

# Chiral Perturbation Theory in the Nuclear Medium Including Short and Long Range Multi-Nucleon Interactions

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1. Introduction
2. Power Counting
3. Pion self energy
4. Conclusions

# 1. Introduction

EFT for nuclear matter with short range+pion mediated nucleon interactions is under development

For a recent review: Furnstahl, Rupad and Schaefer, arXiv:0801.0729

Several groups have already largely exploited meson-baryon interactions in nuclear matter considering chiral Lagrangians

Kaiser, Muehlbauer, Weise Eur. Phys. J. A31, 53 (2007), ETC

Doering, Oset, PRC77,024602 (2008), ETC

AND MANY OTHERS

Many important results and studies of nuclear processes have been accomplished.

Nonetheless, despite all the successes and so much worked deployed:

- i) It still lacks a power counting general enough to include both short- and long-range multi-nucleon interactions.
- ii) This power counting has to take into account three main problems:
  - 1) In the nuclear medium one could have any number of closed nucleon loops arranged in any way.
  - 2) It happens in many instances that nucleon propagators are enhanced and they do not scale as  $\mathcal{O}(p^{-1})$  but as  $\mathcal{O}(p^{-2})$ .
  - 3) The 2N, 3N, ... interactions must be taken into account simultaneously with their reproduction in vacuum.

The multi-nucleon forces are object nowadays of intense interest and study

Bogner, Furnstahl, Ramanan, Schwenk NPA773(2006)203

Bogner, Schwenk, Furnstahl, Nogga NPA763(2006)59

- iii) Meson-baryon interactions have to be included with the same requirement.

In JAO, Phys. Rev. C65, 025204 (2002); Meißner, Wirzba, JAO Ann. Phys. 297, 27(2002) the generating functional in the presence of external sources for a Fermi sea of nucleons interacting through pion exchanges was derived.

These papers will be referred as [1,2] in the following.

Two types of power counting were established:

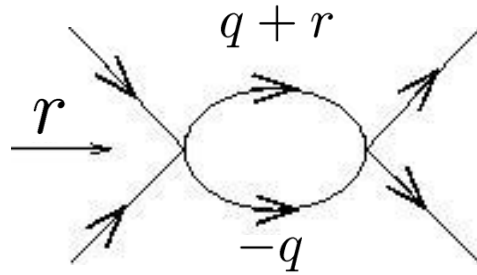
**Standard Case:**  $k_F \sim M_\pi \sim q_\pi \sim \mathcal{O}(p)$

Baryon Propagator:  $\frac{1}{q^0 - E(\vec{p} + \vec{q})} \sim \mathcal{O}(p^{-1})$  for  $q^0 \gg E(\vec{p} + \vec{q}) = \frac{(\vec{q} + \vec{p})^2}{2m}$

**Non-Standard Case:**  $k_F \sim M_\pi \sim q_\pi \sim \mathcal{O}(p)$

Baryon Propagator:  $\frac{1}{q^0 - E(\vec{p} + \vec{q})} \sim \mathcal{O}(p^{-2})$  for  $q^0 \simeq \mathcal{O}\left(\frac{k_F^2}{2m}\right)$

The non-standard case is the one that happens in *vacuum* nucleon-nucleon interactions for the two-nucleon reducible diagrams. S. Weinberg, Nucl. Phys. B363,3 (1991).

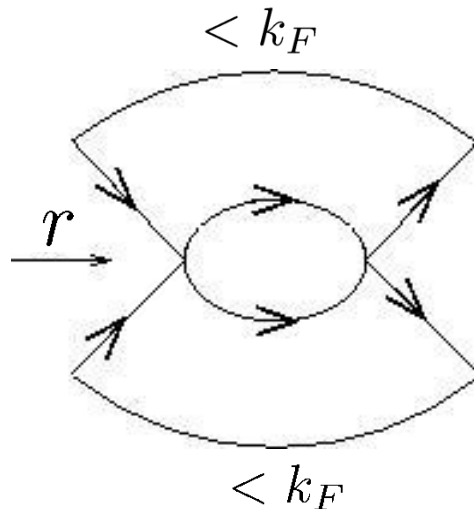


$$i \int \frac{dq^0}{2\pi} \frac{1}{q^0 + r^0 - E(\vec{q} + \vec{r}) + i\epsilon} \frac{1}{-q^0 - E(\vec{q}) + i\epsilon} \longrightarrow \frac{1}{r^0 - E(\vec{q} + \vec{r}) - E(\vec{q}) + i\epsilon}$$

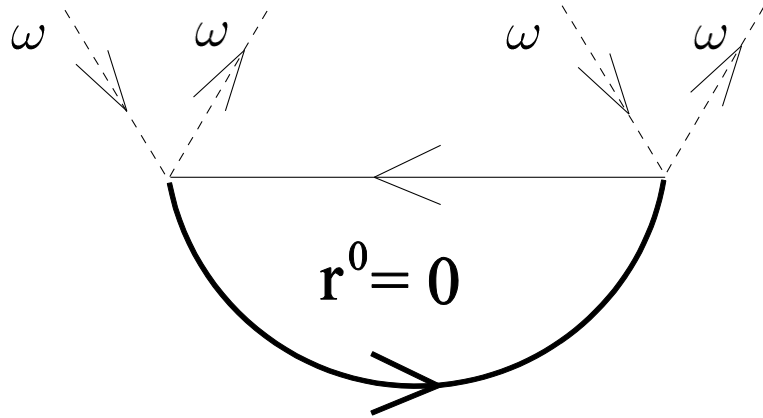
$$\sim \frac{1}{p} \frac{m}{p} \gg \mathcal{O}(p^{-1})$$

$$r^0 = W = E_1 + E_2$$

This of course has a reflection in the nuclear medium, just by closing the lines:



Another example, related to meson properties or meson-nucleus interactions:



$\pi\pi$  scattering in  
the nuclear-medium [2]

$$\frac{1}{E(\vec{k} + \vec{r}) - i\epsilon} \sim \frac{1}{p} \frac{m}{p} \gg \mathcal{O}(p^{-1})$$

One needs a power counting that takes into account the possible presence of such enhanced nucleon propagators which are responsible of non-perturbative effects, like the generation of the deuteron bound state.

