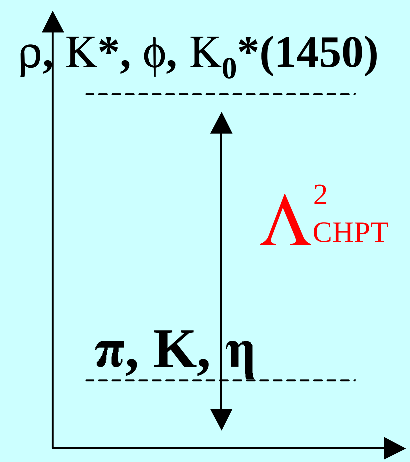

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Perturbative expansion in powers of the external four-momenta of the pseudo-Goldstone bosons over Λ^2_{CHPT}

$$L = L_2 + L_4 + \ldots$$

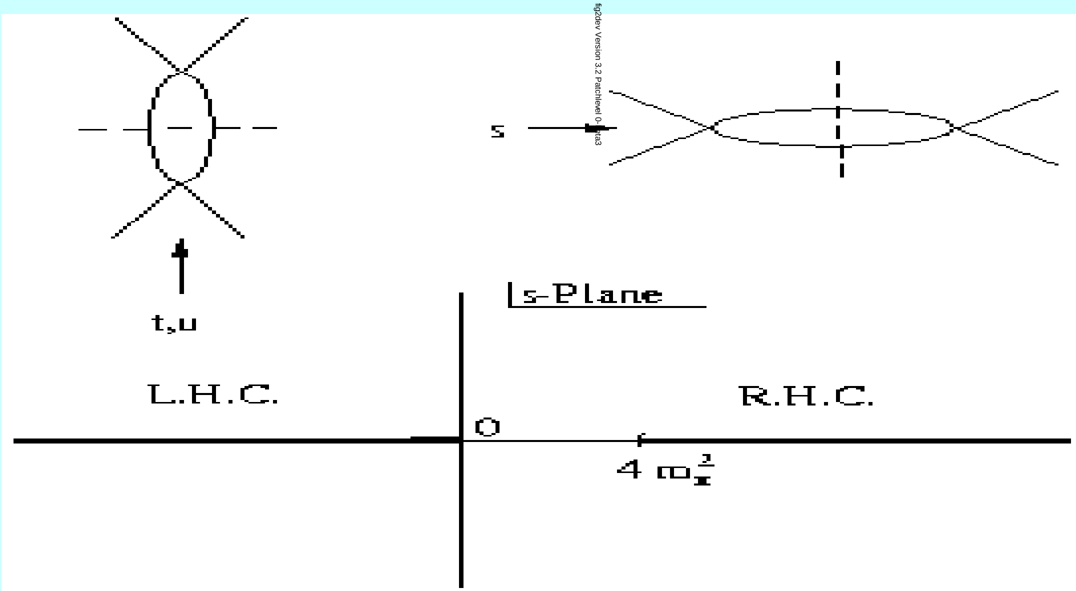
$$\frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda^2_{CHPT}}\right)$$

$$\Lambda_{CHPT} \approx 1\ \mathrm{GeV} \approx M_\rho \approx 4\pi f_\pi \approx 1\ \mathrm{GeV}$$

2. Chiral Unitary Approach

- To use the Chiral Lagrangians when going to higher energies.
- Resummation of CHPT:
  - To match with CHPT (0-, 1-, 2-loops...) at low energies.
  - Able to study Non-Perturbative physics, typical of intermmediate energies $\sqrt{s} \leq 2$ GeV :
    - Resonances.
    - Large unitarity loops.
    - Strong interacting coupled channels, etc.
- Connection with perturbative QCD, $\alpha_S\,(4\text{ GeV}^2)/\pi \approx 0.1$. (OPE). E.g. Imposing high energy QCD constraints, providing phenomenological functions for QCD Sum Rules, etc...

  S-wave $K\pi$ scattering in CHPT with resonances, NPB587,331(00)
  Strangeness-changing scalar form factors, NPB622,279(01)
  Light quark masses from scalar sum rules, EPJC(02) in press.
  By M. Jamin, J.A. Oller, A. Pich

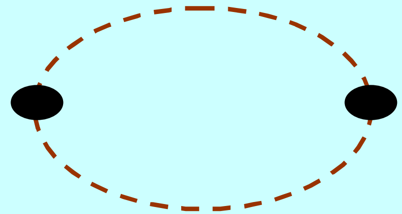


The right hand cut is resummed by considering a dispersion relation of the inverse of the amplitude

$$Im\, T(s)^{-1}_{ij} = -\delta_{ij} \frac{q_i}{8\pi\sqrt{s}} \theta(s - W_i^2)$$

$$T = [\tilde{T}^{-1} + g]^{-1}$$ We have to fix $\tilde{T}(s)$

The function g(s) is the unitarity loop g(s)=



$$g(s) = \frac{1}{4\pi^2}\left(a_{SL} + \sigma(s)\log\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)$$

$$\sigma(s) = \frac{2\,q}{\sqrt{s}}$$

$\tilde{T}(s)$ is fixed by matching with: CHPT or with CHPT+Resonances.

2. Scalar Sector

Leading Order : $\tilde{T}_2 = T_2$

$$T = \frac{1}{\frac{1}{\tilde{T}_2} + g} = \frac{T_2}{1 + T_2 g}$$

Oset, Oller, NPA620,438(97)

$a_{SL} \cong -0.5$ only free parameter, equivalently a three-momentum cut-off $\Lambda \cong 0.9$ GeV

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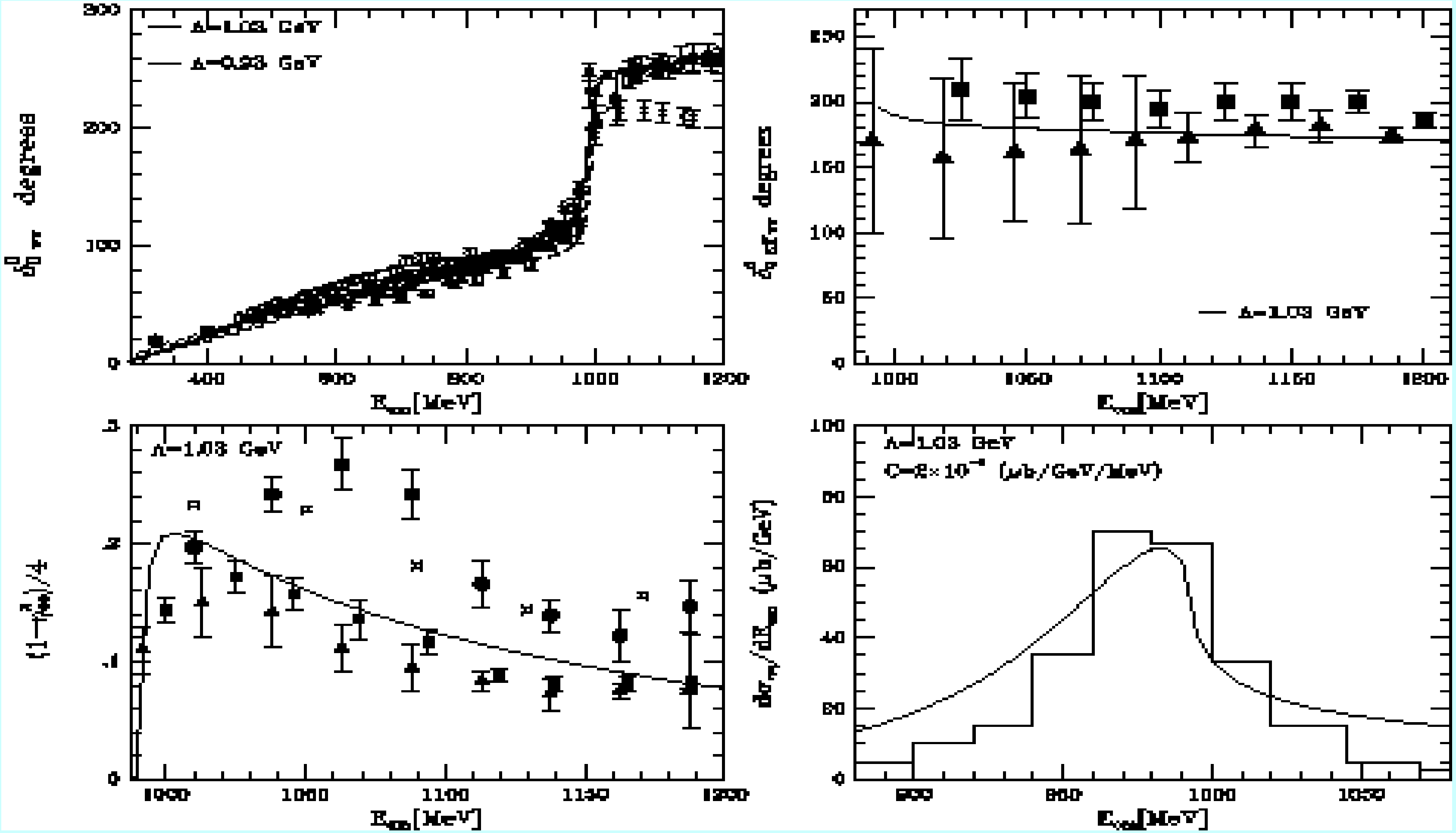


