Granada, November 27th, 2003

The Mixing Angle of the Lightest Scalar Nonet

José A. Oller
Univ. Murcia, Spain

- **Introduction**
- **Chiral Unitary Approach**
- **SU(3) Analyses**
- **Conclusions**

1. Introduction

1) The mesonic scalar sector has the **vacuum quantum numbers** 0^{++}, as any order parameter should. Essencial for the study of Chiral Symmetry Breaking: Spontaneous and Explicit m_u, m_d, m_s .

2) In this sector the **hadrons really interact strongly.**

   1) Large unitarity loops.

   2) Channels coupled very strongly, e.g. $\pi\pi - K\bar{K}$, $\pi\eta - K\bar{K}$ ...

   3) Dynamically generated resonances, ~~Breit-Wigner formulae~~, ~~VMD~~, ...

3) **OZI rule** has large corrections.

   1) No ideal mixing multiplets.

   2) ~~Simple quark~~ model.

Points 2) and 3) imply **large deviations** with respect to **Large Nc QCD.**

4) A precise knowledge of the scalar interactions of the lightest hadronic thresholds, $\pi\pi$ and so on, is often required.

- Final State Interactions (FSI) in ε'/ε , Pich, Palante, Scimemi, Buras, Martinelli,...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- CKM matrix (V_{us})
- Fluctuations in order parameters of SχSB. Stern et al.

5) Recent and accurate experimental data are bringing further evidence on the existence of the σ, κ (E791) and further constrains to the present models (CLOE).

6) The effective field theory of QCD at low energies is Chiral Perturbation Theory (CHPT).

This allows a systematic treatment of pion physics, but only cose to threshold, ($\sqrt{s}$ < 0.4-0.5 GeV).

Chiral Perturbation Theory

Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian | **Hilbert Space Physical States**

u, d, s massless quarks — **Spontaneous Chiral Symmetry Breaking**

$SU(3)_L \otimes SU(3)_R \longrightarrow SU(3)_V$

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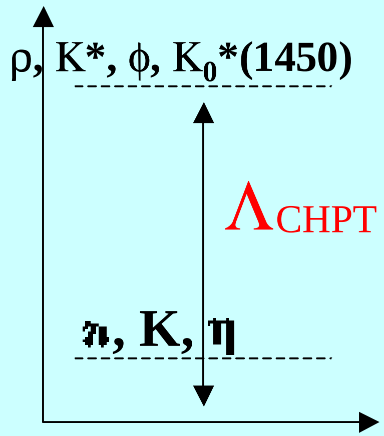
Goldstone Theorem

Octet of massles pseudoscalars
π, K, η
Energy gap



$m_q \neq 0$. Explicit breaking of Chiral Symmetry → **Non-zero masses**
$m_P^2 \propto m_q$

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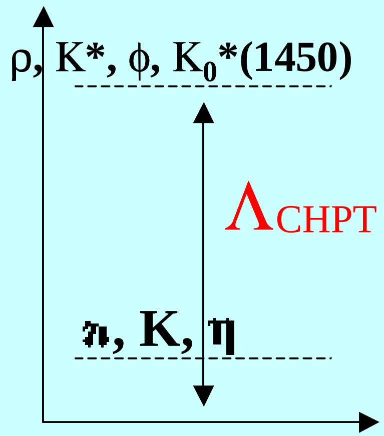

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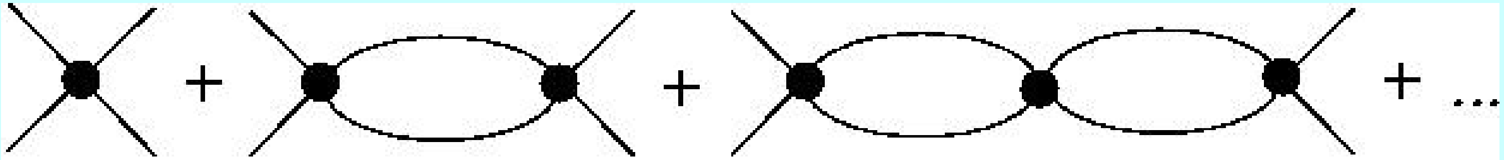
Perturbative expansion in powers of the external four-momenta of the pseudo-Goldstone bosons over Λ^2_{CHPT}

$$L = L_2 + L_4 + \ldots$$

$$\frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda^2_{CHPT}}\right)$$

$$\Lambda_{CHPT} \approx 1\ \text{GeV} \approx M_\rho \approx 4\pi f_\pi \approx 1\ \text{GeV}$$

- New scales or numerical enhancements can appear that makes definitively smaller the overall scale Λ, e.g:
  - Scalar Sector (S-waves) of meson-meson interactions with I=0,1,1/2 the unitarity loops are enhanced by numerical factors.



P-WAVE S-WAVE

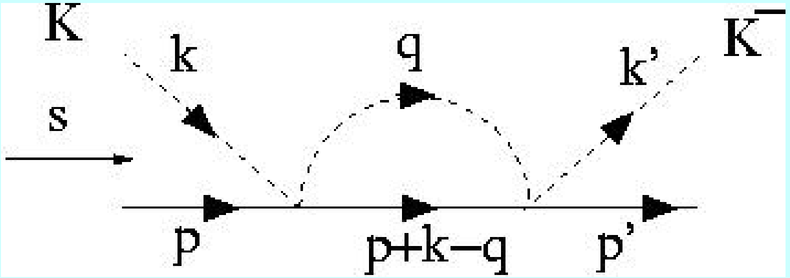
$$\frac{s-4m_\pi^2}{6f^2} \rightarrow \frac{s-m_\pi^2}{f^2}$$

Enhancement by factors 6^L

  - Presence of large masses compared with the typical momenta, e.g. Kaon masses in driving the appearance of the Λ(1405) close to tresholed. This also occurs similarly in the S-waves of Nucleon-Nucleon scattering.

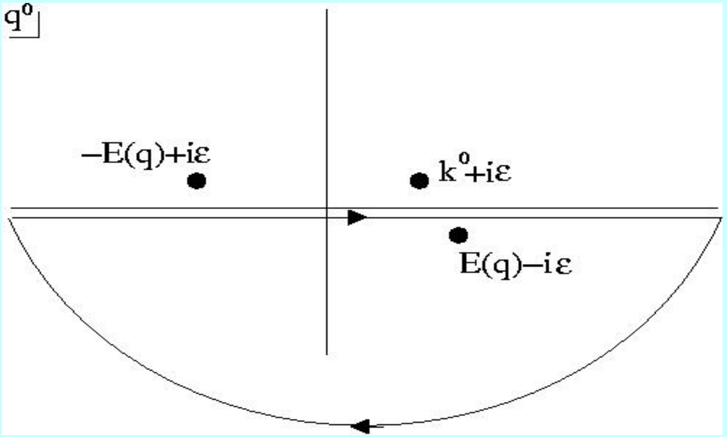
Let us keep track of the kaon mass, $M_K \approx 500$ MeV

We follow similar argumentos to those of S. Weinberg in NPB363,3 (´91) respect to NN scattering (nucleon mass).



Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\varepsilon)(q^0 + \mathrm{E}(q) - i\varepsilon)(q^0 - \mathrm{E}(q) + i\varepsilon)}$$



$$\frac{1}{k^0 - \mathrm{E}(q)} \frac{1}{2\mathrm{E}(q)} \cong \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

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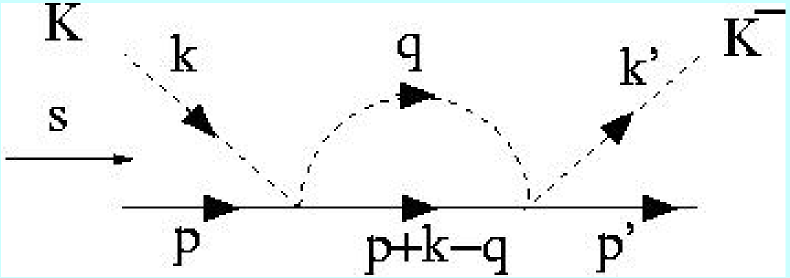

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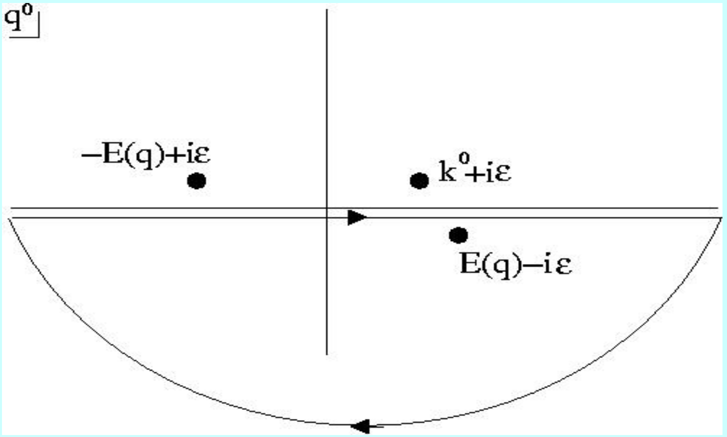

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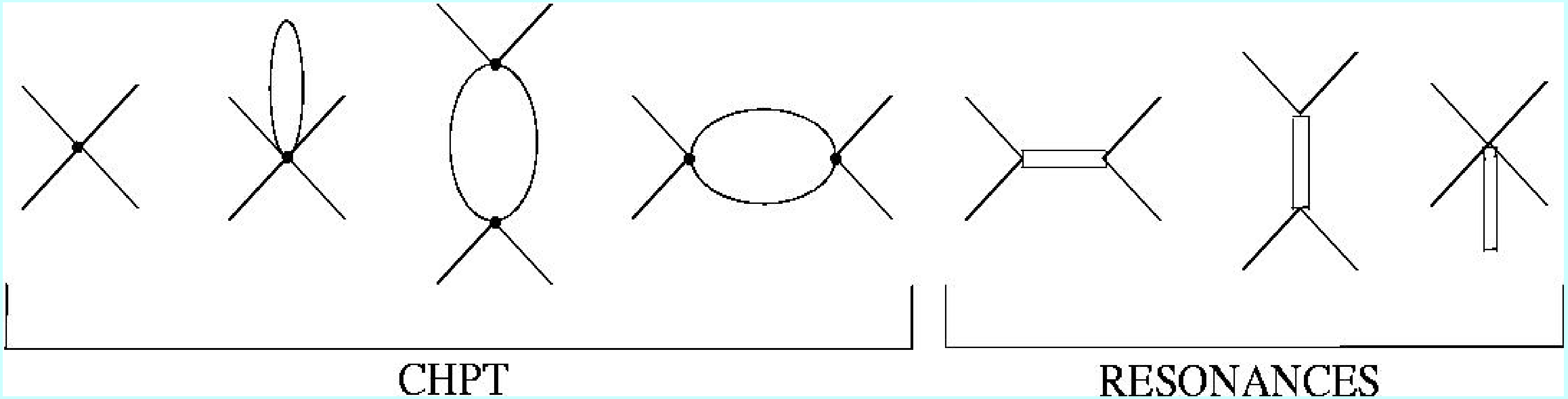
Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

This enhancement takes place in the real part of the uniarity bubble: Analyticity

CHPT+Resonances

Ecker, Gasser, Pich and de Rafael, NPB321, 311 (´98)
Resonances give rise to a resummation of the chiral series at the tree level (local counterterms beyond O(p^4).

$$\frac{1}{M^2-q^2}=\frac{1}{q^2}+\frac{q^2}{M^4}+\frac{q^4}{M^6}+\ldots \quad q^2<<M^2$$



The counting used to perform the matching is a simultaneous one in the number of loops calculated at a given order in CHPT (that increases order by order). E.g:

– Meissner, J.A.O, NPA673,311 (´00) the πN scattering was studied up to one loop calculated at O(p^3) in HBCHPT+Resonances.