# 2. Power Counting

We count a baryon propagator as  $\mathcal{O}(p^{-2})$

$$\frac{1}{E(\mathbf{k}+\mathbf{r})-i\epsilon} = \frac{2m}{(\mathbf{k}+\mathbf{r})^2-i\epsilon} \sim \mathcal{O}(p^{-2})$$

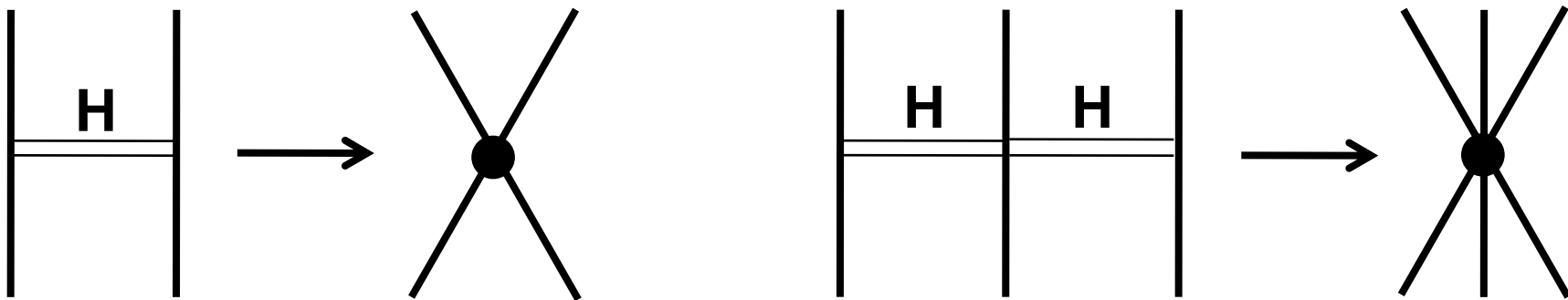
We do not err by ignoring diagrams that would be higher order in the conventional chiral counting.

Despite each baryon propagator is counted as  $\mathcal{O}(p^{-2})$  we show below that the chiral power counting is bounded from below.

In [1,2] only bilinear vertices in the nucleon fields were considered. We now take into account multi-nucleon interactions both local and of long-range.

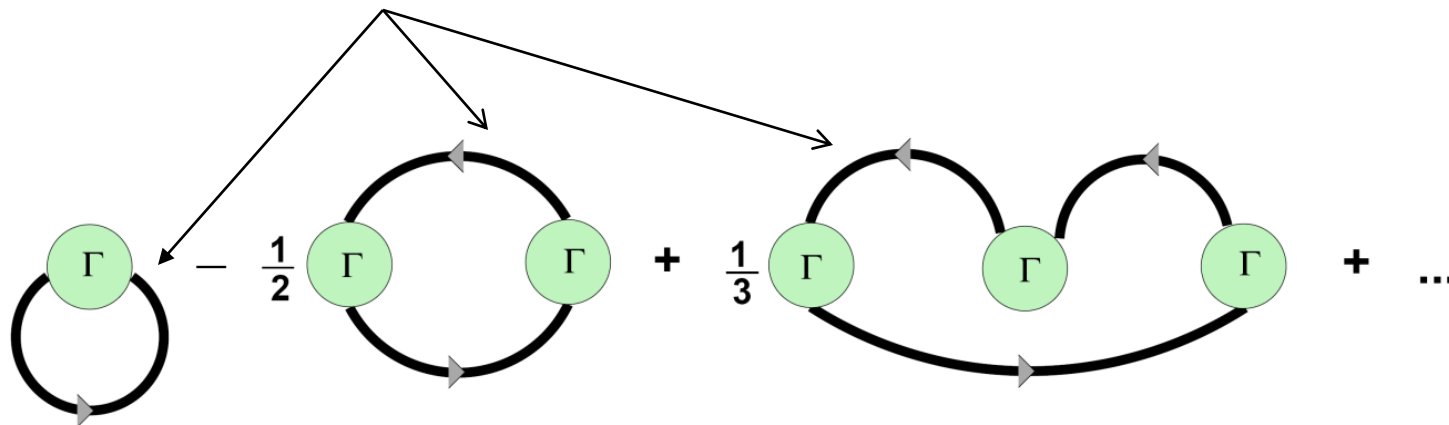
In order to make use of their results let us denote by  $\mathbf{H}$  the heavy mesons whose exchanges give rise to the local nucleon interactions, NN, NNN, ....

The propagator of the heavy mesons  $\mathbf{H}$  is counted as  $\mathcal{O}(p^0)$ .

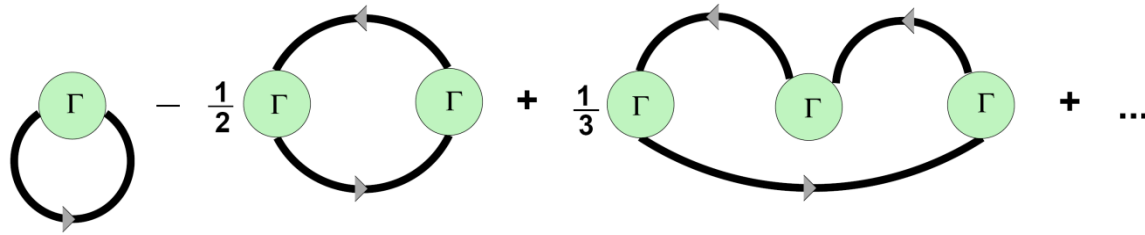


Now everything comes from bilinear vertices.

A useful concept introduced in [1,2] for handling bilinear vertices is that of an **In-medium generalized vertex**







$$\Gamma = -iA + (-iA) \overleftarrow{iD_0^{-1}} (-iA) + (-iA) \overleftarrow{iD_0^{-1}} (-iA) \overleftarrow{iD_0^{-1}} (-iA) + \dots$$

$D_0^{-1}$  is the free baryon propagator

The thick solid lines are Fermi sea insertions (summing over real nucleons in the Fermi sea)

$$\int \frac{d^3 k}{(2\pi)^3} \theta(k_F - k)$$

$-A$  is the interacting Lagrangian made of bilinear nucleon terms involving  $\mathbf{H}$ , pions and external sources

Order of a diagram:  $p^\nu$

$$\nu = 4L_H + 4L_\pi - 2I_\pi + \sum_{i=1}^{V_\rho} \left[ \sum_j d_j - 2m_i \right] + \sum_{i=1}^{V_\pi} \ell_i + 3V_\rho$$