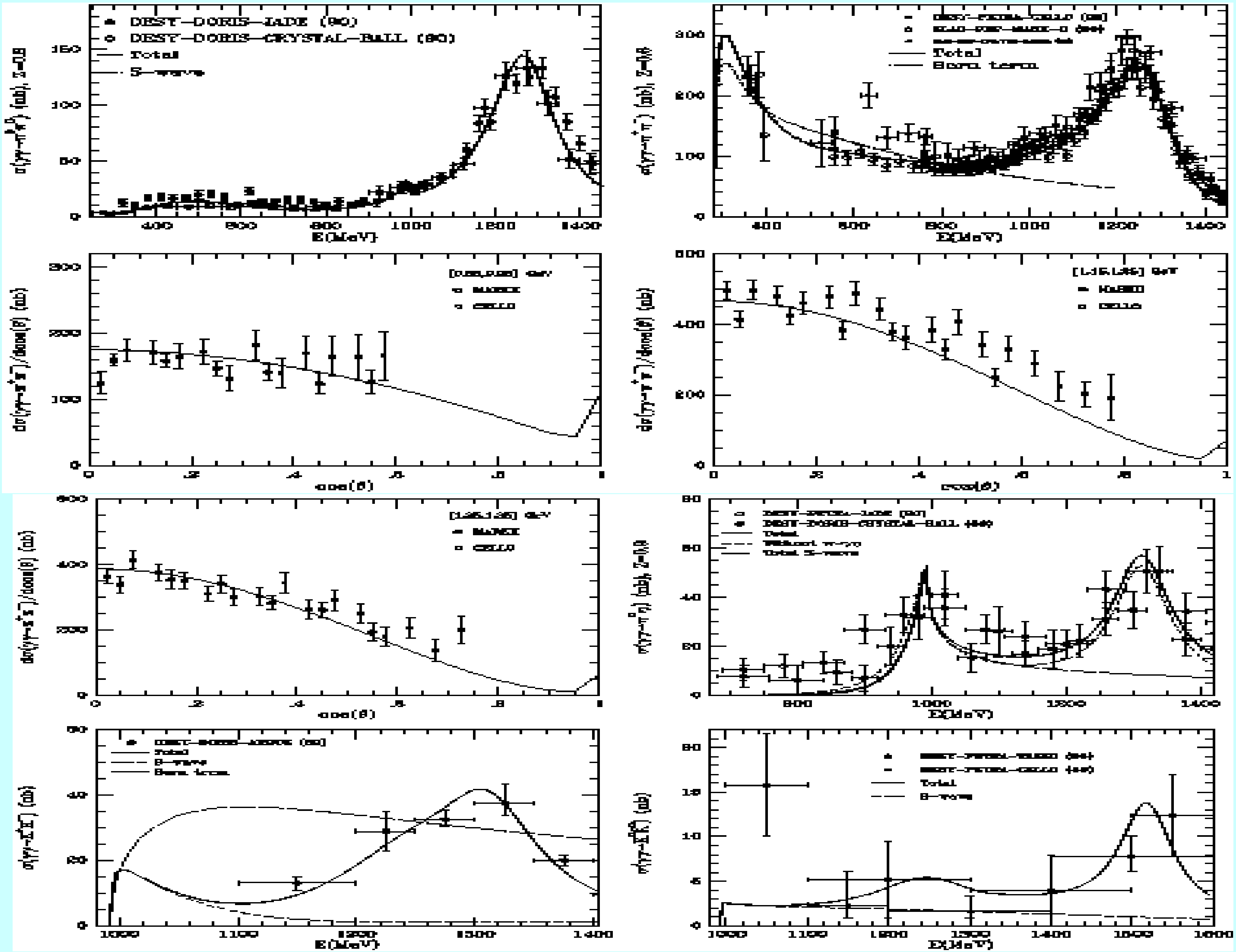
Pole positions and couplings

$f_0(980)$ (GeV)	$a_0(980)$ (GeV)
$0.993 - i\,0.012$	$1.009 - i\,0.056$
$\lvert g^f_{\pi\pi} \rvert = 1.90$	$\lvert g^a_{\pi\eta} \rvert = 3.54$
$\lvert g^f_{KK} \rvert = 3.80$	$\lvert g^a_{KK} \rvert = 5.20$