2. The Chiral Unitary Approach

1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies).
   - S-wave meson-meson scattering: I=0 (σ(500), f_0(980)), I=1 (a_0(980)), I=1/2 (κ(700)). Related by SU(3) symmetry.
   - S-wave Strangeness S=-1 meson-baryon interactions. I=0 Λ(1405) and other resonances.
   - 1S0, 3S1 S-wave Nucleon-Nucleon interactions.
2. Then one can study:
   - Strongly interacting coupled channels.
   - Large unitarity loops.
   - Resonances.
3. This allows as well to use the Chiral Lagrangians for higher energies.
4. The same scheme can be applied to productions mechanisms. Some examples:
   - Photoproduction: $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-, \pi^0\eta, K^+K^-, K^0\overline{K}^0$
   - Decays: $J/\Psi \to \phi(\omega)\pi\pi, K\overline{K}$
     $\phi(1020) \to \gamma K^0\overline{K}^0, \gamma\pi^0\pi^0, \gamma\pi^0\eta$

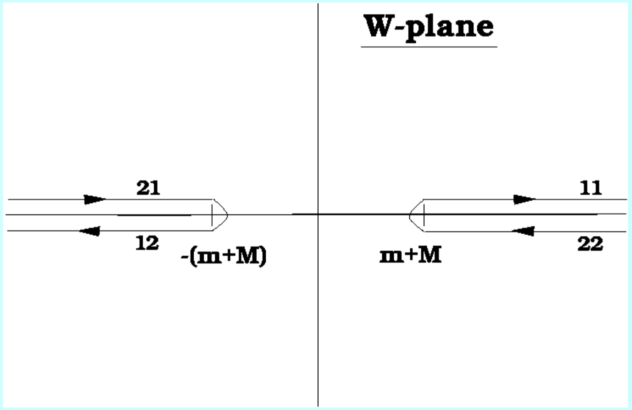
General Expression for a Partial Wave Amplitude

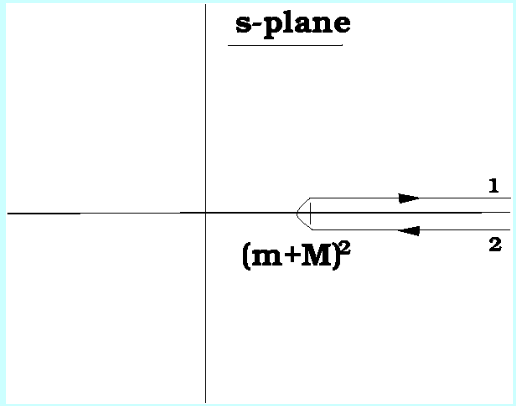
- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$\mathrm{Im}\,T_{ij} = \sum_k T_{ik}\,\rho_k\,T^*_{kj} \quad \rightarrow \quad \mathrm{Im}\,T^{-1}_{ij} = -\rho_i\,\delta_{ij}$$

Unitarity Cut

W=√s



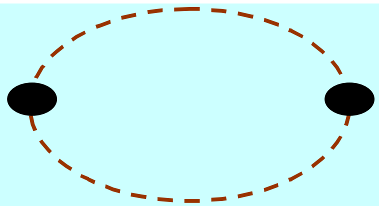


We perform a dispersion relation for the inverse of the partial wave (the unitarity cut is known)

$$T^{-1}_{ij} = R^{-1}_{ij} + \delta_{ij}\left(g(s_0)_i - \frac{s - s_0}{\pi} \int \frac{\rho(s')_i\, ds'}{(s' - s - i0^+)(s' - s_0)} \right)$$

The rest

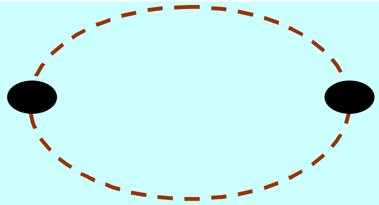
g(s): Single unitarity bubble

g(s)= 

$$g(s)=\frac{1}{4\pi^2}\left(a_{SL}+\sigma(s)\log\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)$$

$$\sigma(s)=\frac{2q}{\sqrt{s}}$$

$$T=\left(R^{-1}+g(s)\right)^{-1}$$

g(s)= 

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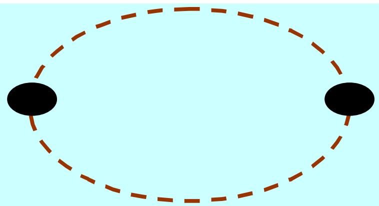
g(s)= 

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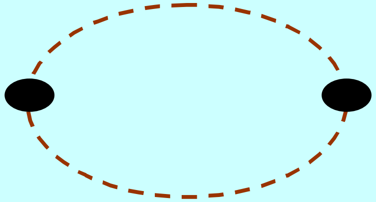
expansion of T

In doing that, one makes use of the CHPT/alike counting for g(s)

The counting/expressions of R(s) are consequences of the known ones for g(s) and T(s) g(s)= 

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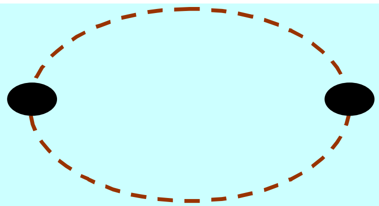
g(s)= 

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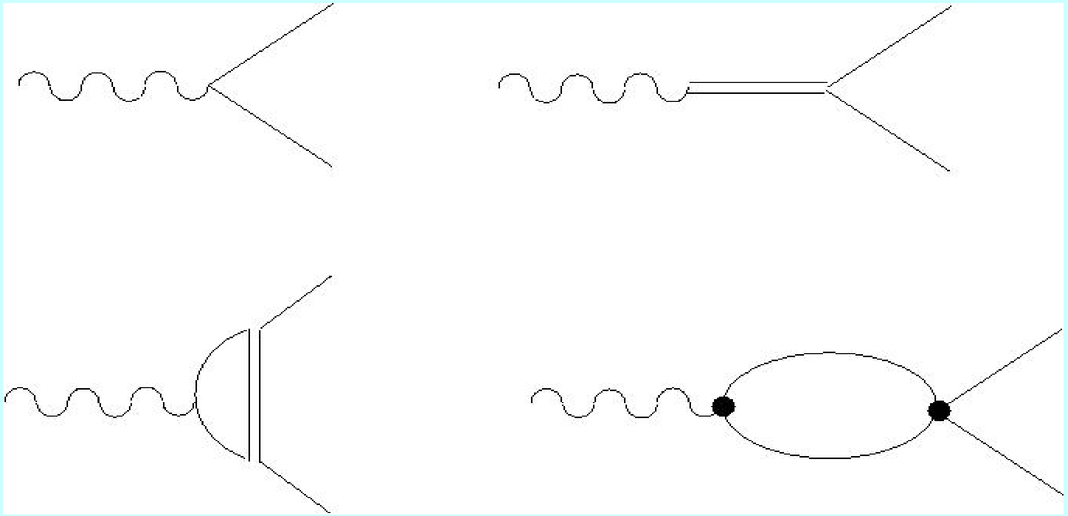
4. The final expressions fulfill unitarity to all orders since R is real in the physical region (T from CHPT fulfills unitarity pertubatively as employed in the matching).

Production Processes

The re-scattering is due to the strong ``final´´ state interactions from some ``weak´´ production mechanism.



$$Im\,F_i = \sum_k F_k\, \rho_k\, T^*_{ki}$$

We first consider the case with only the right hand cut for the strong interacting amplitude, R^{-1} is then a sum of poles (CDD) and a constant. It can be easily shown then:

$$F = (I + R\,g(s))^{-1}\,\xi$$

Finally, ξ is also expanded pertubatively (in the same way as R) by the matching process with CHPT/alike expressions for F, order by order. The crossed dynamics, as well for the production mechanism, are then included pertubatively.

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Meson-Meson Scalar Sector

Let us apply the chiral unitary approach

LEADING ORDER: $T = \left(R^{-1} + g(s) \right)^{-1}$ g is order 1 in CHPT

$T = T_2 = R_2 - R_2 g R_2 + ..$ $R = R_2 = T_2$ Oset, J.A.O, NPA620,438(´97)

I=0 $\pi\pi, K\bar{K}$

I=1 $\pi\eta_8, K\bar{K}$

A three-momentum cut-off was used in the calculation of g(s). The only free parameter.