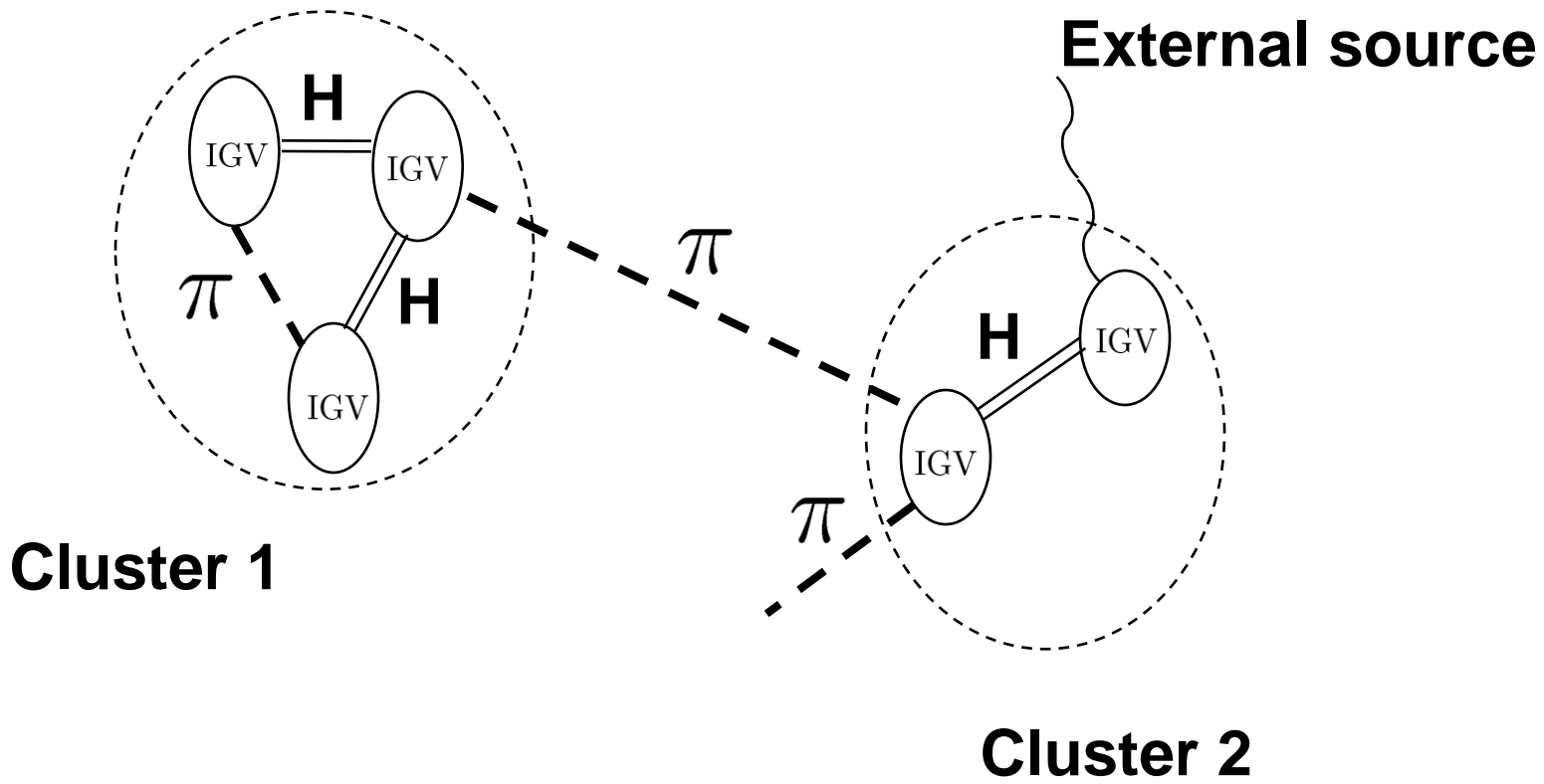
- $V_\rho$  Number of in-medium generalized vertices (IGV)
- $m_i$  Number of baryon propagators in the  $i_{th}$  IGV minus one
- $d_i$  Chiral order of the bilinear vertex
- $\ell_i$  Chiral order of mesonic vertices
- $V_\pi$  Number of pure mesonic vertices
- $L_\pi$  Number of pion loops
- $I_\pi$  Number of internal pion internal lines
- $L_H$  Number of **H** loops

The IGV can be joined by short range nucleon interactions (**H** exchanges) and by longer range interactions due to  $\pi$  exchanges.

We have clusters of IGV that are joined through **H** lines.

The number of these clusters is  $V_\Phi$ .

Inside each set or between them there could be pionic lines.



$$V_{\Phi} = 2$$

$$V_{\rho} = 5$$

$$I_H = 3$$

$$I_{\pi} = 2$$

$$E = 1$$

$$L_H = I_H - \sum_{i=1}^{V_\Phi} (V_{\rho,i} - 1) = I_H - V_\rho + V_\Phi$$

$$L_\pi = I_\pi - V_\pi - V_\Phi + 1$$

$$2I_H + 2I_\pi + E = \sum_{i=1}^V v_i + \sum_{i=1}^{V_\pi} n_i$$

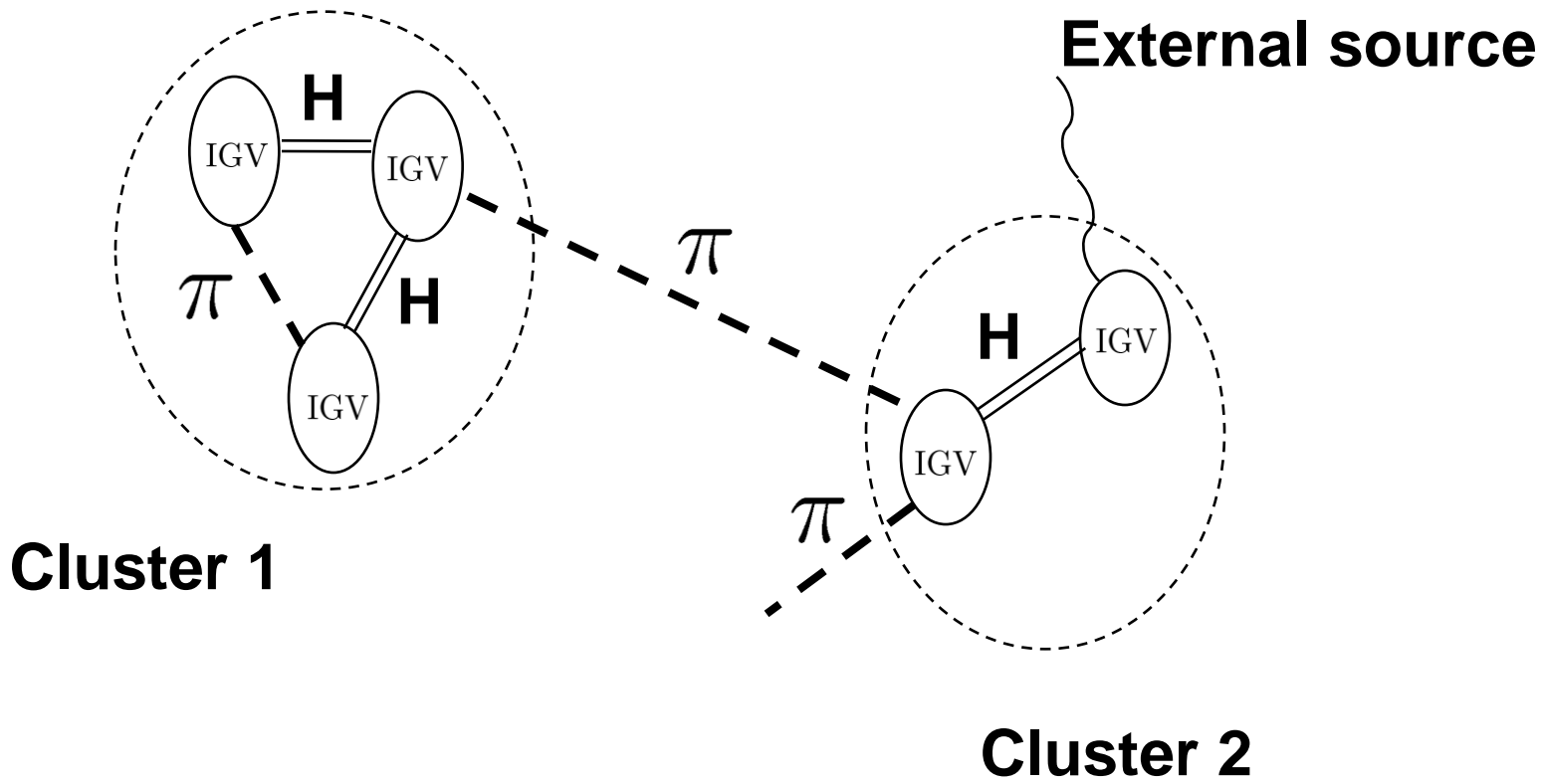
$E$ : external number of pion lines,  $V$ : the total number of bilinear vertices,  $v_i$ : number of meson lines attached to the  $i_{th}$  bilinear vertex,  $n_i$ : number of pions attached to the  $i_{th}$  mesonic vertex.