$Br(f_0(980) \to \pi\pi)=0.70$ **$Br(a_0(980) \to \pi\eta)=0.63$**

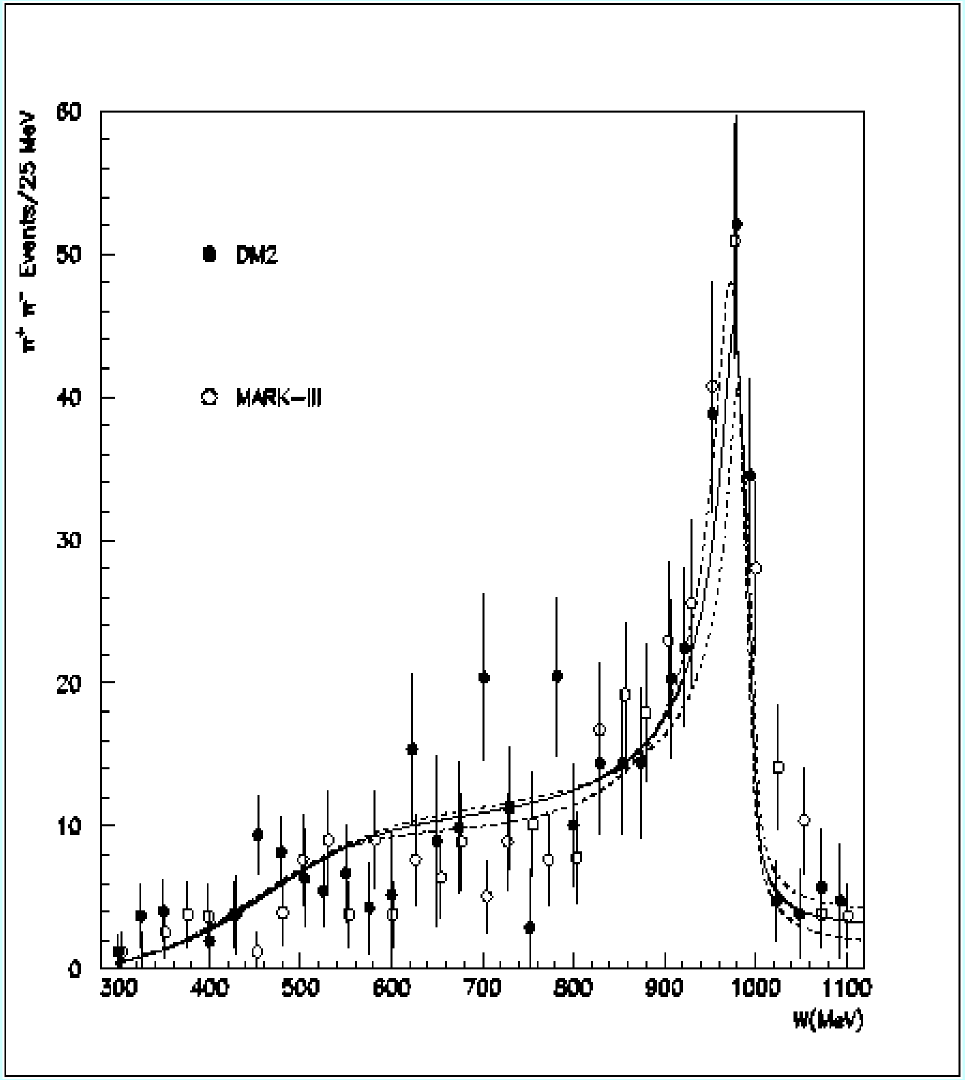
Using these T-matrices we also corrected by Final State Interactions the processes $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-, \pi^0\eta, K^+K^-, K^0\tilde{K}^0$ Where the input comes from CHPT at one loop, plus resonances. There were some couplings and counterterms but were taken from the literature. No fit parameters.

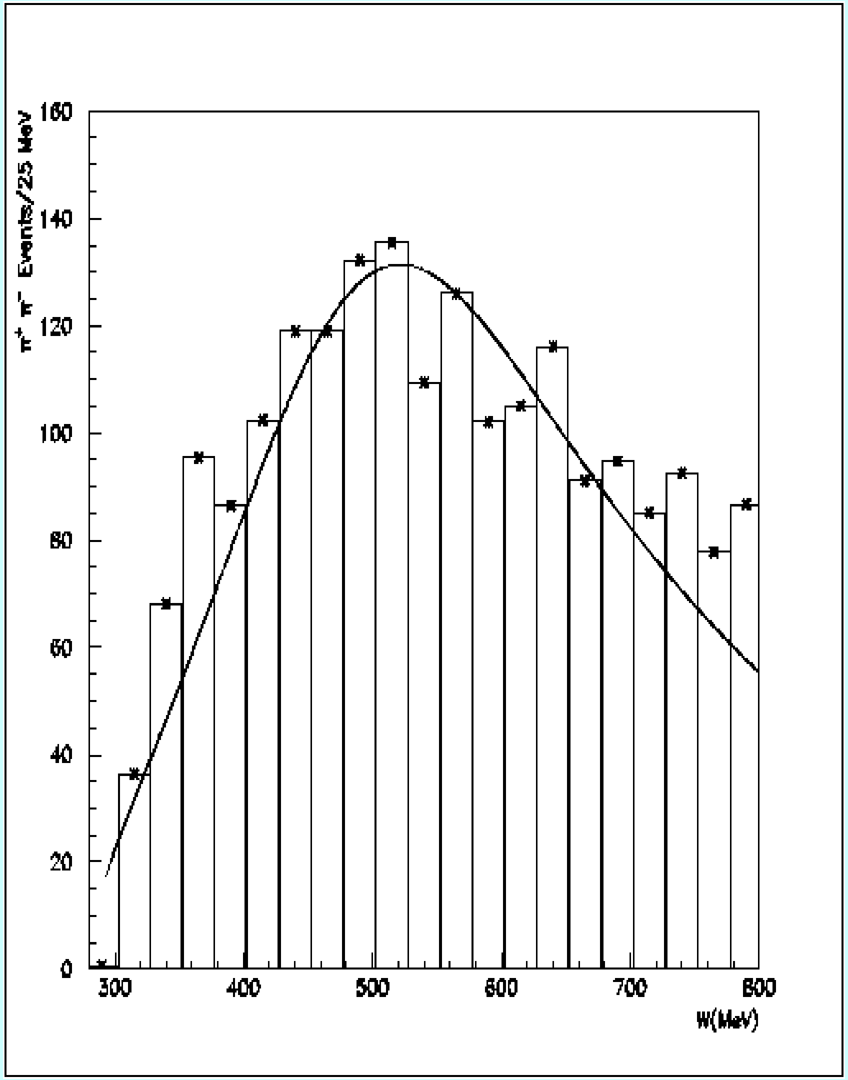
Oset, Oller NPA629,739(98).



DESY–DORIS–JADE (90)
DESY–DORIS–CRYSTAL–BALL (90)
Total
S–wave
E(MeV)
Total
Born term
E(MeV)
cos(θ)
cos(θ)
cos(θ)
E(MeV)
E(MeV)
E(MeV)

$J/\Psi \to \phi(\omega)\pi\pi, K\bar{K}$ Meissner, Oller NPA679,671(01).





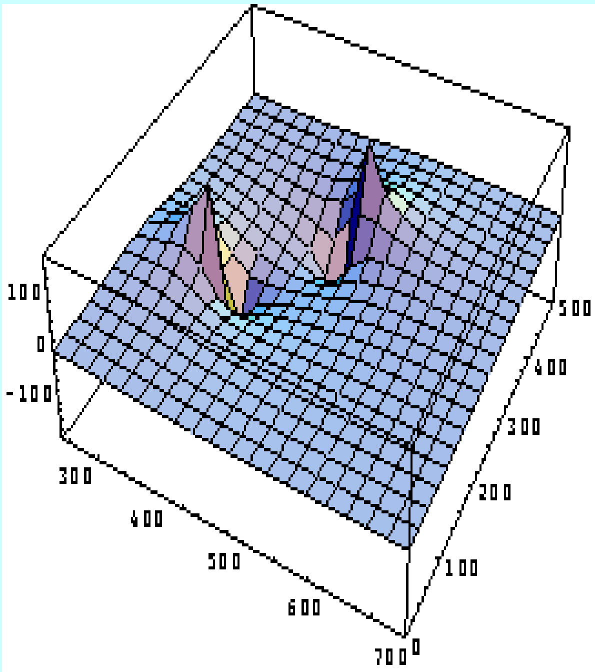
In Oset,Oller PRD60,074023(99) we studied the I=0,1,1/2 S-waves.