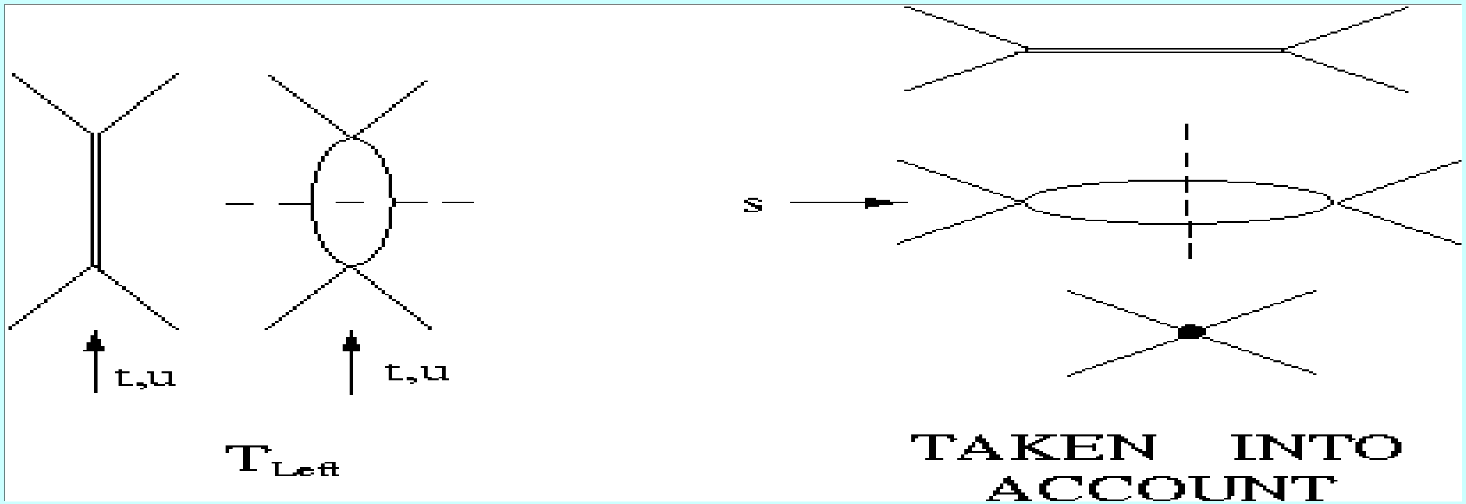
In Oset,Oller PRD60,074023(99):

I=0 ($\pi\pi$, $K\bar{K}$, $\eta_8\eta_8$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

S-waves with the N/D method:

1. No cut-off dependence (subtruction constant) a_{SL},
2. Lowest order CHPT + Resonances (s-channel),
3. The crossed channels were calculated in One Loop CHPT + Resonances. Less than 10% up to 1 GeV compared with 2) .

$$R = T_2 + T_R$$



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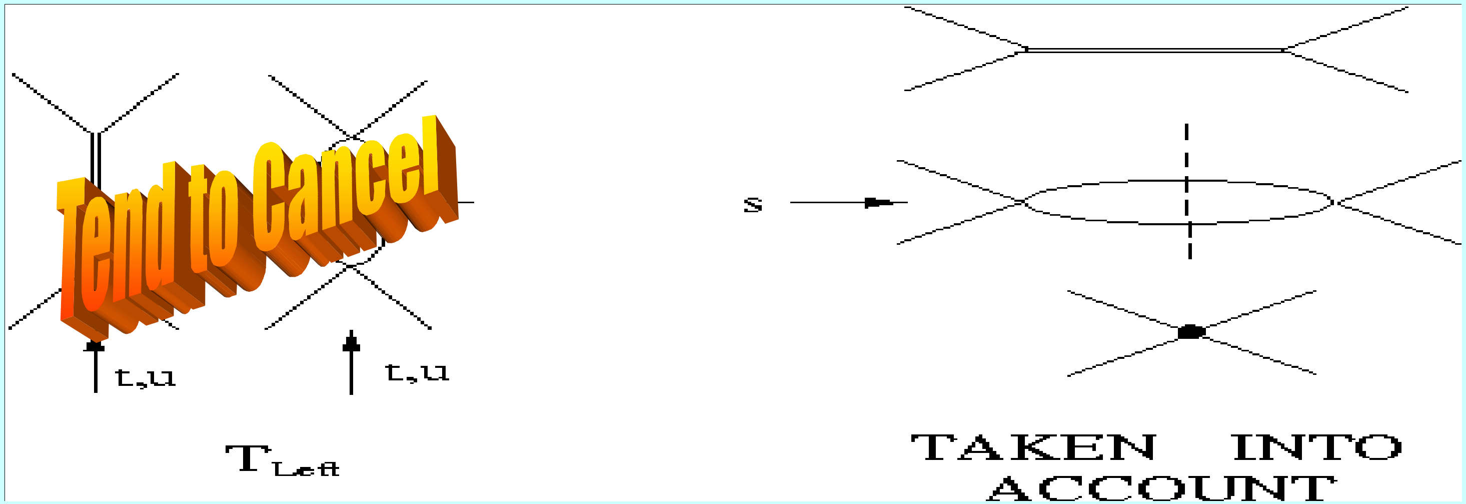
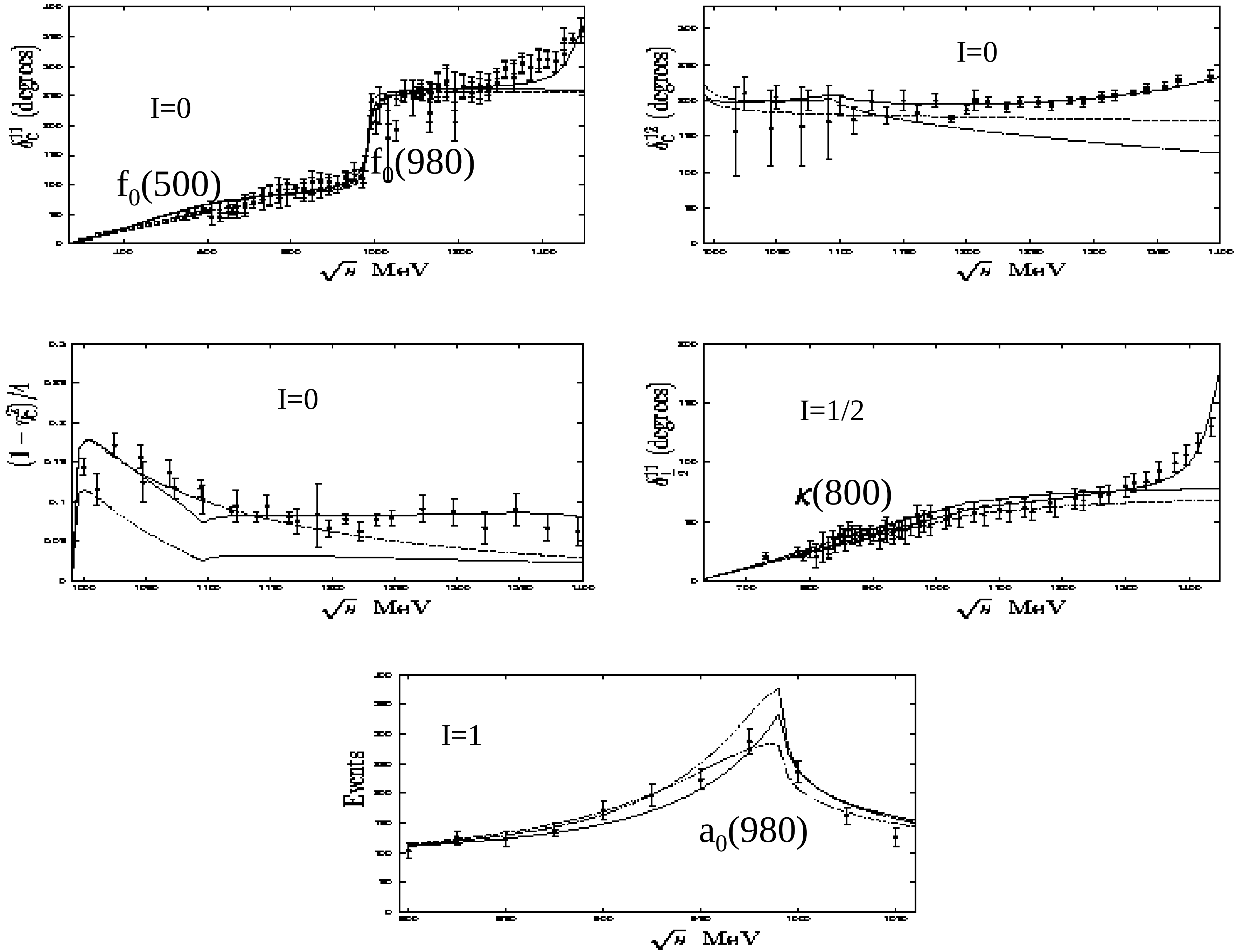




I=0
$f_0(500)$
$f_0(980)$
δ_C^{11} (degrees)
$\sqrt{s}$ MeV
I=0
δ_C^{12} (degrees)
$\sqrt{s}$ MeV
I=0
$(1-\eta_C^2)/4$
$\sqrt{s}$ MeV
I=1/2
$\kappa(800)$
$\delta_{\frac{1}{2}}^{11}$ (degrees)
$\sqrt{s}$ MeV
I=1
$a_0(980)$
Events
$\sqrt{s}$ MeV

Solid lines: I=0 ($\pi\pi$, $K\bar{K}$, $\eta_8\eta_8$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

Singlet 1 GeV: $\tilde{c}_d$=20.9MeV ; $\tilde{c}_m$=10.6MeV; M_1=1021MeV

Octet 1.4 GeV: c_d=19.1MeV ; c_m=15±30MeV; M_8=1390MeV

Subtraction Constant: a=-0.75±0.20

Dashed lines: I=0 ($\pi\pi$, $K\bar{K}$, $\eta_8\eta_8$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

No bare Resonances

Subtraction Constant: a=-1.23

Short-Dashed lines: I=0 ($\pi\pi$, $K\bar{K}$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

No bare Resonances

Several Subtraction Constants:

$$a_{\pi\pi}=-1.14;\ a_{K\bar{K}}=-1.64;\ a_{\pi\eta}=-0.5;\ a_{K\pi}=-0.75;\ a_{K\eta}=-0.75$$

Spectroscopy: Dynamically generated resonances.