$$2I_H = \sum_{i=1}^V w_i, \quad w_i \text{ is the number of H lines in the } i_{th} \text{ bilinear vertex}$$

$$V = m + V_\rho, \quad m = \sum_{i=1}^{V_\rho} m_i$$

$$\nu = 4 - E + \sum_{i=1}^{V_\pi} (n_i + \ell_i - 4) + \sum_{i=1}^V (d_i + w_i - 1) + \sum_{i=1}^m (v_i - 1) + \sum_{i=1}^{V_\rho} v_i$$

**Let us show that  $\nu$  is bounded from below for a given process**



$$V_{\Phi} = 2$$

$$V_{\rho} = 5$$

$$I_H = 3$$

$$I_{\pi} = 2$$

$$E = 1$$

$$L_H = I_H - V_{\rho} + V_{\Phi} = 3 - 5 + 2 = 0$$

$$L_{\pi} = I_{\pi} - V_{\pi} - V_{\Phi} + 1 = 2 - 0 - 2 + 1 = 1$$

$$\nu = 4 - E + \sum_{i=1}^{V_\pi} (n_i + \ell_i - 4) + \sum_{i=1}^V (d_i + w_i - 1) + \sum_{i=1}^m (v_i - 1) + \sum_{i=1}^{V_\rho} v_i$$

- $n_i + \ell_i - 4 \geq 0$  since  $n_i \geq 2$  and  $\ell_i \geq 2$  (modulo external sources which there could be in a finite and fixed number)

- $d_i + w_i - 1 \geq 0$

For pion-nucleon vertices this is clear because  $d_i \geq 1$

For **H**-nucleon vertices  $d_i$  could be zero but  $w_i \geq 1$

- $v_i - 1 \geq 0$  Except:

i) Modulo external sources in meson-baryon vertices that there could be in a finite and fixed number, if any.

ii) Mass renormalization terms. But then  $d_i + v_i - 2 \geq 0$  and for  $d_i = 2$  they can be reabsorbed once for all in the physical nucleon mass.

- $\sum_{i=1}^{V_\rho} v_i$  implies that adding one IGV in a connected diagram rises the counting at least by ONE.

The ways of increasing the number of internal lines in a diagram without increasing the power  $\nu$ :

- Using the lowest order mesonic vertices with two pions,  $\ell_i = n_i = 2$
- Using the lowest order meson-baryon vertices with one pion,  $d_i = v_i = 1$
- Using the Heavy mesonic lines attached to a lowest order bilinear vertex,  $d_i = 0, w_i = 1$ .

It is not free to increase arbitrarily the number of  $2N, 3N, \dots$ , contact interactions because one increases the number of IGV, which then increase the counting by one unit each.

Procedure:

1. Increase step by step  $V_\rho$
2. For a given value of  $V_\rho$  determine those diagrams that do not increase further the order according to the previous rules.
3. In the end not all the baryon propagators are  $\mathcal{O}(p^{-2})$  but some are  $\mathcal{O}(p^{-1})$ . One can then save calculating some diagrams because finally they are higher order contributions.

$\nu$  is a lower bound of the actual power counting of a digram, denoted by  $\mu$ .

$$\mu \geq \nu$$

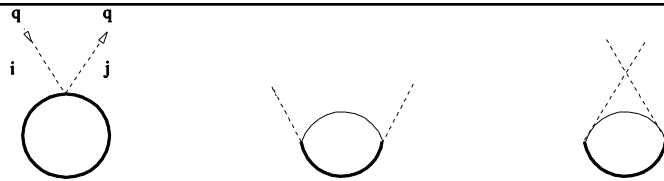
Maybe some nucleon propagators are  $\mathcal{O}(p^{-1})$ .

The important point is that we do not err by ignoring diagrams that are higher order in the conventional chiral counting. Their order is decreased because of the enhanced nucleon propagators and must be evaluated.

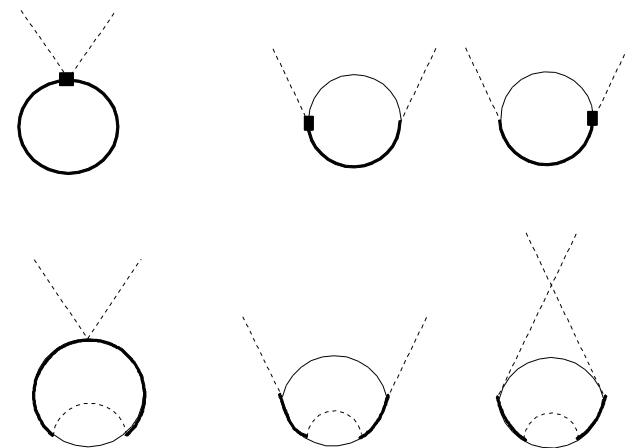


# 3. Pion Self-Energy

$V_\rho = 1$   
 $\mathcal{O}(p^4)$   
 Leading Order

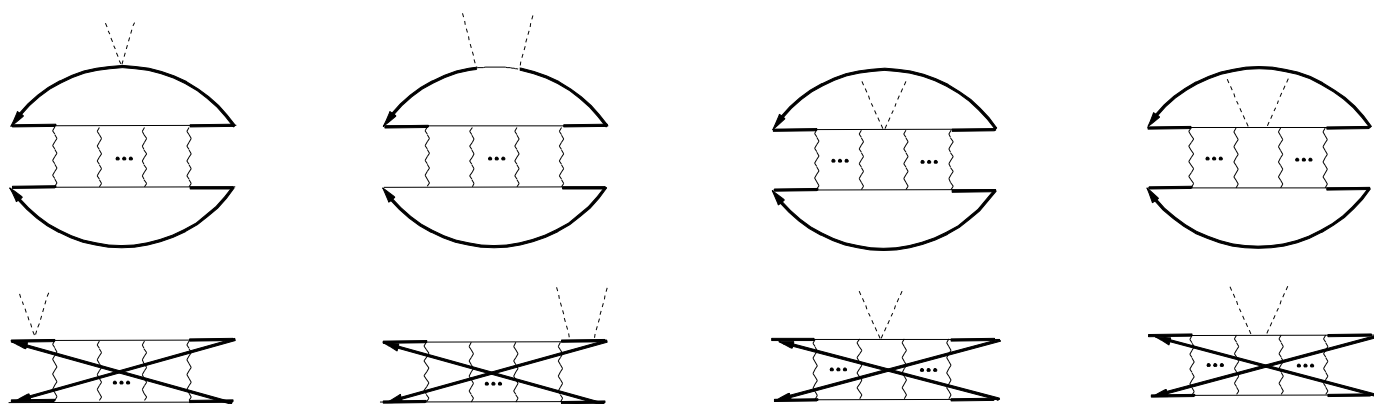


$V_\rho = 1$   
 $\mathcal{O}(p^5)$   
 Next to Leading Order



■  $\mathcal{O}(p^2)$   $\mathcal{L}_{\pi N}$  vertex

$V_\rho = 2$   
 $\mathcal{O}(p^5)$   
 Next to Leading Order



Pion Propagator:  $\Sigma$  is the pion self-energy

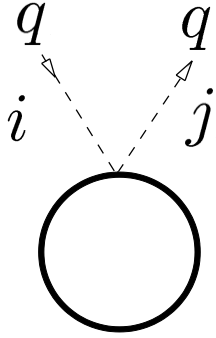
$$iS^{-1}(q) = \frac{1}{q^2 - m_\pi^2 + \Sigma}$$