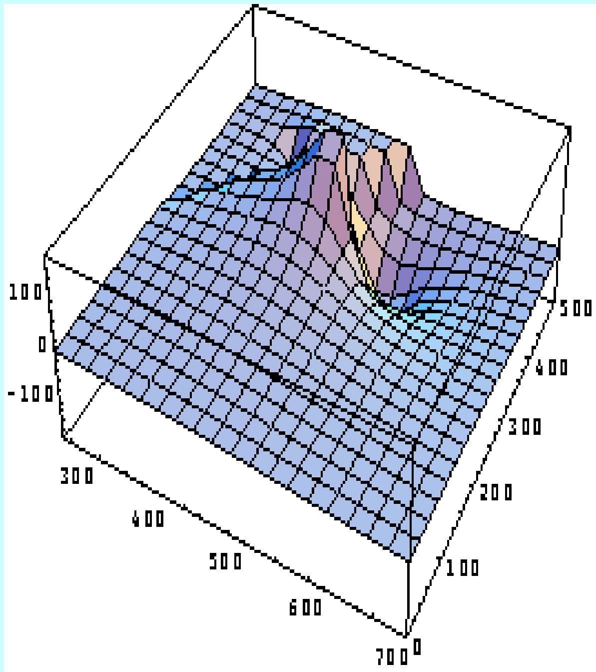
The input included next-to-leading order CHPT plus resonances:

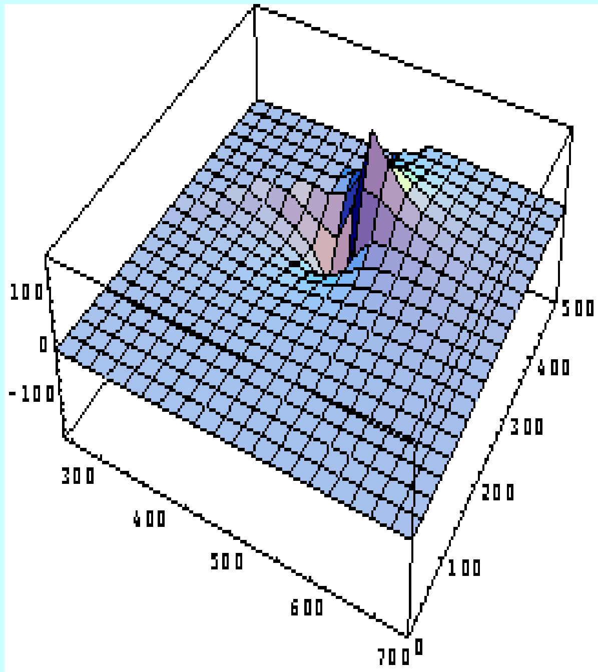
1. Cancellation between the crossed channel loops and crossed channel resonance exchanges. (Large Nc violation).
2. Dynamically generated renances. The tree level or preexisting resonance move higher in energy (octet around 1.4 GeV). Singlet contribution to the physical $f_0(980)$ appart of the main $K\bar{K}$ bound state contribution.

$$\sqrt{s_\kappa} = 0.78 - i0.33\ GeV;\ g^{\kappa}_{K\pi} = 4.99\ ,\ g^{\kappa}_{K\eta} = 2.97\ \ GeV$$

3. In the SU(3) limit we have a degenerate octet plus a singlet of dynamically generated resonances







First Conclusions.

- Dynamically generated nonet of scalar resonances σ, $f_0(980)$, $a_0(980)$, κ.
- The $f_0(980)$ has as well a preexisting singlet contribution.
- Main interacting Kernel: Lowest order CHPT.
- Resummation of the right hand cut.
- Perturbative effects of the left hand cut in the s-physical region.
- Octet of more standard resonances around 1.4-1.5 GeV, e.g. $K^0(1450)$

3. Φ Radiative Decays. FSI

One has to study FSI in production processes where these can modify the Born terms by orders of magnitude.

- $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-, \pi^0\eta, K^+K^-, K^0\overline{K}^0$ Oset,Oller NPA629,438(98)
- $J/\Psi \to \phi(\omega)\pi\pi, K\overline{K}$ Meissner, Oller NPA679,671(01).
- $\phi(1020) \to \gamma K^0\overline{K}^0, \gamma\pi^0\pi^0, \gamma\pi^0\eta$ Oller,PLB426,7(98) Marco,Hirenzaki,Oset,Toki, PLB470,20 (98) Oller, hep-ph/0205121

We now consider the $\phi(1020) \to \gamma f_0(980)$, $\gamma a_0(980)$ decays

$$\phi(1020) \to \gamma K^0\overline{K}^0, \gamma\pi^0\pi^0, \gamma\pi^0\eta$$

There are recent data from the:

- CMD-2 Collaboration PLB462,380(99)
- SND Collaboration PLB485,442(00),PLB479,53(00)
- High statistic data from the KLOE Collaboration are presented in PLB536(02)209 and hep-ex/0204012.

	KLOE 2002	SND 2000	CMD-2 1999
$Br(\phi\to\gamma f_0(980)) \cong 3\ Br(\phi\to\gamma\,\pi^0\pi^0)$ $1.09\pm0.03\pm0.05 10^{-4}$	$3.27\ 10^{-4}$	$3.5 \pm 0.3 \pm 1.3\ 10^{-4}$	$2.90\pm0.21\pm1.54\ 10^{-4}$
$Br(\phi\to\gamma a_0(980)) \cong Br(\phi\to\gamma\pi^0\eta)$	$0.74 \pm 0.07\ 10^{-4}$	$0.88 \pm0.17\ 10^{-4}$	

$Br(\phi\to\gamma f_0(980))= Br(\phi\to\gamma\,\pi^0\pi^0)/ Br(f_0(980)\to\pi^0\pi^0)= 3\ Br(\phi\to\gamma\,\pi^0\pi^0)$

taking $Br(f_0(980)\to\pi^0\pi^0)=1/3$ from two Pion threshold up to M_ϕ .

$$Br(\phi\to\gamma f_0(980))/ Br(\phi\to\gamma a_0(980)) = 4.1 \pm 0.2;\ \ 3.3 \pm 2.0$$

It has been claimed that large isospin violations are necessary to interpret the numbers above. F. Close, A. Kirk, PLB515,13(01); Black,Harada,Schechter,hep-ph/02022069.

Recent controversy Achasov,Kiselev, Phys. Lett. B534, 83 (02).

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We want to stress that:

1. One does not need to violate isospin symmetry in the couplings of the scalar resonances (R) to the kaons.
2. One has to treat carefully the finite widths of the $f_0(980)$ and $a_0(980)$ resonances because of:
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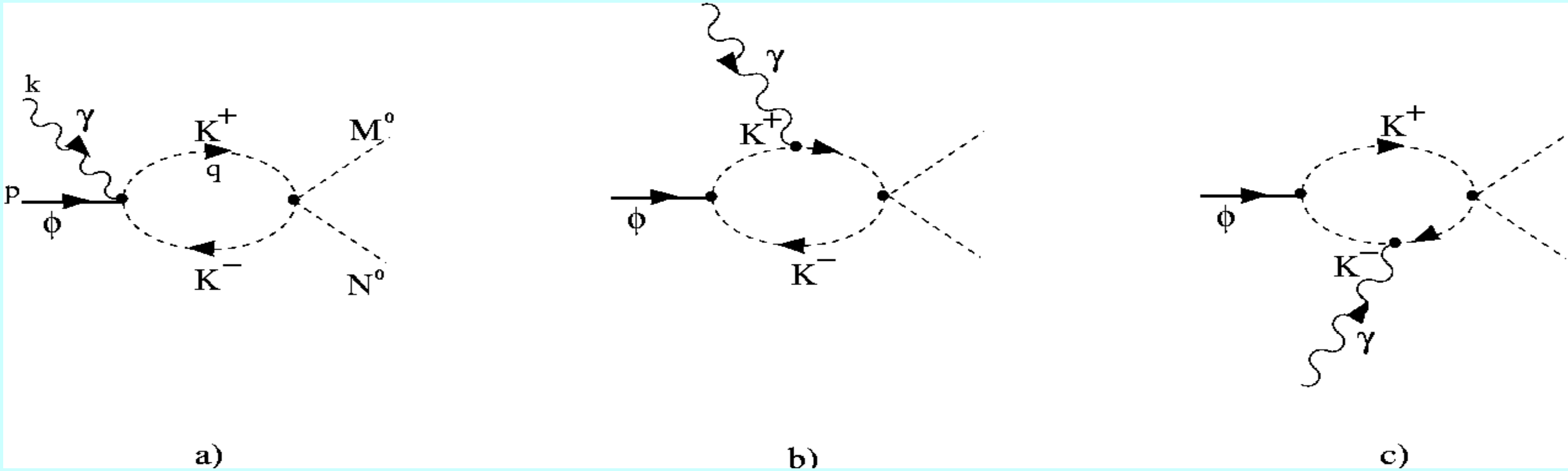
   - A change of 10 MeV from 986 MeV in the masses of the resonances gives rise to a factor 5 in the resulting value of $\Gamma(\phi \to \gamma R)$.