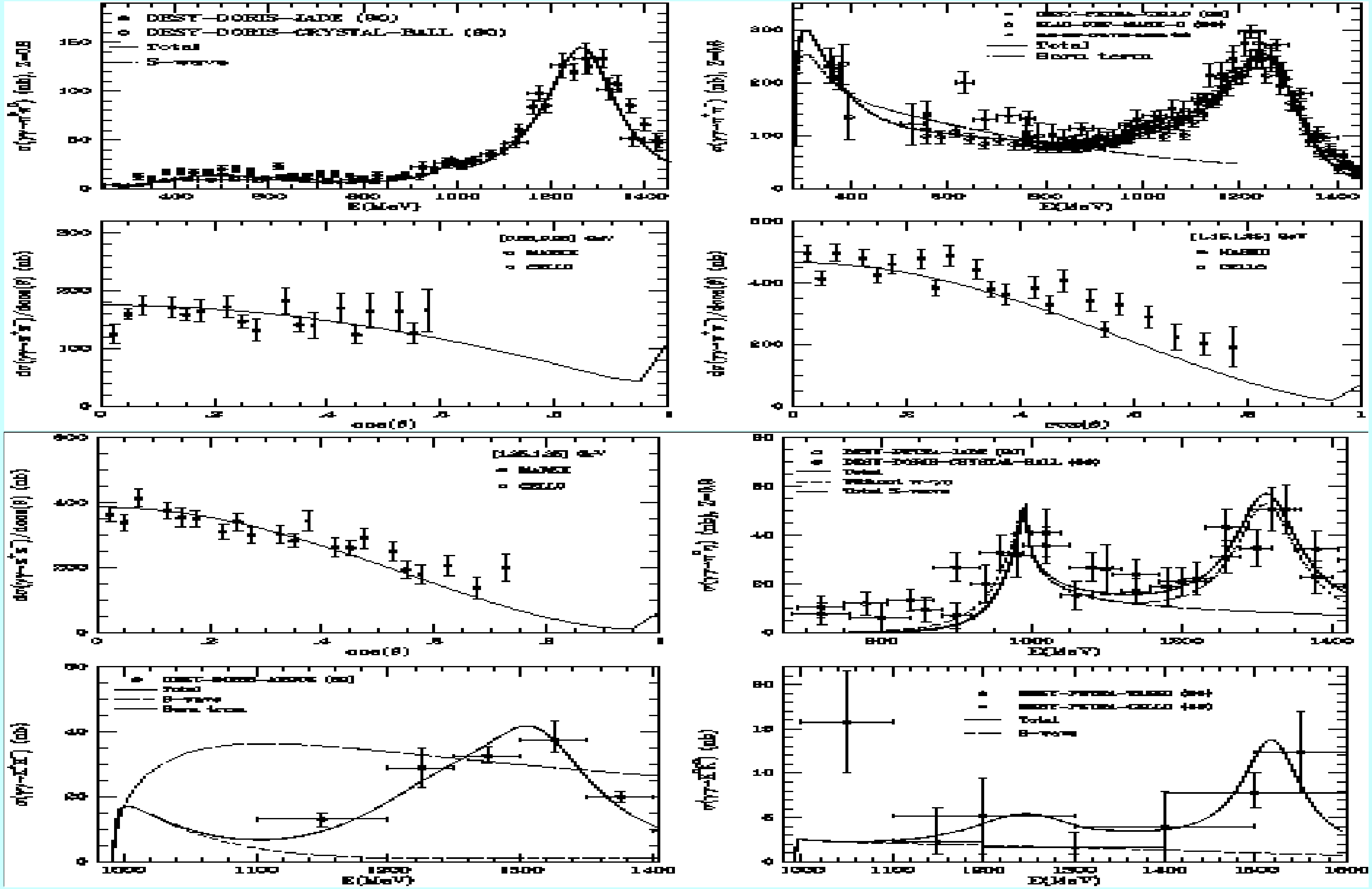
TABLE 1

σ	$0.445 - i\,0.220$ $\lvert g_{\pi\pi}\rvert = 3.01$ $\lvert g_{K\bar{K}}\rvert = 1.09$ $\lvert g_{\eta_8\eta_8}\rvert = 0.09$	$0.443 - i\,0.213$ $\lvert g_{\pi\pi}\rvert = 2.94$ $\lvert g_{K\bar{K}}\rvert = 1.30$ $\lvert g_{\eta_8\eta_8}\rvert = 0.04$	$0.442 - i\,214$ $\lvert g_{\pi\pi}\rvert = 2.95$ $\lvert g_{K\bar{K}}\rvert = 1.34$
$f_0(980)$	$0.988 - i\,0.014$ $\lvert g_{\pi\pi}\rvert = 1.33$ $\lvert g_{K\bar{K}}\rvert = 3.63$ $\lvert g_{\eta_8\eta_8}\rvert = 2.85$	$0.983 - i\,0.007$ $\lvert g_{\pi\pi}\rvert = 0.89$ $\lvert g_{K\bar{K}}\rvert = 3.59$ $\lvert g_{\eta_8\eta_8}\rvert = 2.61$	$0.987 - i\,0.011$ $\lvert g_{\pi\pi}\rvert = 1.18$ $\lvert g_{K\bar{K}}\rvert = 3.83$
$a_0(980)$	$1.055 - i\,0.025$ $\lvert g_{\pi\eta_8}\rvert = 3.88$ $\lvert g_{K\bar{K}}\rvert = 5.50$	$1.032 - i\,0.042$ $\lvert g_{\pi\eta_8}\rvert = 3.67$ $\lvert g_{K\bar{K}}\rvert = 5.39$	$1.030 - i\,0.086$ $\lvert g_{\pi\eta_8}\rvert = 4.08$ $\lvert g_{K\bar{K}}\rvert = 5.60$
κ	$0.784 - i\,0.327$ $\lvert g_{K\pi}\rvert = 5.02$ $\lvert g_{K\eta_8}\rvert = 3.10$	$0.804 - i\,0.285$ $\lvert g_{K\pi}\rvert = 4.93$ $\lvert g_{K\eta_8}\rvert = 2.96$	$0.774 - i\,0.338$ $\lvert g_{K\pi}\rvert = 4.89$ $\lvert g_{K\eta_8}\rvert = 3.00$

** Unitarity Normalization for $\pi\pi$, $\eta\eta$, extra $1/\sqrt{2}$ factor

$$\gamma\gamma \rightarrow \pi^0\pi^0\,, \pi^+\pi^-, K^0\overline{K}^0, K^+K^-, \pi^0\eta$$

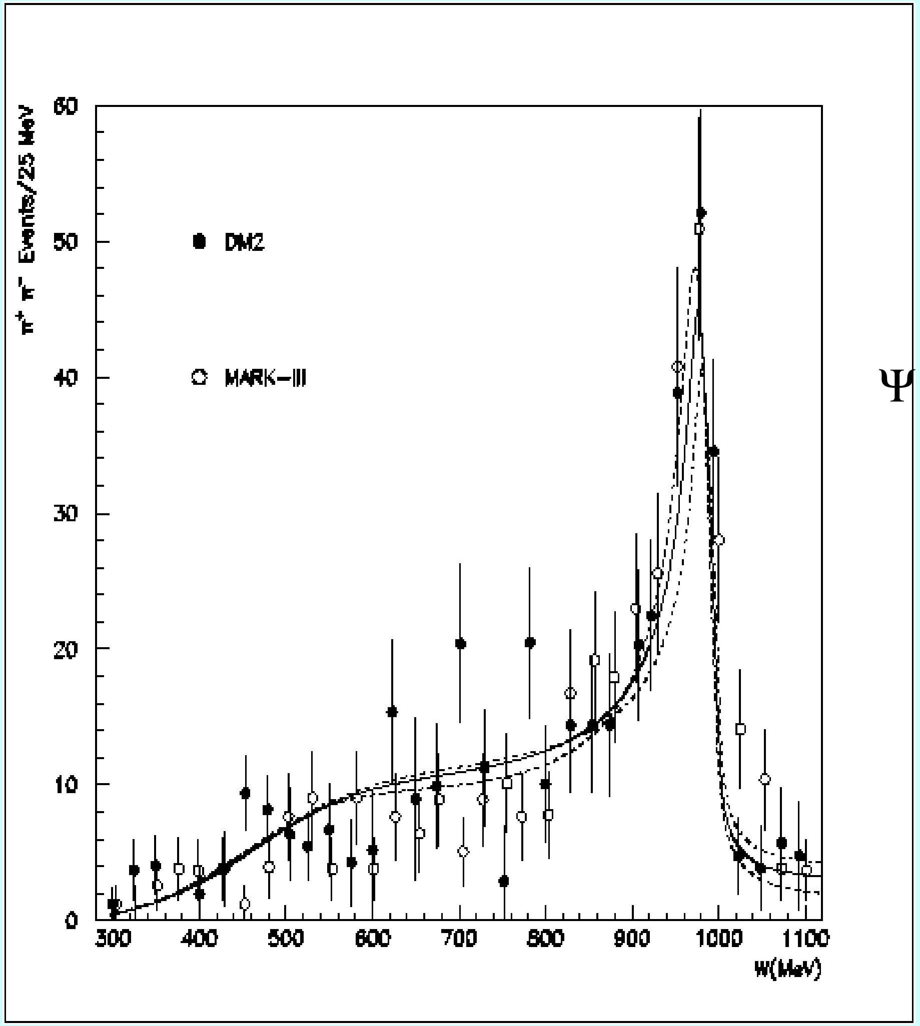
Oset, J.A.O NPA629,739(´99).



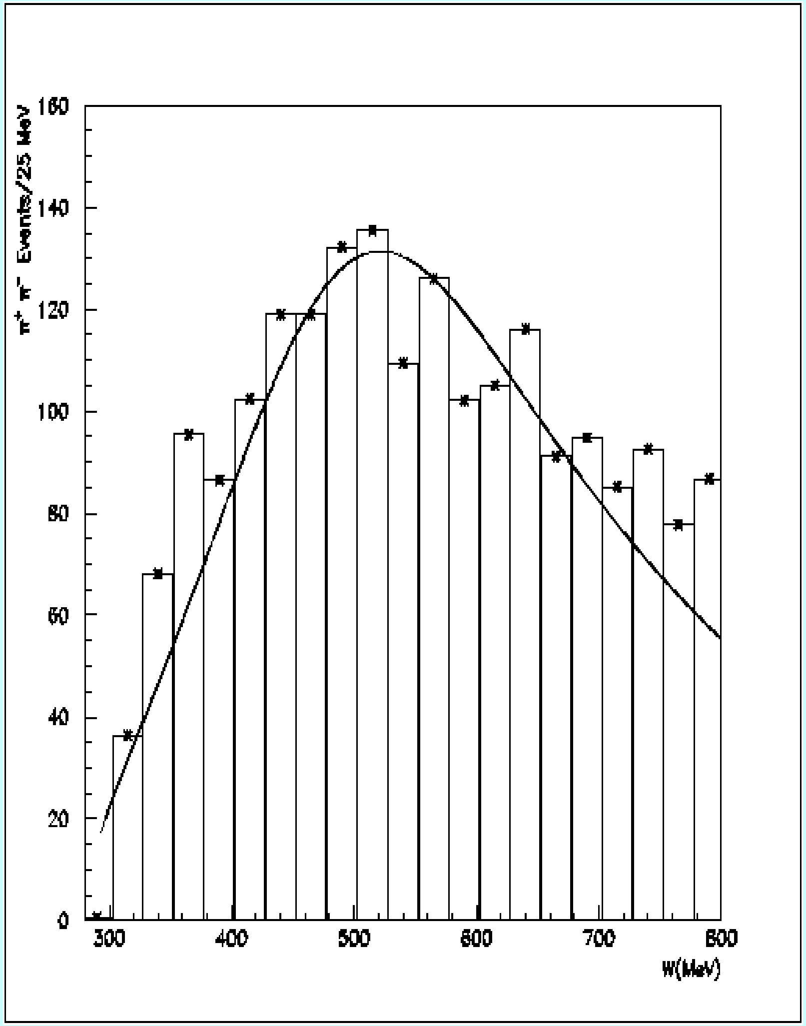
$J/\Psi \rightarrow \phi(\omega)\pi\pi, K\bar{K}$

Meissner, J.A.O NPA679,671(´01).