In-medium nucleon propagator

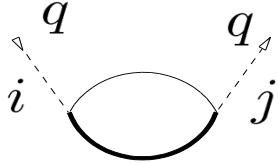
$$\begin{aligned} \text{-----} & \frac{\theta(\xi_{i3} - |\mathbf{k}|)}{k^0 - E(\mathbf{k}) - i\epsilon} + \frac{\theta(|\mathbf{k}| - \xi_{i3})}{k^0 - E(\mathbf{k}) + i\epsilon} \\ & = \frac{1}{k^0 - E(\mathbf{k}) + i\epsilon} + i(2\pi)\theta(\xi_{i3} - k)\delta(k^0 - E(\mathbf{k})) \end{aligned}$$

$$\text{—————} \frac{\theta(\xi_{i3} - k)}{k^0 - E(\mathbf{k}) - i\epsilon}$$

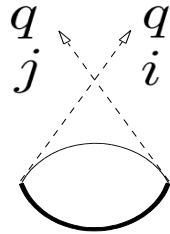
# Terms linear in the nuclear density



$$\begin{aligned}\Sigma_1 &= \frac{-iq^0}{2f^2} \varepsilon_{ijk} \int \frac{d^3k}{(2\pi)^3} \text{Tr} \left[ \tau^k \left\{ \frac{1+\tau_3}{2} \theta(\xi_p - |\mathbf{k}|) + \frac{1-\tau_3}{2} \theta(\xi_n - |\mathbf{k}|) \right\} \right] \\ &= \frac{-iq^0}{2f^2} \varepsilon_{ij3} (\rho_p - \rho_n)\end{aligned}$$

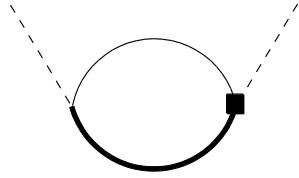
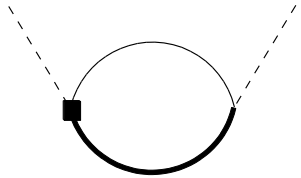


a)

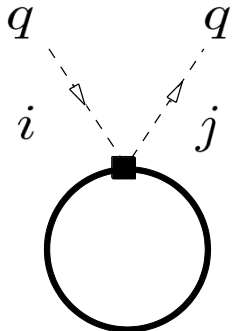


b)

$$\begin{aligned}\Sigma_2^{\text{LO}} &= \frac{ig_A^2 \mathbf{q}^2}{2f^2 q^0} \varepsilon_{ij3} (\rho_p - \rho_n) , \\ \Sigma_2^{\text{NLO}} &= \frac{-g_A^2 (\mathbf{q}^2)^2}{4f^2 m q^0{}^2} \delta_{ij} (\rho_p + \rho_n)\end{aligned}$$

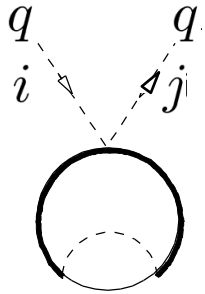


$$\Sigma_3 = \frac{g_A^2 \mathbf{q}^2}{2mf^2} (\rho_p + \rho_n) \delta_{ij}$$

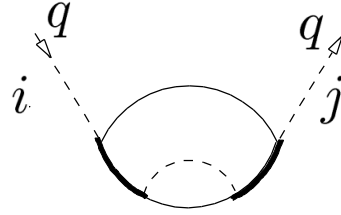


$$\Sigma_4 = \frac{-2\delta_{ij}}{f^2} \left( 2c_1 m_\pi^2 - q_0^2 (c_2 + c_3 - \frac{g_A^2}{8m}) + c_3 \mathbf{q}^2 \right) (\rho_p + \rho_n)$$

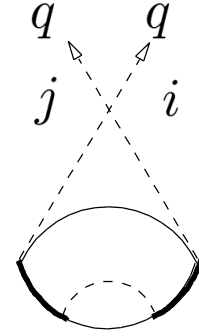
# Pion-loop nucleon self-energy contributions



a)



b)



c)

$$\Sigma^\pi = \frac{1 + \tau_3}{2} \Sigma_p^\pi + \frac{1 - \tau_3}{2} \Sigma_n^\pi$$

$$\theta(\xi_p - |\mathbf{k}|) \equiv \theta_p^- , \quad \theta(\xi_n - |\mathbf{k}|) \equiv \theta_n^- .$$

$$\Sigma_{i3}^\pi(k^0, \mathbf{k})$$

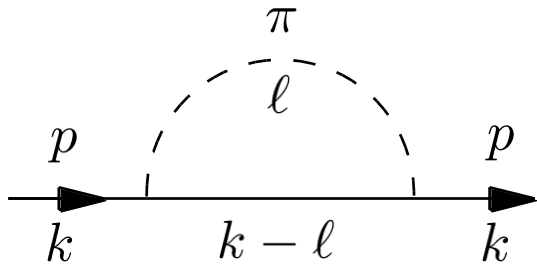
Diagram a)

$$\Sigma_5 = \frac{iq^0}{f^2} \varepsilon_{ijk} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\partial \Sigma_p^\pi}{\partial k^0} \theta_p^- - \frac{\partial \Sigma_n^\pi}{\partial k^0} \theta_n^- \right)_{k^0=E(\mathbf{k})}$$

Diagrams b)+c):

$$\Sigma_6 = \frac{-ig_A^2 \mathbf{q}^2}{f^2 q^0} \varepsilon_{ij3} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\partial \Sigma_p^\pi}{\partial k^0} \theta_p^- - \frac{\partial \Sigma_n^\pi}{\partial k^0} \theta_n^- \right)_{k^0=E(\mathbf{k})}$$

$$- \frac{g_A^2 \mathbf{q}^2}{f^2 q^0{}^2} \delta_{ij} \int \frac{d^3k}{(2\pi)^3} \left( \Sigma_p^\pi \theta_p^- + \Sigma_n^\pi \theta_n^- \right)$$



$$\Sigma_N^\pi = \Sigma_{N,f}^\pi + \Sigma_{N,m}^\pi = \mathcal{O}(p^3)$$

$\frac{\partial \Sigma_N^\pi}{\partial k^0} = \mathcal{O}\left(\frac{p^3}{p^2}\right)$  because  $k^0 = \mathcal{O}(p^2)$ , it is a nucleon kinetic energy

$$\Sigma_5 = \frac{iq^0}{f^2} \varepsilon_{ijk} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\partial \Sigma_p^\pi}{\partial k^0} \theta_p^- - \frac{\partial \Sigma_n^\pi}{\partial k^0} \theta_n^- \right)_{k^0=E(\mathbf{k})}$$