$\phi \to \gamma M^0 N^0$ Decays



For the contact term on the left vertex of the figure a) we have two contributions from the chiral Lagrangians with resonances of Gasser, Ecker, Pich, de Rafael, NPB 321, 311 (89).

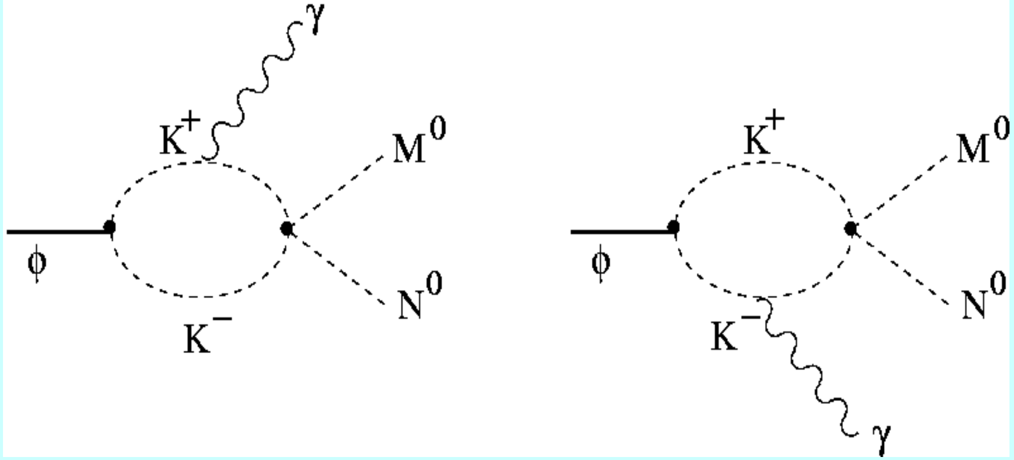
$$\frac{\sqrt{2}eM_\Phi G_V}{f^2}\varepsilon(\gamma)\cdot\varepsilon(\Phi) \quad \frac{\sqrt{2}e}{f^2 M_\Phi}\left(\frac{F_V}{2}-G_V\right)p_\alpha\,\varepsilon(\Phi)_\beta\left(k^\alpha\varepsilon(\gamma)^\beta - k^\beta\varepsilon(\gamma)^\alpha\right)$$

The blue one requires the Bremsstrahlung diagrams b) and c). Oller, PLB426,7(98)

The second one is gauge invariant by itself and requires its own treatment.

Vector Meson Dominance predicts $F_V/2 = G_V$ and vanishes.

$$\frac{\sqrt{2}eM_{\Phi}G_V}{f^2}\varepsilon(\gamma)\cdot\varepsilon(\Phi)$$



Standard treatments making use of Gauge invariance: Achasov, Ivachenko NPB315,465 (89);Lucio,Pestieau, Truong, Nussinov,Bramon,Grau,Panchieri,Close, Isgur,Kumano,Oller...

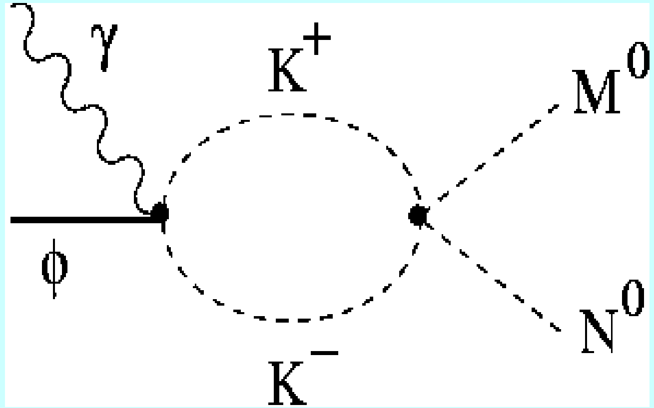
In Oller PLB426,7(98) the vertex of the right corresponds to the full strong S-wave T-matrices of Oset,Oller NPA620,438(97) taking into account the off-shell effects in the calculation of the loops.

By gauge invariance the $\phi\to\gamma M^0 N^0$ amplitude can be written as:

$$\mathbf{M}\left[\Phi\to\gamma(k)M^0(q_1)N^0(q_2)\right]=\mathbf{H}\left(Q^2,\ kq_1\right)[g^{\alpha\beta}(pk)-p^{\alpha}k^{\beta}]\varepsilon(\gamma)_{\alpha}\varepsilon(\Phi)_{\beta}$$

$$H=\frac{\sqrt{2}eM_{\Phi}G_V}{4\pi^2 f^2 m_{K^+}^2}\mathbf{I}(a,b)\,t_{K^+K^-\to M^0N^0}(Q^2) \qquad a=\frac{m_{K^+}^2}{M_{\Phi}^2}\ ;\ b=\frac{Q^2}{M_{\Phi}^2}$$

$$V_I(\phi\gamma K\bar{K}) = -\frac{\sqrt{2}e\zeta_I}{f^2 M_\Phi} p_\alpha \varepsilon(\Phi)_\beta (k^\alpha \varepsilon(\gamma)^\beta - k^\beta \varepsilon(\gamma)^\alpha)$$



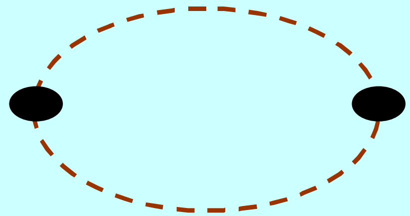
Pointed out by Marco,Hirenzaki,Oset,Toki PLB470,20,

we extend the analysis of this contribution.

The off-shell parts of the full S-wave T-matrix of the vertex on the right renormalizes the coupling as in the pure S-wave scattering analysed in Oset,Oller NPA620,438(97). ζ_I is a free parameters and reabsorbes this renormalization process.

Similar structures for $\phi\gamma\pi\pi$ and $\phi\gamma\pi\eta$ are not considered because the OZI rule.

We are left with a logarithmic divergence.
$G_I'(s)$ can differ in a subtraction constant from g(s) of strong interactions δG_I.



$T=[1+K\cdot g]^{-1}\cdot K$; $H'(Q^2)=[1+K\cdot g]^{-1}\cdot R$

K, R have no cuts.