Oset, Li, Vacas nucl-th/0305041
J/ψ decays to N anti-N meson-meson



Ψ



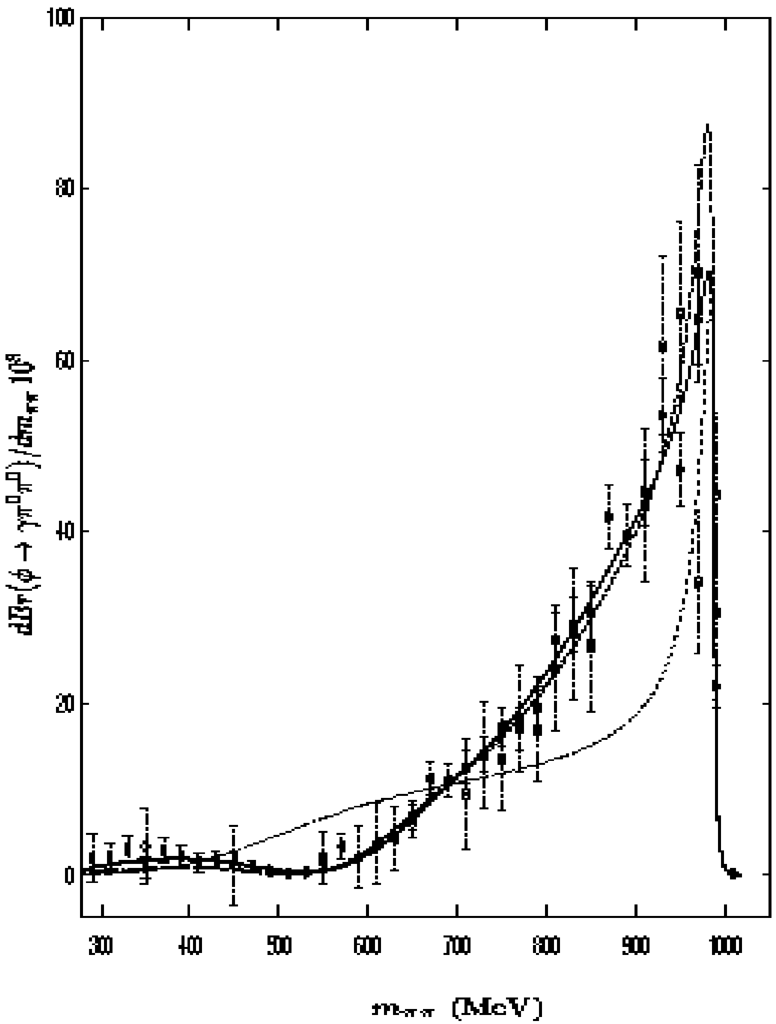
$\phi(1020) \rightarrow \gamma K^0 \bar{K}^0, \gamma \pi^0 \pi^0, \gamma \pi^0 \eta$

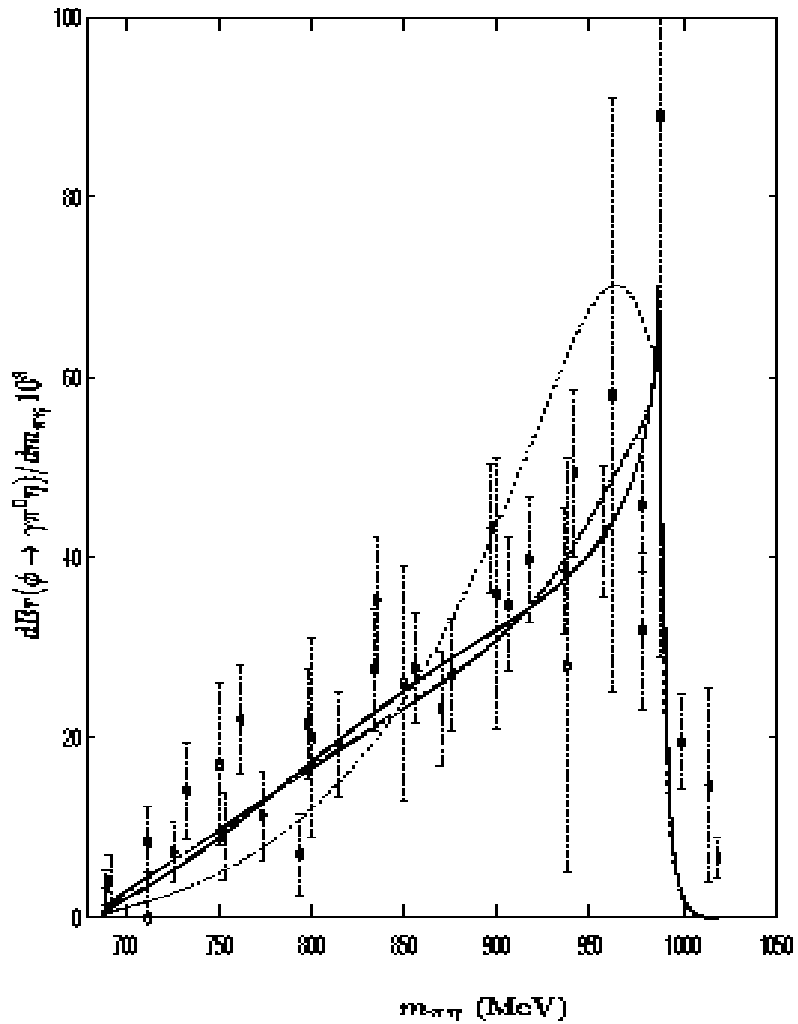
J.A.O. NPA714, 161 (´02)

Oset, Palomar, Roca, Vacas
hep-ph/0306249

BS: $\zeta_0 = -\zeta_1 = +180.83$ MeV, $\delta G_0 = \delta G_1 = 1.42/16\pi^2$.

IAM: $\zeta_0 = -\zeta_1 = +146.42$ MeV, $\delta G_0 = \delta G_1 = 1.54/16\pi^2$.





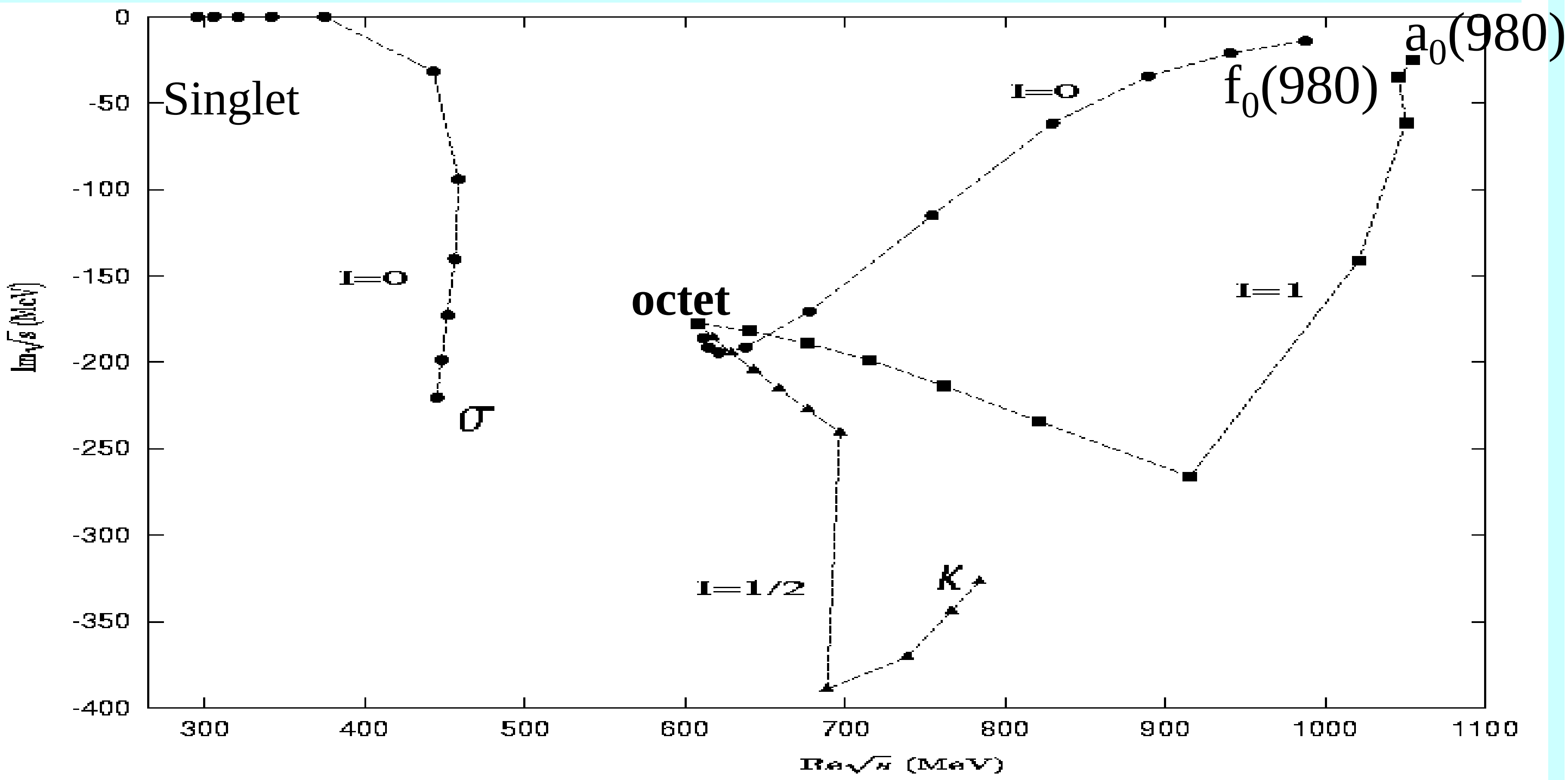
3. SU(3) Analyses

J.A.O. NPA727(03)353; hep-ph/0306031

Continuous movement from a SU(3) symmetric point: from a SU(3) symmetric point with equal masses, which implies equal subtraction constants (Jido,Oset,Ramos,Meissner, J.A.O NPA725(03)181) to the physical limit

$$m_0=300 \ \ MeV$$

$$m_\pi(\lambda)=m_\pi+\lambda(m_0-m_\pi);\ m_K(\lambda)=m_K+\lambda(m_0-m_K);\ m_\eta(\lambda)=m_\eta+\lambda(m_0-m_\eta)$$



