S-wave $\gamma\gamma \to \pi\pi$, $f_0(980) \to \pi\pi$, scalar glueball

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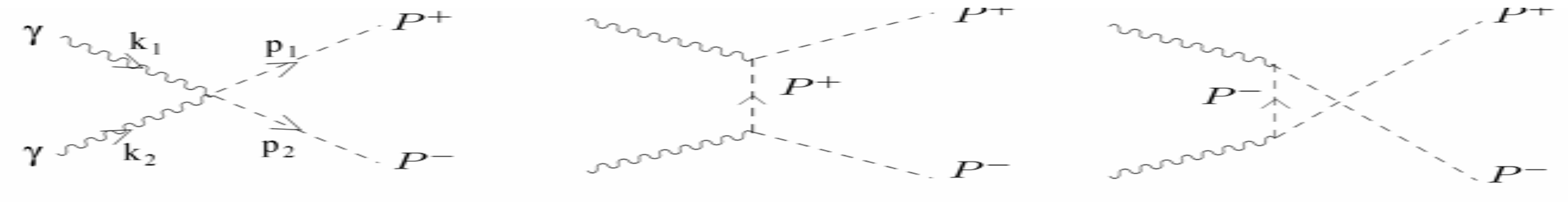
1. Introduction

L. Roca, C. Schat and J.A.O., PLB659,201(2008); Forthcoming

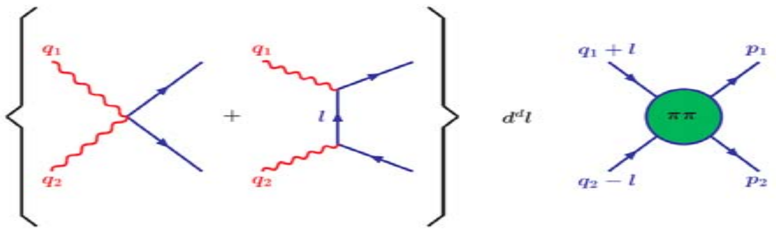
There is no Born term as π^0 is neutral



Final state interactions are enhanced

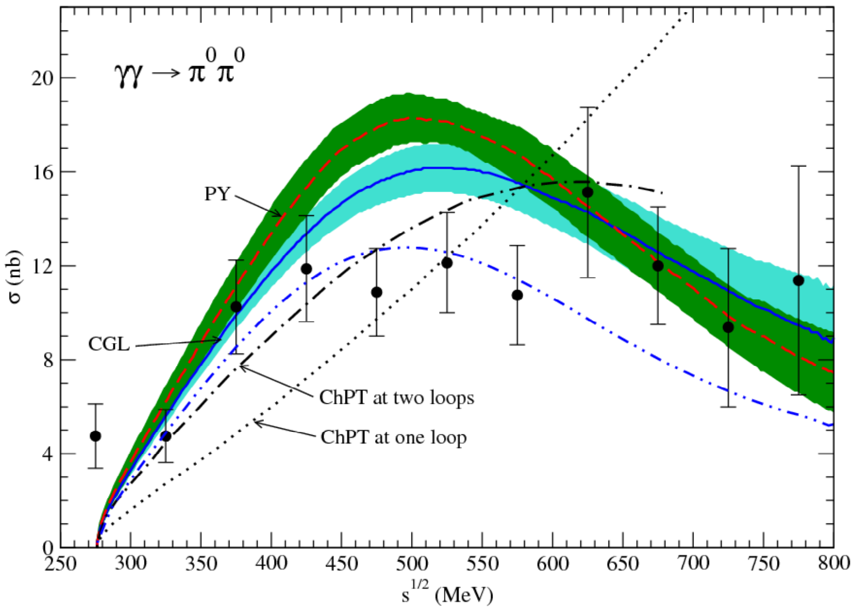
The final state is a two body hadronic state