It is expected to be  $\mathcal{O}(p^5)$  or NLO

$$\Sigma_6 = \frac{-ig_A^2 \mathbf{q}^2}{f^2 q^0} \varepsilon_{ij3} \int \frac{d^3k}{(2\pi)^3} \left( \frac{\partial \Sigma_p^\pi}{\partial k^0} \theta_p^- - \frac{\partial \Sigma_n^\pi}{\partial k^0} \theta_n^- \right)_{k^0=E(\mathbf{k})}$$

$$- \frac{g_A^2 \mathbf{q}^2}{f^2 q^0{}^2} \delta_{ij} \int \frac{d^3k}{(2\pi)^3} \left( \Sigma_p^\pi \theta_p^- + \Sigma_n^\pi \theta_n^- \right)$$

$\mathcal{O}(p^6)$  or N<sup>2</sup>LO

$\Sigma_{N,f}^\pi$  is calculated in HBCHPT Meißner, Bernard, Kaiser IJM E4('95)193

$$\frac{\partial \Sigma_{N,f}^\pi}{\partial k^0} = \frac{3g_A^2}{32\pi^2 f^2} \left[ m_\pi^2 + k_0^2 - 3k^0 \sqrt{b} \left( i \log \frac{k^0 + i\sqrt{b}}{-k^0 + i\sqrt{b}} + \pi \right) \right] \quad b = m_\pi^2 - k_0^2$$

Hence, because  $\partial \Sigma_{p(n),f}^\pi / \partial k^0 = \mathcal{O}(p^2)$  when inserted in  $\Sigma_5$  and  $\Sigma_6$  it gives rise to  $\mathcal{O}(p^6)$  or N<sup>2</sup>LO contributions.

$\Sigma_{N,m}^\pi$  involves the integral

$$I_m = 2\pi \int \frac{d^4 \ell}{(2\pi)^4} \frac{\vec{\ell}^2 \delta(k^0 - \ell^0) \theta(\xi_{i_3} - |\mathbf{k} - \vec{\ell}|)}{\ell^2 - m_\pi^2 + i\epsilon} = - \int \frac{d^3 \ell}{(2\pi)^3} \frac{\vec{\ell}^2 \theta(\xi_{i_3} - |\mathbf{k} - \vec{\ell}|)}{b + \vec{\ell}^2 - i\epsilon}$$

It only depends on  $k^0$  through the variable  $b$ . Then

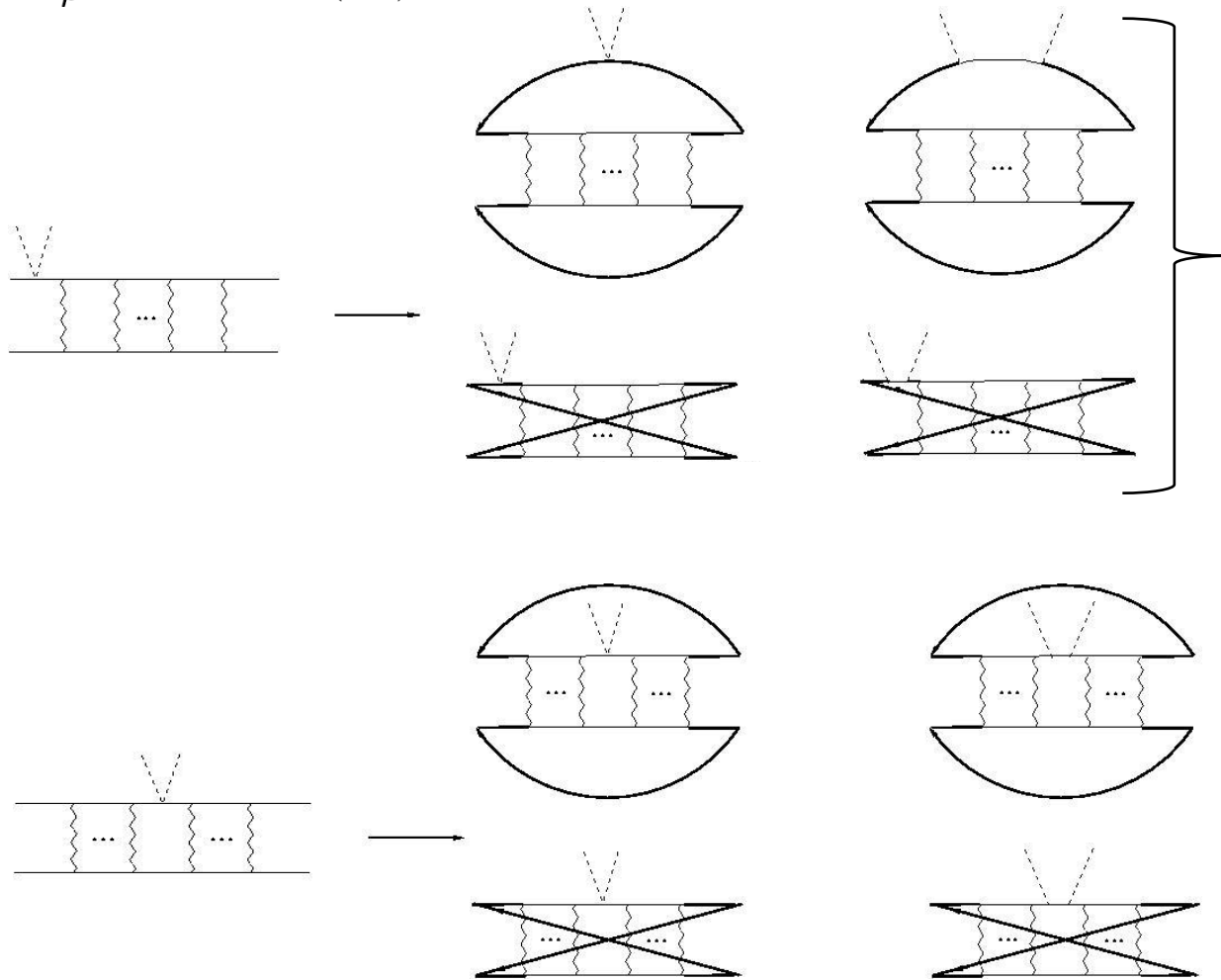
$$\partial I_m / \partial k^0 = -2k^0 \partial I_m / \partial b = \mathcal{O}(p^3)$$

Its contributions to the pion self-energy are  $\mathcal{O}(p^7)$  or N<sup>3</sup>LO