$a_0(980)$, κ : Pure Octet States

$$g(a_0 \to K\bar{K}_1) = -\sqrt{\frac{3}{10}}\, g_8 \qquad g(a_0 \to \pi\eta_8) = \frac{1}{\sqrt{5}}\, g_8$$

$$g(\kappa \to K\pi) = \frac{3}{\sqrt{20}}\, g_8 \qquad g(\kappa \to K\eta_8) = -\frac{1}{\sqrt{20}}\, g_8$$

$$\frac{g(a_0 \to \pi\eta_8)}{g(a_0 \to K\bar{K}_1)} = 0.82 \quad 0.70\pm0.04 \qquad \frac{g(\kappa \to K\pi_{\frac{1}{2}})}{g(\kappa \to K\eta_8)} = 3$$

1.64±0.05 from table 1

2.5±0.25 from Jamin,Pich,J.A.O NPB587(00)331

$$\frac{g(a_0 \to K\bar{K}_1)}{g(\kappa \to K\pi_{\frac{1}{2}})} = 0.82 \quad 1.10\pm0.03$$

This is an indication that systematic errors are larger than really shown in table 1 from the 3 T-matrices

From $g(a_0 \to K\bar{K}_1)$, $g(a_0 \to \pi\eta_8)$, $g(\kappa \to K\pi_{\frac{1}{2}})$ we calculate:

$$|g_8| = 8.7 \pm 1.3 \text{ GeV}$$

σ, $f_0(980)$ System: Mixing

$$\sigma = \cos\theta \, S_1 + \sin\theta \, S_8$$

$$f_0 = -\sin\theta \, S_1 + \cos\theta \, S_8$$

σ, $f_0(980)$ System: Mixing

$$\sigma = \cos\theta\, S_1 + \sin\theta\, S_8$$

$$f_0 = -\sin\theta\, S_1 + \cos\theta\, S_8$$

Ideal Mixing: $\cos^2\theta = \frac{2}{3} = 0.67$

If $\bar{q}q$ $\quad \sigma = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}}$; $\quad f_0(980) = \bar{s}s$

Morgan, PLB51,71(´74); $f_0(980)$, $a_0(980)$, $\kappa(\sim1200)$, $f_0(\sim1100)$ $\quad \cos^2\theta = 0.13$

Jaffe, PRD15,267(´77); $f_0(980)$, $a_0(980)$, $\kappa(\sim900)$, $f_0(\sim700)$ $\quad \cos^2\theta = 0.33$

Dual Ideal Mixing

Scadron, PRD26,239(´82); $f_0(980)$, $a_0(980)$, $\kappa(800)$, $f_0(750)$ $\quad \cos^2\theta = 0.39$

Black, Fariborz, Sannino, Schechter PRD59,074026(´99) (Syracuse group);
$f_0(980)$, $a_0(980)$, $\kappa(800)$, $f_0(600)$ $\quad \cos^2\theta = 0.63$

Napsuciale (hep-ph/9803396),+ Rodriguez Int.J.Mod.Phys.A16,3011(´01);
$f_0(980)$, $a_0(980)$, $\kappa(900)$, $f_0(500)$ $\quad \cos^2\theta = 0.87$

$$
\begin{aligned}
g(\sigma \to (\pi\pi)_0) &= -\frac{\sqrt{3}}{4}\cos\theta g_1 - \sqrt{\frac{3}{10}}\sin\theta g_8 , \\
g(\sigma \to (K\bar{K})_0) &= -\frac{1}{2}\cos\theta g_1 + \frac{1}{\sqrt{10}}\sin\theta g_8 , \\
g(\sigma \to (\eta_8\eta_8)_0) &= \frac{1}{4}\cos\theta g_1 - \frac{1}{\sqrt{10}}\sin\theta g_8 , \\
g(f_0 \to (\pi\pi)_0) &= \frac{\sqrt{3}}{4}\sin\theta g_1 - \sqrt{\frac{3}{10}}\cos\theta g_8 , \\
g(f_0 \to (K\bar{K})_0) &= \frac{1}{2}\sin\theta g_1 + \frac{1}{\sqrt{10}}\cos\theta g_8 , \\
g(f_0 \to (\eta_8\eta_8)_0) &= \frac{1}{4}\sin\theta g_1 - \frac{1}{\sqrt{10}}\cos\theta g_8 .
\end{aligned}
$$

Standard Unitary Symmetry Model analysis in vector and tensor nonets of coupling constants.

Okubo, PL5,165(´63)

Glashow,Socolow,PRL15,329(´65)

$$g(\sigma \to \eta_8 \eta_8)=0 \longrightarrow \frac{g_1}{g_8} = \sqrt{\frac{8}{5}}\, tan\,\theta$$

STRATEGY I: We take the most important couplings for the σ and $f_0(980)$ to determining g_8 and θ

$$\frac{g(\sigma \to \pi\pi_0)}{g_8} = -2\sqrt{\frac{3}{10}}\,\sin\theta$$

$$\frac{g(f_0 \to K\bar{K}_0)}{g_8} = \frac{1}{\sqrt{10}}\cos\theta\,(1 + 2\,tan^2\theta)$$

$$|g_8| = 10.5 \pm 0.5 \;\; GeV$$

$$\cos^2\theta = 0.93 \pm 0.01$$

$$|\theta| = (14.9 \pm 0.8)^\circ$$

STRATEGY II: We take the 6 ratios between the σ and $f_0(980)$ couplings to determining $\tan\theta$

$$\underbrace{(g_{\sigma\pi\pi}/g_{f_0\pi\pi})}_{\lambda_{11}};\ \underbrace{(g_{\sigma\pi\pi}/g_{f_0KK})}_{\lambda_{12}};\ \underbrace{(g_{\sigma\pi\pi}/g_{f_0\eta\eta})}_{\lambda_{13}}$$

$$\underbrace{(g_{\sigma KK}/g_{f_0\pi\pi})}_{\lambda_{21}};\ \underbrace{(g_{\sigma KK}/g_{f_0KK})}_{\lambda_{22}};\ \underbrace{(g_{\sigma KK}/g_{f_0\eta\eta})}_{\lambda_{23}}$$

$$\lambda_{11} = \frac{2\tan\theta}{\tan^2\theta - 1} \quad \textit{and so on}$$

We generate 12 numbers: calculating $\tan\theta$ for the maximum and minimum values of λ_{ij} from table 1

	$\tan\theta$	
λ_{11}	0.75	0.65
λ_{12}	0.28	0.25
λ_{13}	0.38	0.33
λ_{21}	0.83	0.71
λ_{22}	0.71	0.36
λ_{23}	1.00	0.46

$\tan\theta = 0.56 \pm 0.25$

STRATEGY II: CALCULATION OF g_8

$$g_{\sigma\pi\pi} = -g_8\sqrt{\frac{6}{5}}\sin\theta \;\; ; \;\; g_{f_0 KK} = \frac{g_8\cos\theta(1+2\tan^2\theta)}{\sqrt{10}} \quad \textit{and so on}$$

- We take the 5 couplings $g_{\sigma\pi\pi}$, $g_{\sigma KK}$, $g_{f0\pi\pi}$, g_{f0KK} , $g_{f0\pi\pi}$ and generate 20 numbers for g_8 .

- From every coupling we take the extrem values for this coupling from table 1 for $\tan\theta+\epsilon$ $(\tan\theta)=0.81$ and $\tan\theta-\epsilon$ $(\tan\theta)=0.31$, giving rise to four numbers/coupling.

$$g_8 = 6.8\pm2.3 \text{ GeV}$$

$\tan\theta=0.56\pm0.25 \Longrightarrow \cos^2\theta=0.76\pm0.16$ $\quad$ $|\theta|$: $(16.4\text{-}39.2)°$

STRATEGY III: χ^2 calculation between the couplings in the table and those predicted by SU(3)

$$\chi^2 = \sum_{R=(\phi\, f_0)\, PQ} \frac{\left(|g(R\to PQ)| - |g(R\to PQ)_{SU(3)}|\right)^2}{\sigma^2_{R,PQ}}$$

III.1) We multiply the error obtained from table 1 for each coupling ***g(R→PQ)*** by a common factor so that $\chi^2_{dof} = 1$.

$\tan\theta = 0.35 \pm 0.10 \quad \Longrightarrow \quad \cos^2\theta = 0.89 \pm 0.06$

$g8 = (8.2 \pm 0.8)$ GeV

III.2) We take the same error for all the couplings ***g(R→PQ)*** such that $\chi^2_{dof} = 1$.

$\tan\theta = 0.59 \pm 0.15 \quad \Longrightarrow \quad \cos^2\theta = 0.86 \pm 0.13$

$g8 = (7.7 \pm 0.8)$ GeV

III.1) We multiply the error obtained from table 1 for each coupling $g_{R\to PQ}$ by a common factor so that $\chi^2_{dof} = 1$.

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$\tan\theta = 0.59 \pm 0.15 \Longrightarrow \cos^2\theta = 0.86 \pm 0.13$

$g8 = (7.7 \pm 0.8)$ GeV

III.3) $\mathrm{Error}[g_{R\to PQ}] = \sqrt{(0.2\, g_{R\to PQ})^2 + \epsilon^2}$ and ϵ is fixed such that $\chi^2_{dof} = 1$.