Good reaction to study the $I = 0$ $\pi\pi$ S-wave



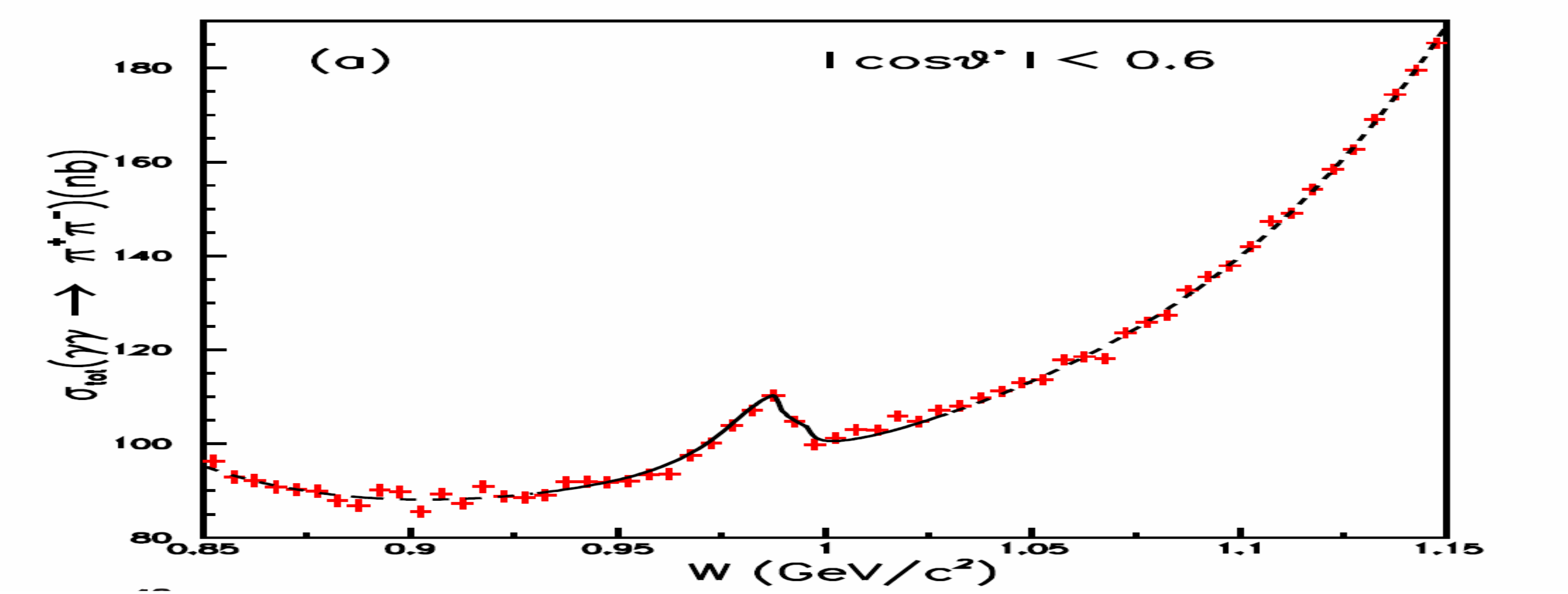
Silver mode for χPT

- One loop χPT is the leading contribution: Bijnens, Cornet NPB296,557(1988); PRD37,2423(1988)
- No counterterms, pure χPT quantum prediction (the agreement with data was not satisfactory)
- Two loop calculation in χPT was performed: Bellucci, Gasser, Sainio NPB423,80 (1994), revised in Gasser,Ivanov, Sainio NPB728,31 (2005).
- Better agreement with data. The three counterterms were fixed according to the resonance saturation hypothesis.



Data points:
$\sigma(\gamma\gamma \to \pi^0\pi^0)$ was measured
for $|\cos\theta| < 0.8$ by Crystal Ball Collaboration,
H. Marsiske *et al.* PRD41,3324 (1990)

New accurate data on $\gamma\gamma \to \pi^+\pi^-$ $|\cos\theta| < 0.6$
T. Mori *et al.* [Belle Coll.] PRD75,051101 (2007)
Remarkable resolution of the $f_0(980)$ resonance.