At NLO there are no contributions from the pion-loop nucleon self-energy

# In-medium NN scattering contributions

$$V_\rho = 2 \quad \mathcal{O}(p^5)$$

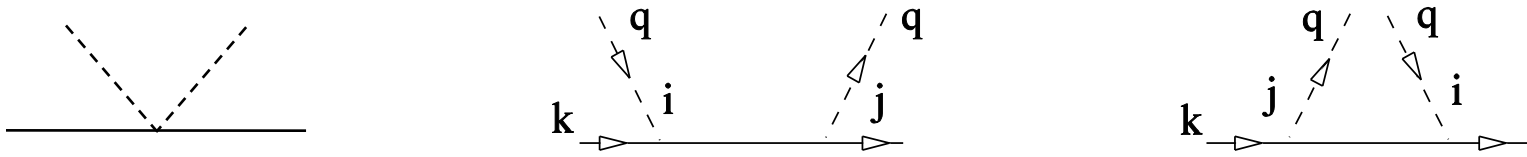


$$\text{Wavy line} = \text{Contact } C_S, C_T + \text{OPE}$$

$\mathcal{O}(p^0)$

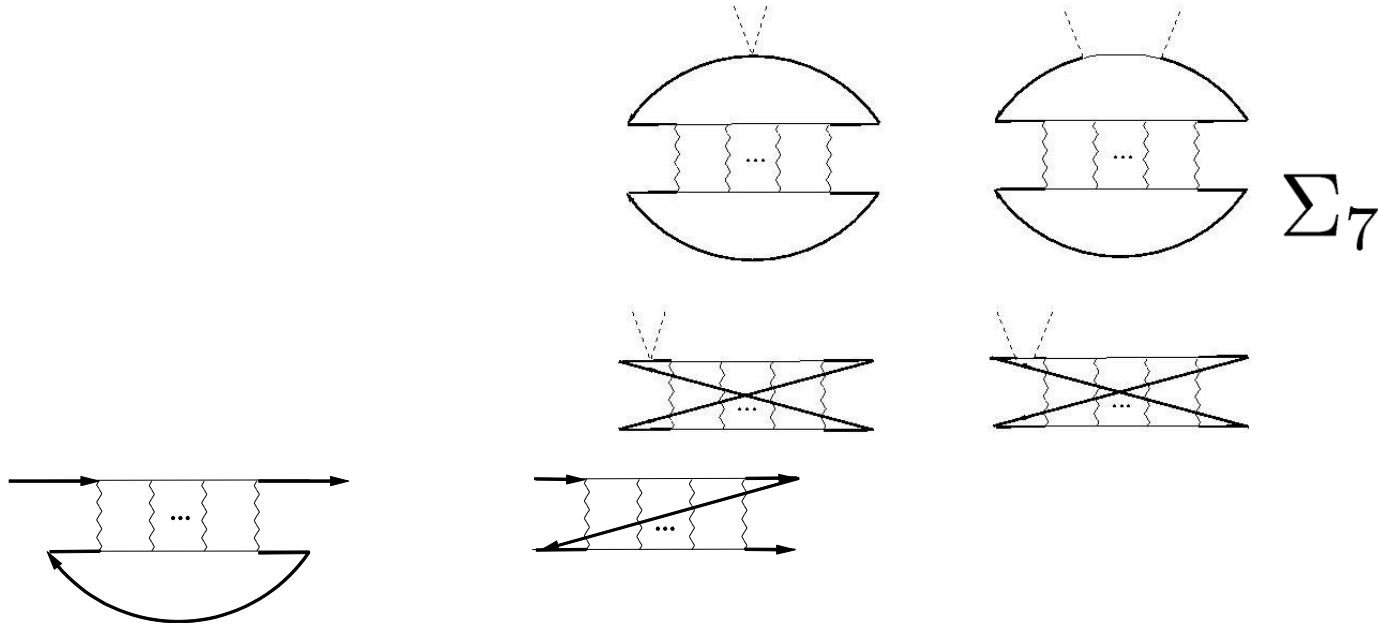
These diagrams originate because of the nucleon in-medium self-energy due to the nucleon-nucleon interactions

Nucleon-Nucleon Initial and Final State Interactions in the medium



The leading contribution to this sum is  $-\frac{iq^0}{2f^2} \left(1 - g_A^2 \frac{\mathbf{q}^2}{q_0^2}\right) \varepsilon_{ijk} \tau^k$

Thus, we can discuss simultaneously the diagrams on the left and right columns of the previous figure



$$\Sigma_{i_3, NN} = \sum_{\alpha_2, \sigma_2} \int \frac{d^3 k_2}{(2\pi)^3} \theta(\xi_{\alpha_2} - |\mathbf{k}_2|)_A \langle \mathbf{k}_1 \sigma_1 \alpha_1, \mathbf{k}_2 \sigma_2 \alpha_2 | T_{NN} | \mathbf{k}_1 \sigma_1 \alpha_1, \mathbf{k}_2 \sigma_2 \alpha_2 \rangle_A$$

$$\Sigma_{NN} = \frac{1 + \tau_3}{2} \Sigma_{p, NN} + \frac{1 - \tau_3}{2} \Sigma_{n, NN}$$



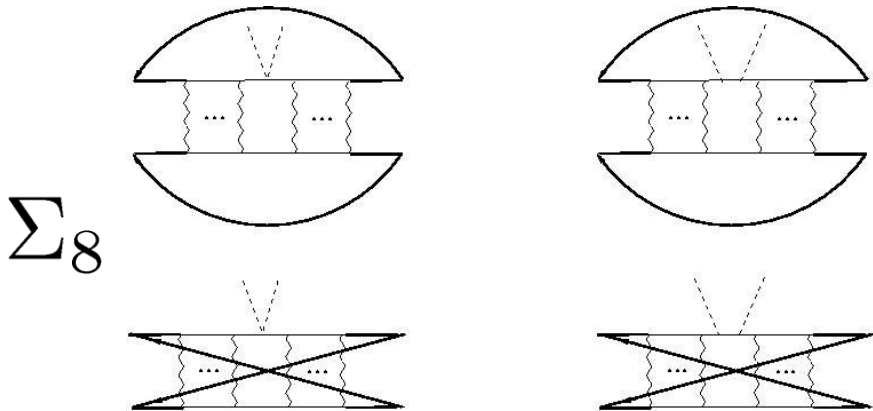
$$\Sigma_7 = \frac{q^0}{2f^2} \varepsilon_{ijk} \left(1 - g_A^2 \frac{\mathbf{q}^2}{q_0^2}\right) \int \frac{d^4 k_1}{(2\pi)^4} \text{Tr} \left\{ \tau^k \left( \frac{1 + \tau_3}{2} \theta_p^- + \frac{1 - \tau_3}{2} \theta_n^- \right) \Sigma_{NN} \left( \frac{1 + \tau_3}{2} \theta_p^- + \frac{1 - \tau_3}{2} \theta_n^- \right) \right\}$$

$$\times \frac{1}{(k_1^0 - E(\mathbf{k}) - i\epsilon)^2}$$

$$\Sigma_7 = \frac{iq^0 \left(1 - g_A^2 \frac{\mathbf{q}^2}{q_0^2}\right) \varepsilon_{ij3}}{2f^2} \sum_{\sigma_1, \sigma_2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{\partial}{\partial k_1^0}$$