$\tan\theta = 0.67 \pm 0.15 \Longrightarrow \cos^2\theta = 0.69 \pm 0.10$

$g8 = (7.0 \pm 0.8)$ GeV

Paper

I: $g_{\sigma\pi\pi}$, g_{f0KK}	g_8=10.5±0.5 GeV	$\cos^2\theta$=0.93±0.06
II: All couplings	g_8=6.8±2.3 GeV	$\cos^2\theta$=0.76±0.16
III: Fit, multiplying errors	g_8=8.2±0.8 GeV	$\cos^2\theta$=0.89±0.06
III: Fit, Global error	g_8=7.7±0.8 GeV	$\cos^2\theta$=0.74±0.10
III: Fit, 20% + systematic error	g_8=7.0±0.8 GeV	$\cos^2\theta$=0.69±0.10
From $a_0(980)$, κ	g_8=8.7±1.3 GeV	

We average by taking the extrem values from each entry $x+\epsilon(x)$ and $x-\epsilon(x)$ 12 numbers g_8 10 numbers $\cos^2\theta$

g_8 = 8.2±1.8 GeV

$\cos^2\theta$=0.80±0.15

$|\theta|$: (13-34.4)°

g_1 = 4.69±2.7 GeV

Paper → I, III: Fit, multiplying errors

I: $g_{\sigma\pi\pi}$, g_{f0KK}	g_8=10.5±0.5 GeV	$\cos^2\theta$=0.93±0.06
II: All couplings	g_8=6.8±2.3 GeV	$\cos^2\theta$=0.76±0.16
III: Fit, multiplying errors	g_8=8.2±0.8 GeV	$\cos^2\theta$=0.89±0.06
III: Fit, Global error	g_8=7.7±0.8 GeV	$\cos^2\theta$=0.74±0.10
III: Fit, 20% + systematic error	g_8=7.0±0.8 GeV	$\cos^2\theta$=0.69±0.10
From $a_0(980)$, κ	g_8=8.7±1.3 GeV	

g_8 = 8.2±1.8 GeV 21%

$\cos^2\theta$=0.80±0.15 18%

$|\theta|$: (13-34.4)°

We average the extrem values for g_8 and $\cos^2\theta$ ⇒

g_8 = 8±3 GeV (37%)

$\cos^2\theta$=0.8±0.2 (0.25%)

$|\theta|$: (0-39)°

σ	$g_{\sigma\pi\pi}=2.94-3.01$ $g_{\sigma KK}=1.09-1.30$ $g_{\sigma\eta\eta}=0.04-0.09$	$g_{\sigma\pi\pi}=3.6\pm1.5$ $g_{\sigma KK}=1.0\pm0.4$ $g_{\sigma\eta\eta}=0$	$g_{\sigma\pi\pi}=3.0\pm2.5$ $g_{\sigma KK}=0.9\pm0.7$ $g_{\sigma\eta\eta}=0.$
$f_0(980)$	$g_{f0\pi\pi}=0.89-1.33$ $g_{f0KK}=3.59-3.83$ $g_{f0\eta\eta}=2.61-2.85$	$g_{f0\pi\pi}=3.0\pm1.1$ $g_{f0KK}=3.4\pm0.8$ $g_{f0\eta\eta}=2.9\pm0.6$	$g_{f0\pi\pi}=3.1\pm1.7$ $g_{f0KK}=3.3\pm1.3$ $g_{f0\eta\eta}=2.8\pm0.9$
$a_0(980)$	$g_{a0\pi\eta}=3.67-4.08$ $g_{a0KK}=5.39-5.60$	$g_{a0\pi\eta}=3.7\pm1.4$ $g_{a0KK}=4.5\pm1.4$	$g_{a0\pi\eta}=3.6\pm1.9$ $g_{a0KK}=4.4\pm2.3$
κ	$g_{\kappa K\pi}=4.89-5.02$ $g_{\kappa K\eta}=2.96-3.10$ 1.1-2.1	$g_{\kappa K\pi}=5.5\pm1.7$ $g_{\kappa K\eta}=1.8\pm0.6$	$g_{\kappa K\pi}=5.4\pm2.8$ $g_{\kappa K\eta}=1.8\pm1.0$

We remove the values from Strategy I (higher than the rest)

A) $g_8=7.7\pm1.7$ 22% , $\cos^2\theta=0.76\pm0.15$ 20%, $|\theta|$: (17-39)°

σ	$g_{\sigma\pi\pi}=2.94\text{-}3.01$ $g_{\sigma KK}=1.09\text{-}1.30$ $g_{\sigma\eta\eta}=0.04\text{-}0.09$	$g_{\sigma\pi\pi}=4.0\pm1.4$ $g_{\sigma KK}=1.2\pm0.4$ $g_{\sigma\eta\eta}=0.$	$g_{\sigma\pi\pi}=3.6\pm1.5$ $g_{\sigma KK}=1.0\pm0.4$ $g_{\sigma\eta\eta}=0$	$g_{\sigma\pi\pi}=3.0\pm2.5$ $g_{\sigma KK}=0.9\pm0.7$ $g_{\sigma\eta\eta}=0.$
$f_0(980)$	$g_{f0\pi\pi}=0.89\text{-}1.33$ $g_{f0KK}=3.59\text{-}3.83$ $g_{f0\eta\eta}=2.61\text{-}2.85$	$g_{f0\pi\pi}=2.4\pm1.1$ $g_{f0KK}=3.5\pm0.9$ $g_{f0\eta\eta}=2.81\pm0.6$	$g_{f0\pi\pi}=3.0\pm1.1$ $g_{f0KK}=3.4\pm0.8$ $g_{f0\eta\eta}=2.9\pm0.6$	$g_{f0\pi\pi}=3.1\pm1.7$ $g_{f0KK}=3.3\pm1.3$ $g_{f0\eta\eta}=2.8\pm0.9$
$a_0(980)$	$g_{a0\pi\eta}=3.67\text{-}4.08$ $g_{a0KK}=5.39\text{-}5.60$	$g_{a0\pi\eta}=3.4\pm1.1$ $g_{a0KK}=4.22\pm1.1$	$g_{a0\pi\eta}=3.7\pm1.4$ $g_{a0KK}=4.5\pm1.4$	$g_{a0\pi\eta}=3.6\pm1.9$ $g_{a0KK}=4.4\pm2.3$
κ	$g_{\kappa K\pi}=4.7\text{-}5.02$ $g_{\kappa K\eta}=2.96\text{-}3.10$ 1.1-2.1	$g_{\kappa K\pi}=5.2\pm1.6$ $g_{\kappa K\eta}=1.7\pm0.5$	$g_{\kappa K\pi}=5.5\pm1.7$ $g_{\kappa K\eta}=1.8\pm0.6$	$g_{\kappa K\pi}=5.4\pm2.8$ $g_{\kappa K\eta}=1.8\pm1.0$

B) $g_8 = 8.2\pm1.8$ GeV 21%
$\cos^2\theta=0.80\pm0.15$ 18%
θ: (13-34.4)°

Second SU(3) Analysis:

Couplings of the resonances with SU(3) two pseudosclar eigenstates.

σ	$\lvert g(\sigma \to 1)\rvert = 3.65$ $\lvert g(\sigma \to 8_s)\rvert = 2.67$	$\lvert g(\sigma \to 1)\rvert = 3.86$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$	$\lvert g(\sigma \to 1)\rvert = 3.89$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$
$f_0(980)$	$\lvert g(f_0(980) \to 1)\rvert = 5.35$ $\lvert g(f_0(980) \to 8_s)\rvert = 4.15$	$\lvert g(f_0(980) \to 1)\rvert = 5.07$ $\lvert g(f_0(980) \to 8_s)\rvert = 3.90$	$\lvert g(f_0(980) \to 1)\rvert = 4.20$ $\lvert g(f_0(980) \to 8_s)\rvert = 2.45$
$a_0(980)$	$\lvert g_8\rvert = 8.95$	$\lvert g_8\rvert = 8.83$	$\lvert g_8\rvert = 9.01$
κ	$\lvert g_8\rvert = 8.12$	$\lvert g_8\rvert = 7.94$	$\lvert g_8\rvert = 7.90$

Second SU(3) Analysis:

Couplings of the resonances with SU(3) two pseudosclar eigenstates.

σ	$\lvert g(\sigma \to 1)\rvert = 3.65$ $\lvert g(\sigma \to 8_s)\rvert = 2.67$	$\lvert g(\sigma \to 1)\rvert = 3.86$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$	$\lvert g(\sigma \to 1)\rvert = 3.89$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$
$f_0(980)$	$\lvert g(f_0(980) \to 1)\rvert = 5.35$ $\lvert g(f_0(980) \to 8_s)\rvert = 4.15$	$\lvert g(f_0(980) \to 1)\rvert = 5.07$ $\lvert g(f_0(980) \to 8_s)\rvert = 3.90$	$\lvert g(f_0(980) \to 1)\rvert = 4.20$ $\lvert g(f_0(980) \to 8_s)\rvert = 2.45$
$a_0(980)$	$\lvert g_8\rvert = 8.95$	$\lvert g_8\rvert = 8.83$	$\lvert g_8\rvert = 9.01$
κ	$\lvert g_8\rvert = 8.12$	$\lvert g_8\rvert = 7.94$	$\lvert g_8\rvert = 7.90$

Averaging $a_0(980)$, κ couplings: $\lvert g_8 \rvert = 8.5 \pm 0.5 \text{ GeV}$

Second SU(3) Analysis:

Couplings of the resonances with the SU(3) two pseudosclar eigenstates.

σ	$\lvert g(\sigma \to 1)\rvert = 3.65$ $\lvert g(\sigma \to 8_s)\rvert = 2.67$	$\lvert g(\sigma \to 1)\rvert = 3.86$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$	$\lvert g(\sigma \to 1)\rvert = 3.89$ $\lvert g(\sigma \to 8_s)\rvert = 2.38$
$f_0(980)$	$\lvert g(f_0(980) \to 1)\rvert = 5.35$ $\lvert g(f_0(980) \to 8_s)\rvert = 4.15$	$\lvert g(f_0(980) \to 1)\rvert = 5.07$ $\lvert g(f_0(980) \to 8_s)\rvert = 3.90$	$\lvert g(f_0(980) \to 1)\rvert = 4.20$ $\lvert g(f_0(980) \to 8_s)\rvert = 2.45$
$a_0(980)$	$\lvert g_8\rvert = 8.95$	$\lvert g_8\rvert = 8.83$	$\lvert g_8\rvert = 9.01$
κ	$\lvert g_8\rvert = 8.12$	$\lvert g_8\rvert = 7.94$	$\lvert g_8\rvert = 7.90$

Averaging $a_0(980)$, κ couplings: $|g_8| = 8.5 \pm 0.5$ GeV

A) $g(\sigma \to 1) = g_1 \cos\theta = 4.7 \pm 1.7$

$g(\sigma \to 8) = g_8 \sin\theta = 3.6 \pm 1.3$

$g(f_0 \to 1) = - g_1 \sin\theta = 2.5 \pm 1.2$

$g(f_0 \to 8) = g_8 \cos\theta = 6.6 \pm 1.5$

A) $g_8 = 7.7 \pm 1.7$ 22% , $\cos^2\theta = 0.76 \pm 0.15$ 20%, θ: (17-39)°

$g1 = 5.5 \pm 2.3$

1. The σ is mainly the singlet state. The $f_0(980)$ is mainly the I=0 octet state.
2. Very similar to the mixing in the pseudoscalar nonet but inverted.