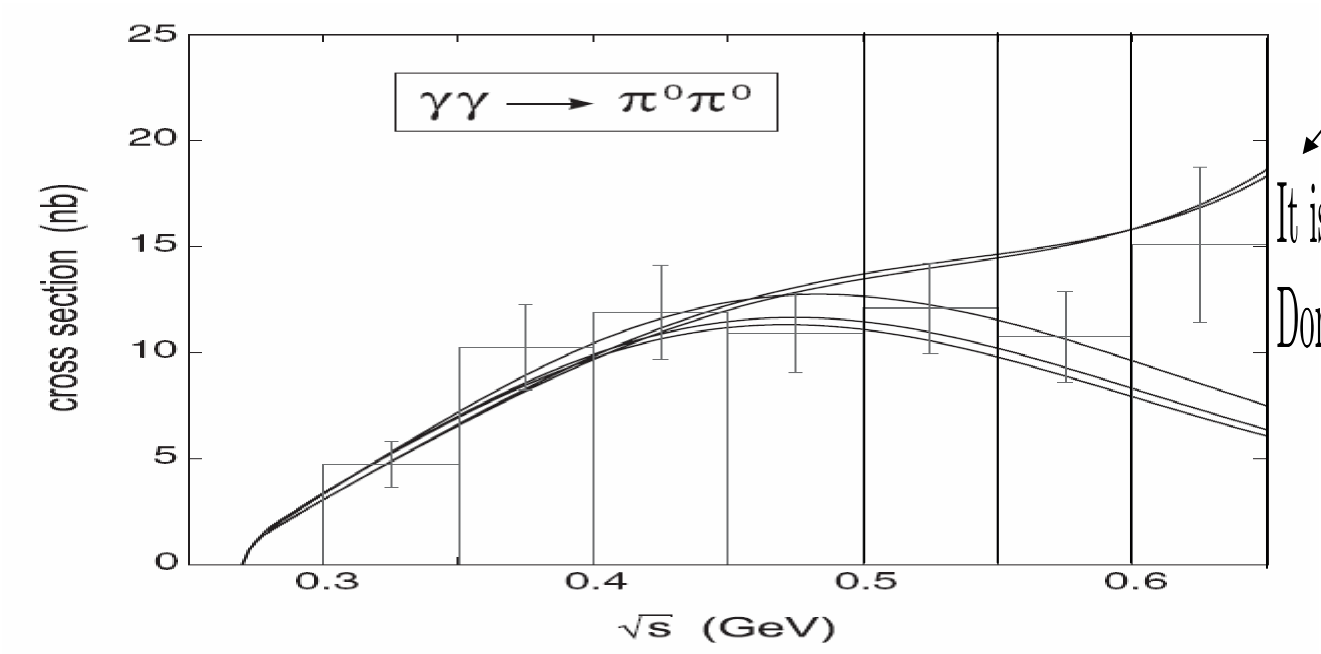
$$M = 985.6^{+1.2}_{-1.6}(stat)^{+1.1}_{-1.6}(sys)\ \text{MeV}\ ,\ \Gamma_{\pi^+\pi^-} = 34.2^{+13.9}_{-11.8}(stat)^{+8.8}_{-2.5}(sys)\ \text{MeV}$$

$$\Gamma(f_0(980) \to \gamma\gamma) = 205^{+95}_{-83}(stat)^{+147}_{-117}(sys)\ \text{eV}$$

E.Oset,J.A.O, NPA629,739(1998): $\Gamma(f_0(980) \to \gamma\gamma)$=0.2 KeV

Recently:

Pennington, PRL97,011601(2006) employs a dispersive method to calculate the S-wave $\gamma\gamma \to \pi\pi$ for low energies.
This approach was settled in Pennington,Morgan PLB272,134(1991), revised in Pennington DAFNE Physics Handbook, Vol.1



Explodes at around 1 GeV

It is also the case for Donoghue and Holstein, PRD48,137(1993)

Large ambiguity because of the phase of the $\gamma\gamma \to \pi\pi$ $I = 0$ S-wave for $s > 4m_K^2$
$\sqrt{s} = 0.5$, 0.55, 0.6, 0.65 GeV one has 20, 45, 92 and 200% error, respectively

$\Gamma(\sigma \to \gamma\gamma) = 4.09 \pm 0.29$ KeV only a 7% error.

- Is it possible to reduce such large uncertainty for $\sqrt{s} \gtrsim 0.5$ GeV?