Meissner,Oller NPA679, 671 (01)
Oset,Palomar,Oller, PRD63,114009 (01)

$\lambda_I \propto (0, \zeta_I)$, first channel $\pi\pi$(I=0) or $\pi\eta$(I=1), second $K\bar{K}$

$H'(Q^2)=\lambda - T\cdot G'\cdot\lambda=[1+K\cdot g]^{-1}(1-K\cdot\delta G)\cdot\lambda$; $G'=g+\delta G_I$

$$\Gamma(\Phi \to \gamma\pi^0\pi^0)=\int d\sqrt{Q^2}\,\frac{\alpha|k|^3|p_M|}{6\pi^2 f^4}\left|\left(\frac{M_\Phi G_V}{4\pi^2 f^4}\mathrm{I}(a,b)-\frac{\sqrt{2}\zeta_0}{M_\Phi}G'_0(Q^2)\right)t^0_{K^+K^-\to\pi^0\pi^0}\right|^2$$

$$\Gamma(\Phi \to \gamma\pi^0\eta)=\int d\sqrt{Q^2}\,\frac{\alpha|k|^3|p_M|}{6\pi^2 f^4}\left|\left(\frac{M_\Phi G_V}{4\pi^2 f^4}\mathrm{I}(a,b)-\frac{\sqrt{2}\zeta_1}{M_\Phi}G'_1(Q^2)\right)t^1_{K^+K^-\to\pi^0\eta}\right|^2$$

$$\Gamma(\Phi \to \gamma K^0\bar{K}^0)=\int d\sqrt{Q^2}\,\frac{\alpha|k|^3|p_M|}{6\pi^2 f^4}\left|\frac{\zeta_0-\zeta_1}{\sqrt{2}M_\Phi}+\sum_I\left(\frac{M_\Phi G_V}{4\pi^2 f^4}\mathrm{I}(a,b)-\frac{\sqrt{2}\zeta_I}{M_\Phi}G'_I(Q^2)\right)t^I_{K^+K^-\to\pi^0\eta}\right|^2$$

G_V=55 MeV from $\phi \to K^+ K^-$

This is our calculated Final State Contribution to $\phi\to\gamma\, \mathbf{M^0\, N^0}$ due to the S-wave meson-meson scattering.

We take the strong S-wave T-matrices from:
BS: E. Oset, J.A.O, NPA620,438 (97).
Dashed lines.
IAM: E. Oset, J.R. Peláez, J.A.O, PRD59,074001 (99). (Inverse Amplitude Method).
Solid lines.

For $\phi\to\gamma\, \pi^0\pi^0$ the background $\phi\to\pi^0\rho\to\gamma\pi^0\pi^0$ is not negligible, particularly for low energies. We have included its interference with the vector piece taken from Bramon,Escribano,Lucio, Napsuciale and Panchieri, hep-ph/0204339 (VMD) with out own scalar amplitudes.

BS: $\zeta_0=+164.12$ MeV, $\delta G_0=1.46/16\pi^2$

$\zeta_1=-165.87$ MeV, $\delta G_1=1.36/16\pi^2$

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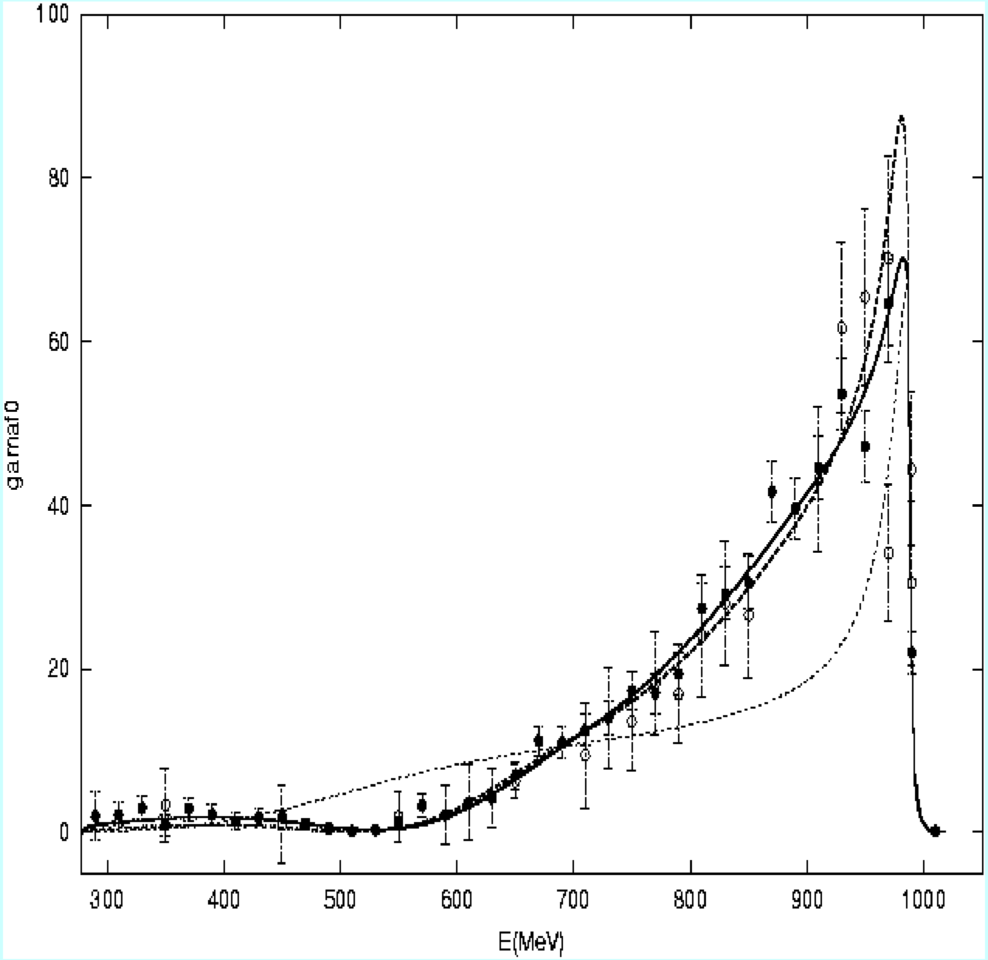

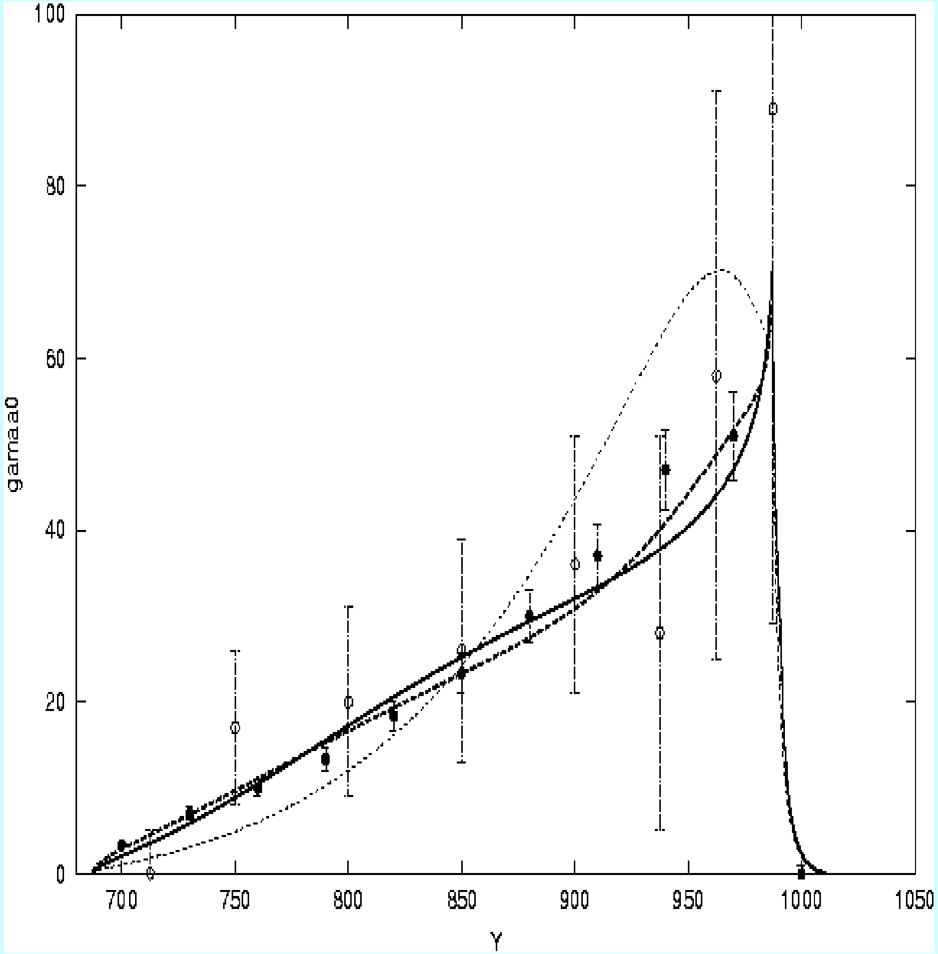

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$Br(\phi\to\gamma\pi^0\pi^0) = 1.09\ 10^{-4}$