η Octet $\longrightarrow$ σ Singlet ; η' Singlet $\longrightarrow$ $f_0(980)$ Octet. In this model θ is positive.

3. Scalar QCD Sum rules, Bijnens, Gamiz, Prades, JHEP 0110 (01) 009.

LINEAR MASS RELATION

$4\, m_\kappa - m_{a0} = 3\,(m_{f0} \cos^2\theta + m_\sigma\ \mathrm{sen}^2\theta)$

2.0-2.2 2.57±0.2

1.3-1.5 2.4±0.4 Quadratic Mass Relation

Sign of θ

Non-strange, strange basis: $ns=\frac{1}{\sqrt{2}}(V_1^1+V_2^2)$ $\quad s=V_3^3$

$$\langle 0|\bar{n}n|\pi\pi\rangle = \sqrt{2}\,B_0\,\Gamma_1^n(s)\ ,$$
$$\langle 0|\bar{n}n|K\overline{K}\rangle = \sqrt{2}\,B_0\,\Gamma_2^n(s)\ ,$$
$$\langle 0|\bar{s}s|\pi\pi\rangle = \sqrt{2}\,B_0\,\Gamma_1^s(s)\ ,$$
$$\langle 0|\bar{s}s|K\overline{K}\rangle = \sqrt{2}\,B_0\,\Gamma_2^s(s)\ .$$

Scalar Form Factors I=0

U.-G. Meissner, J.A.O NPA679(00)671

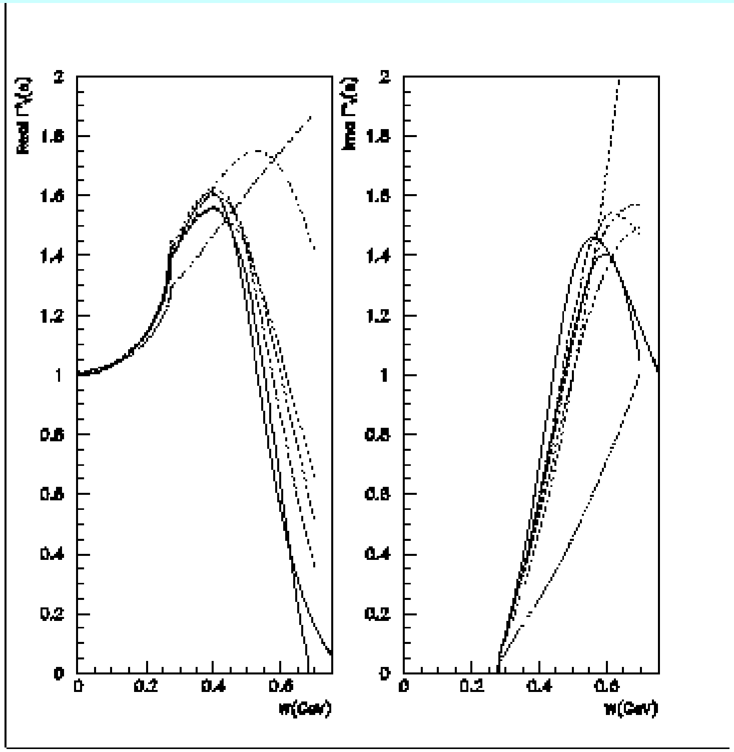
$$\left|\frac{\Gamma_2^s}{\Gamma_2^{\bar{n}n}}\right| = 5.5$$

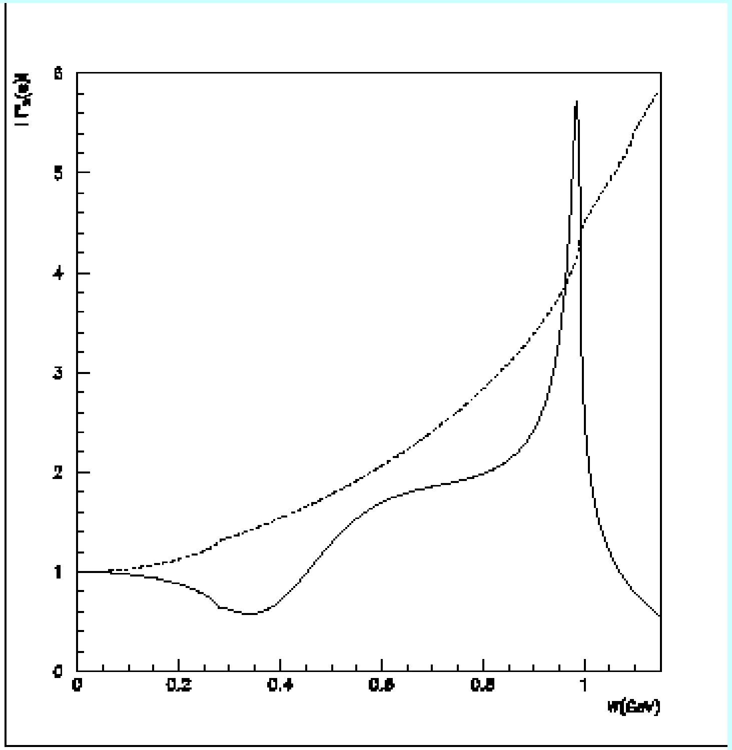
At the $f_0(980)$ peak. Clearly the $f_0(980)$ should be mainly strange and then $\theta>0$

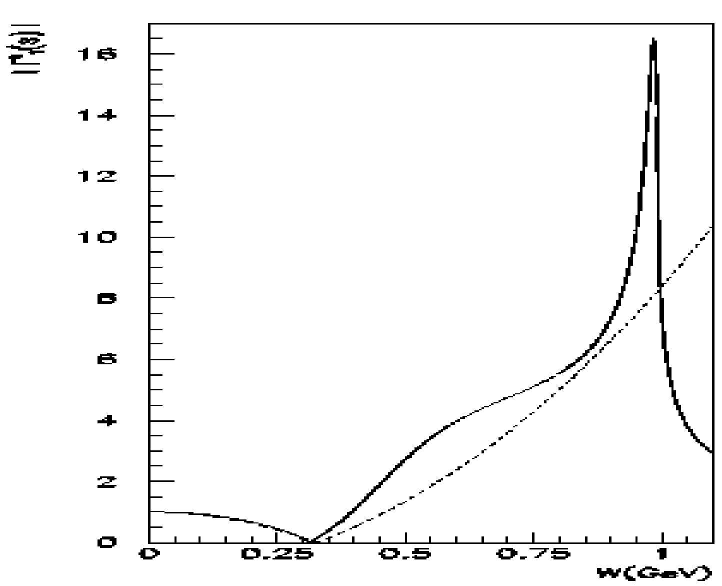
$$\frac{<0|\bar{s}s|f_0>}{<0|\bar{n}n|f_0>} = \frac{\sin\theta\ y + \frac{\cos\theta}{\sqrt{3}}}{\frac{\sin\theta\ y}{\sqrt{3}} + \cos\theta}$$

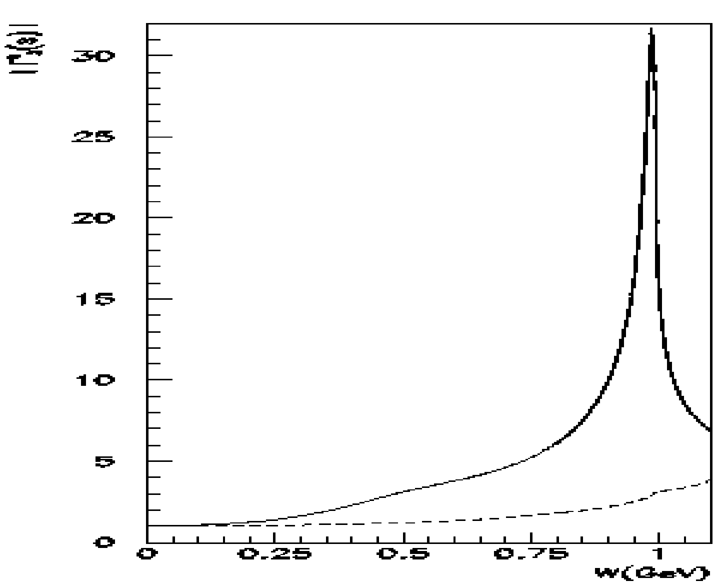
$$y = \frac{<0|s_1|S_1>}{<0|s_8|S_8>} \approx 1 \quad \textit{in}\ \ \mathrm{U(3)}\ \ \textit{symmetry}$$

A) $\theta>0$ ratio= 5.0±3 $\qquad$ $\theta<0$ ratio= 0.3±0.2


w(GeV)
w(GeV)


w(GeV)


w(GeV)


w(GeV)

4. Conclusions

1. $f_0(980)$, $a_0(980)$, $\kappa(900)$, $f_0(600)$ or σ form the lightest scalar nonet.
2. They evolve continuously from the physical situation to a SU(3) symmetric limit and give rise to a degenerate octet of poles and a singlet pole.
3. Several different SU(3) analyses of the scalar resonance couplings constants. They are consistent among them within 20%.
4. $\cos^2\theta=0.77\pm0.15$; $\theta=28 \pm11$; σ is mainly a singlet and the $f_0(980)$ is mainly the I=0 octet state.
5. These scalar resonances satisfy a Linear Mass Relation.
6. The value of the mixing angle is compatible with the one of the Syracuse group, ideal mixing, of the U(3)xU(3) with $U_A(1)$ breaking model of Napsuciale from the U(2)xU(2) model of t´Hooft.