- Revise the value and given error for the $\Gamma(\sigma \to \gamma\gamma)$ width

- How to extend the dispersive formalism to study more resonances apart from the σ, e.g. the $f_0(980)$?

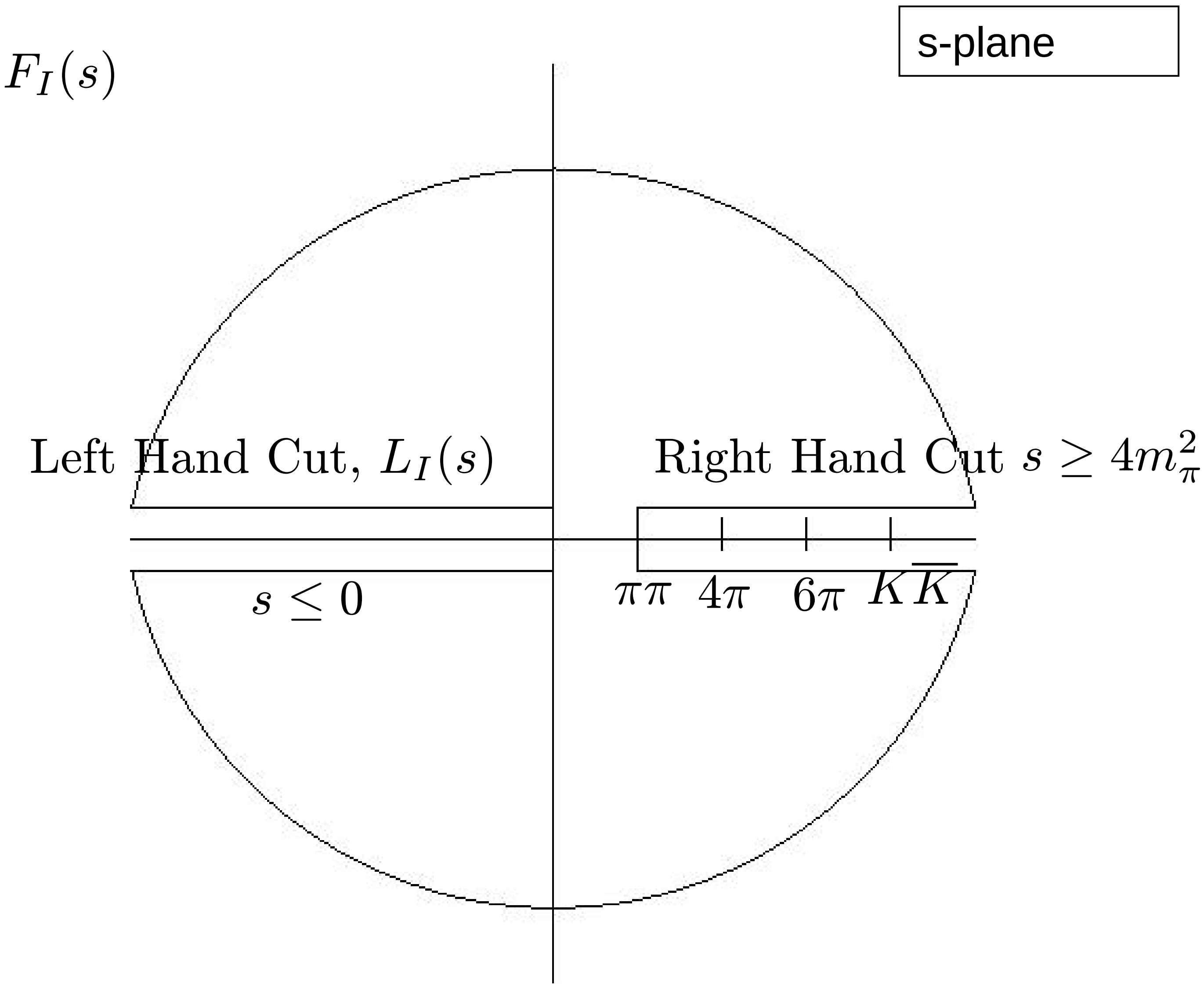
2. Dispersive approach

$F_I(s)$ is the S-wave $\gamma\gamma \to \pi\pi$ with isospin I, $I = 0$, 2

On the complex s-plane $F_I(s)$ is an analytic function except for two cuts along the real axis:
Unitarity cut: $s \geq 4m_\pi^2$
Left hand cut: $s \leq 0$.