Experiment: KLOE 1.09±0.03±0.05 10^{-4}

$Br(\phi\to\gamma\pi^0\eta) = 0.72\ 10^{-4}$

Experiment: KLOE 0.796 ± 0.07 10^{-4}

$$V(\Phi\gamma K^+K^-)\ \alpha\ -\frac{\zeta_1+\zeta_0}{\sqrt{2}} \cong 0$$

We do not expect any contribution from the K^+K^- physical state. When passing from the isospin basis to the one of physical states this occurs because: $G'_1\zeta_1 + G'_0\zeta_0 \cong 0$

Since $G'_1 = G'_0$ because $\delta G_1 = \delta G_0$

CONSISTENCY IN THE FITTED VALUES OF THE PARAMETERS WITH ISOSPIN SYMMETRY!!

$$V(\Phi\gamma K^0\bar{K}^0)\ \alpha\ \frac{\zeta_1-\zeta_0}{\sqrt{2}} \cong \sqrt{2}\zeta_1$$

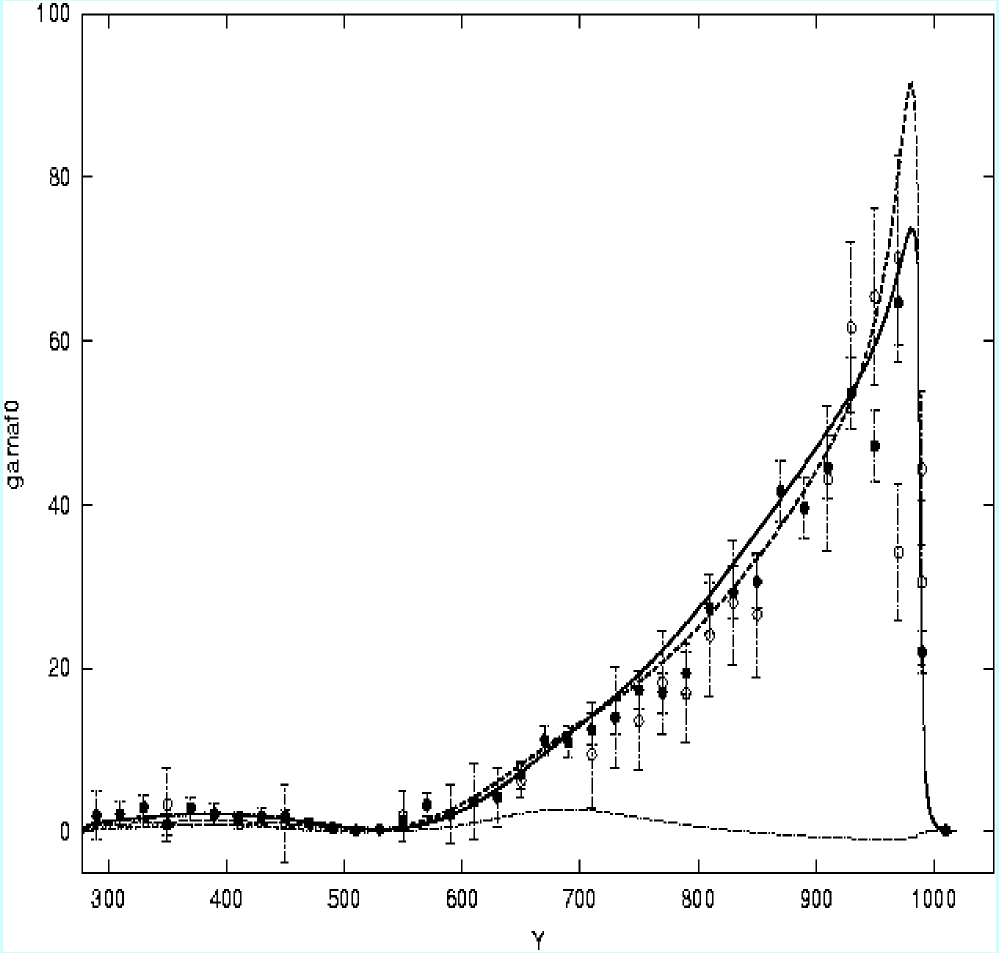
$$K^0\bar{K}^0 : G'_1\zeta_1 - G'_0\zeta_0 \cong 2G'_1\zeta_1$$

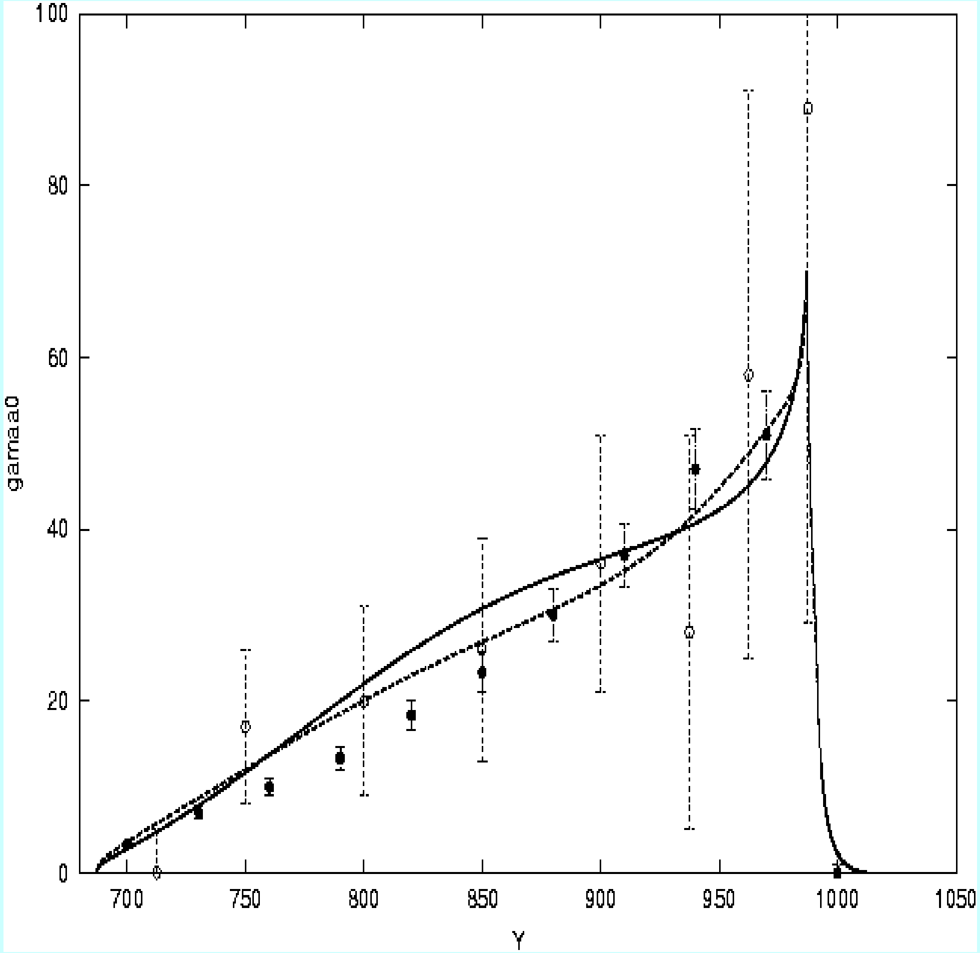
BS: $\zeta_0 = -\zeta_1 = +180.83$ MeV,

$\delta G_0 = \delta G_1 = 1.42/16\pi^2$.