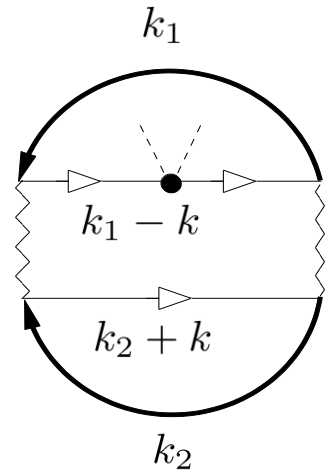
$$\times \left[ \theta(\xi_p - |\mathbf{k}_1|) \theta(\xi_p - |\mathbf{k}_2|) {}_A \langle \mathbf{k}_1 \sigma_1 p, \mathbf{k}_2 \sigma_2 p | T_{NN} | \mathbf{k}_1 \sigma_1 p, \mathbf{k}_2 \sigma_2 p \rangle_A \right.$$

$$\left. - \theta(\xi_n - |\mathbf{k}_1|) \theta(\xi_n - |\mathbf{k}_2|) {}_A \langle \mathbf{k}_1 \sigma_1 n, \mathbf{k}_2 \sigma_2 n | T_{NN} | \mathbf{k}_1 \sigma_1 n, \mathbf{k}_2 \sigma_2 n \rangle_A \right]_{k_1^0 = E(\mathbf{k}_1)}$$



These diagrams consist of the pion-nucleon scattering in a two-nucleon reducible loop that it is corrected by initial and final state interactions (ISI and FSI).

$\Sigma_8$  and  $\Sigma_7$  cancel each other



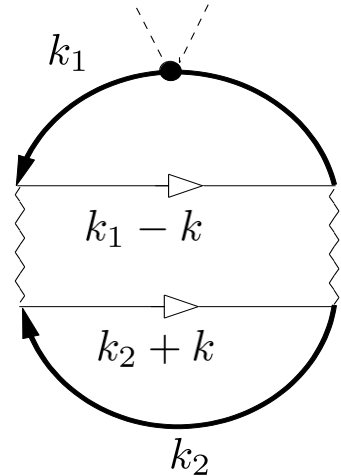
$$\Sigma_8^L = \frac{iq_0 \left(1 - g_A^2 \frac{\mathbf{q}^2}{q_0^2}\right) \varepsilon_{ij3}}{2f^2} \sum_{\sigma_1, \sigma_2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{\partial}{\partial k_1^0} \times \left[ \theta(\xi_p - |\mathbf{k}_1|) \theta(\xi_p - |\mathbf{k}_2|) \Pi_p - \theta(\xi_n - |\mathbf{k}_1|) \theta(\xi_n - |\mathbf{k}_2|) \Pi_n \right]_{k_1^0 = E(\mathbf{k}_1)}$$

$$\Pi_{i_3} = i \int \frac{d^4 k}{(2\pi)^4} V Pro(i_3, k_1 - k) Pro(i_3, k_2 + k) V$$

The derivative appears because of the propagator squared

$$\left[ \frac{\theta(\xi_o - |\mathbf{k}_1 - \mathbf{k}|)}{k_1^0 - k^0 - E(\mathbf{k}_1 - \mathbf{k}) - i\epsilon} + \frac{\theta(|\mathbf{k}_1 - \mathbf{k}| - \xi_o)}{k_1^0 - k^0 - E(\mathbf{k}_1 - \mathbf{k}) + i\epsilon} \right]^2$$

$$= -\frac{\partial}{\partial k_1^0} \left[ \frac{\theta(\xi_o - |\mathbf{k}_1 - \mathbf{k}|)}{k_1^0 - k^0 - E(\mathbf{k}_1 + \mathbf{k}) - i\epsilon} + \frac{\theta(|\mathbf{k}_1 - \mathbf{k}| - \xi_o)}{k_1^0 - k^0 - E(\mathbf{k}_1 + \mathbf{k}) + i\epsilon} \right]$$

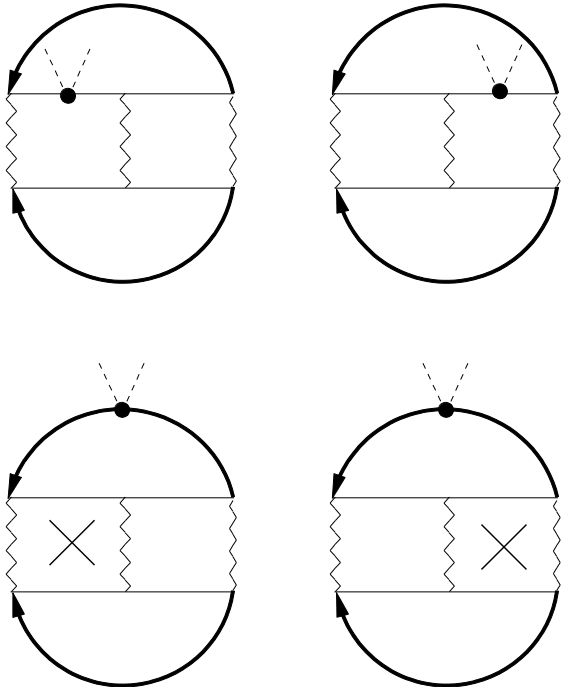


This loop is given by the first iterated wiggly line exchange that gives  $-\Pi_{i_3}$

$$\Sigma_7^L = -\frac{iq^0 \left(1 - g_A^2 \frac{\mathbf{q}^2}{q_0^2}\right) \varepsilon_{ij3}}{2f^2} \sum_{\sigma_1, \sigma_2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{\partial}{\partial k_1^0} \times \left[ \theta(\xi_p - |\mathbf{k}_1|) \theta(\xi_p - |\mathbf{k}_2|) \Pi_p - \theta(\xi_n - |\mathbf{k}_1|) \theta(\xi_n - |\mathbf{k}_2|) \Pi_n \right]_{k_1^0 = E(\mathbf{k}_1)}$$

$$\Sigma_7 + \Sigma_8 = 0$$

The basic simple reason for such cancellation is that while for  $\Sigma_7$  the presence of a nucleon propagator squared gives rise to  $+i\partial/\partial k_1^0$ , for  $\Sigma_8$  it yields  $-i\partial/\partial k_1^0$



All the NLO corrections to the linear density approximation for the pion self-energy vanish

# 4. Conclusions

- Promising scheme for an EFT in the nuclear medium including both short-range and pion mediated multi-nucleon interactions

- We have established a new chiral power counting which is bounded from below. It requires the resummation of some infinite strings of two-nucleon reducible diagrams.

Non-perturbative methods for this are given in [Lacour, Oller, Meißner](#), “Non-perturbative methods for the EFT of nuclear matter”

- The pion self-energy in asymmetric nuclear matter has been calculated up to NLO or  $\mathcal{O}(p^5)$ .

There are no corrections to the linear density approximation at this order.

The first corrections are N<sup>2</sup>LO. There are many of them, e.g. the well-known and time-honoured Ericson-Ericson-Pauli rescattering effect.