IAM: $\zeta_0 = -\zeta_1 = +146.42$ MeV,

$\delta G_0 = \delta G_1 = 1.54/16\pi^2$.





For $\phi \to \gamma K^0 \overline{K}^0$ Br($\phi \to \gamma K^0 \overline{K}^0$) $\lesssim 10^{-6}$ to measure CP violation ϵ'/ϵ

We update the calculation of Oller,PLB426,7(98) . It was obtained 5 10^{-8} .
Now we obtain:
BS,   Br($\phi \to \gamma K^0 \overline{K}^0$) = 3.7 10^{-8}
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Finite width effects in $\Gamma(\phi \to \gamma R)$.

$$M(\Phi \to \gamma R(Q)) = \tilde{M} g^R_{K^+K^-} = \tilde{H}(Q^2)\, g^R_{K^+K^-}\, [g^{\alpha\beta} pk - p^\alpha k^\beta]\, \varepsilon(\gamma)_\alpha \varepsilon(\Phi)_\beta$$

$$\Gamma(\Phi \to \gamma R) = \frac{1}{3}\Sigma \int \frac{d^3k}{(2\pi)^3 2|k|} \int \frac{dQ^0}{2Q^0} f_R(Q^0) \frac{|\tilde{M} g_{K^+K^-}|^2}{2M_\Phi} (2\pi)\delta(M_\Phi - |k| - Q^0)$$

We introduce the energy distribution $f_R(Q^0)$

$f_R(Q^0) = \delta(Q^0 - \sqrt{\bar{Q}^2 + m_R^2})$ Standard two body decay formula with fixed R mass.

$$2\,\mathrm{Im}\left[t^R_{K^+K^-\to K^+K^-}(Q^2)\right] = \langle K^+K^-|T^+T|K^+K^-\rangle$$

Saturation by the exchange of the scalar resonance R

$$2\,\mathrm{Im}\left[t^R_{K^+K^-\to K^+K^-}(Q^2)\right] = \int \frac{d^4q}{(2\pi)^4 2q^0} f_R(q^0)(2\pi)^4\delta^4(Q-q)|g^R_{K^+K^-}|^2$$

$$= \frac{\pi}{Q^0} f_R(Q^0)\,|g^R_{K^+K^-}|^2$$

Three choices

$$\mathrm{Im}\left[t^R_{K^+K^-\to K^+K^-}(Q^2)\right]$$

from BS, IAM.

$$\mathrm{Im}\left[t^R_{K^+K^-\to K^+K^-}(Q^2)\right] = -\,\mathrm{Im}\left[\frac{(g^R_{K^+K^-})^2}{D_{R(Q^2)}}\right]$$

$$D_R(Q^2) = Q^2 - m_R^2 - \mathrm{Re}\left[\Pi(m_R^2)\right] + \Pi(Q^2)$$

Parameterization used in the experimental analyses of CMD-2, SND, KLOE collaborations from

Achasov, Ivanchenko NPB315,465(89)

$$\Gamma(\Phi\to\gamma R) = \int d|k| \frac{4\alpha|k|^3 M_\Phi}{3\pi f^4}\,\mathrm{Imag}\left[t^R_{K^+K^-}(Q^2)\right]\,\left|\left(\frac{M_\Phi G_V}{4\pi^2 m^2_{K^+}}\right)\mathrm{I}(a,b) - \frac{\sqrt{2}\zeta_I}{M_\Phi} G'_I(Q^2)\right|^2$$

$\phi \to \gamma R \to \gamma M^0 N^0$

Because of unitarity the prior expression fulfills, $\frac{d\Gamma(\Phi \to \gamma M^0 N^0)}{d|k|} = \frac{d\Gamma(\Phi \to \gamma R)}{d|k|} \mathrm{Br}(R \to M^0 N^0)$

Above $K\bar{K}$ threshold a very specific form of $\mathrm{Br}(R \to M^0 N^0)$ has to be used.

Calculated Branching Ratios

$\mathrm{Br}(\phi \to \gamma f_0(980)) = 3.15\ 10^{-4}$

$\mathrm{Br}(\phi \to \gamma a_0(980)) = 0.73\ 10^{-4}$

$$\frac{\mathrm{Br}(\phi \to \gamma f_0(980))}{\mathrm{Br}(\phi \to \gamma a_0(980))} = 4.32$$

We have also applied this equation with the experimental parameterization for $f_R(Q^0)$, with the free parameters (2)masses and (4)couplings, from the best fits of CMD-2 and SND.

$$\Gamma(\Phi \to \gamma R) = \int d|k| \frac{4\alpha |k|^3 M_\Phi}{3\pi f^4} \mathrm{Imag}\left[t^R_{K^+K^-}(Q^2)\right] \left| \left(\frac{M_\Phi G_V}{4\pi^2 m^2_{K^+}}\right) I(a,b) - \frac{\sqrt{2}\zeta_I}{M_\Phi} G'_I(Q^2) \right|^2$$

	KLOE 2002	SND 2000	CMD-2 1999
$Br(\phi \to \gamma f_0(980))$	$3.27\ 10^{-4}$ BS: $3.19\ 10^{-4}$ IAM: $3.11\ 10^{-4}$	$3.5 \pm 0.3 \pm 1.3\ 10^{-4}$	$2.90 \pm 0.21 \pm 1.54\ 10^{-4}$
$Br(\phi \to \gamma a_0(980))$	$0.74 \pm 0.07\ 10^{-4}$ BS: $0.73\ 10^{-4}$ IAM: $0.73\ 10^{-4}$	$0.88 \pm 0.17\ 10^{-4}$	

$Br(\phi \to \gamma f_0(980)) / Br(\phi \to \gamma a_0(980)) = 4.37, 4.26$: KLOE: 4.1 ± 0.2

$Br(\phi \to \gamma f_0(980)) = 3.15 \ 10^{-4}$

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	CMD-2 fit(1) $\gamma\pi^0\pi^0, \gamma\pi^+\pi^-$ $Br(\phi \to \gamma f_0(980))$ 10^4	CMD-2 fit(2) $\gamma\pi^0\pi^0$ $Br(\phi \to \gamma f_0(980))$ 10^4	SND $Br(\phi \to \gamma f_0(980))$ 10^4	SND $Br(\phi \to \gamma a_0(980))$ 10^4
reported	2.90 ±0.21 ±0.65	3.05 ±0.25 ±0.72	$4.6 \pm 0.3^{+1.3}_{-0.5}$	0.88 ±0.17
$\zeta_I=0$	3.21	3.51	4.8	0.96

4. Conclusions

- No large isospin breaking corrections. One can understand the experimental rates $Br(\phi \to \gamma f_0(980))$, $Br(\phi \to \gamma a_0(980))$, $Br(\phi \to \gamma \pi^0 \pi^0)$ and $Br(\phi \to \gamma \pi^0 \eta)$ without any isospin violation.

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- Kinematical isospin breaking, using the average kaon mass for calculating I(a,b) and G´(s) instead of the K^+ one, amounts just to around a 10% in $\Gamma(\phi\to\gamma f_0(980))$ and $\Gamma(\phi\to\gamma a_0(980))$. If the finite widht effects were not taken into account this would be around a 100%.