$L_I(s)$ is the left hand cut contribution of $F_I(s)$

$F_I(s) - L_I(s)$ has no left hand cut


s-plane
$F_I(s)$
Left Hand Cut, $L_I(s)$
Right Hand Cut $s \geq 4m_\pi^2$
$s \leq 0$
$\pi\pi$
4π
6π
$K\overline{K}$

First, we consider the approach of
Pennington, Morgan PLB272,134 (1991)

1) Build the auxiliary function $w_I(s)$

$$w_I(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\phi_I(s')}{s'(s'-s)}ds'\right]$$

$\phi_I(s)$ is the phase of $F_I(s)$, modulo π.
It must be continuous and $\phi_I(4m_\pi^2) = 0$

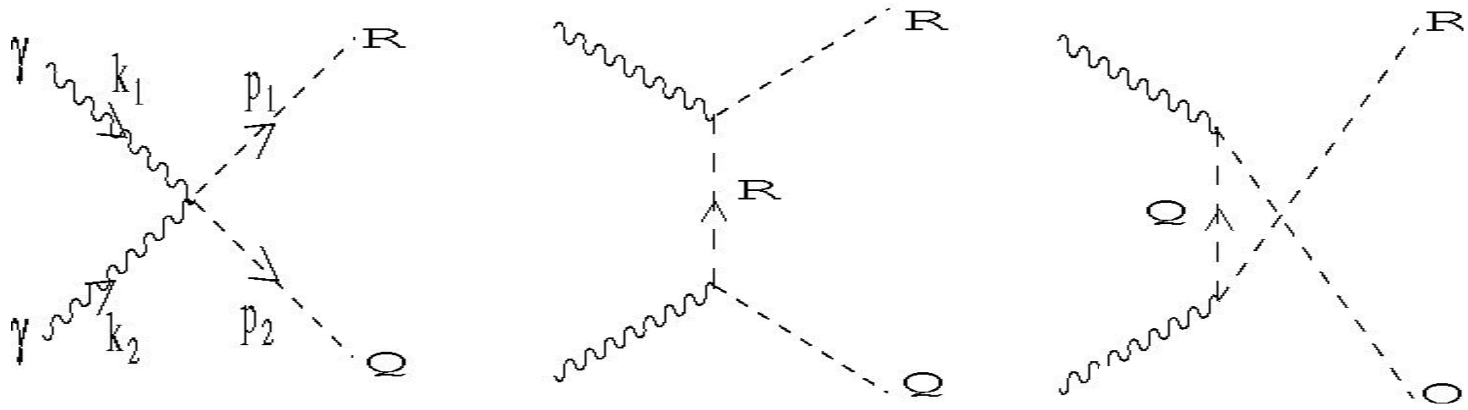
$F_I(s)/\omega_I(s)$ has no right hand cut

2) Take a twice subtracted dispersion relation for $(F_I(s) - L_I(s))/\omega_I(s)$

$$F_I(s) = L_I(s) + a_I\,\omega_I(s) + c_I\, s\,\omega_I(s) + \frac{s^2}{\pi}\omega_I(s)\int_{4m_\pi^2}^{\infty} \frac{L_I(s')\sin\phi_I(s')}{s'^2(s'-s)|\omega_I(s')|}ds'$$

Low's theorem $F_I(s) \to B_I(s) + \mathcal{O}(s)$ for $s \to 0$ then $a_I = 0$.

$B_I(s)$ is the Born term, included in $L_I(s)$



Watson final state theorem

- Elastic case, only $\pi\pi$. Above threshold,

$$\mathrm{Im}F(t) = F(t)\rho(t)T^*_{\pi\pi}(t)$$

Since the left hand side is real then the phases of $F(t)$ ($\phi(s)$) and $T_{\pi\pi}(t)$ ($\varphi(s)$) are equal (modulo π)

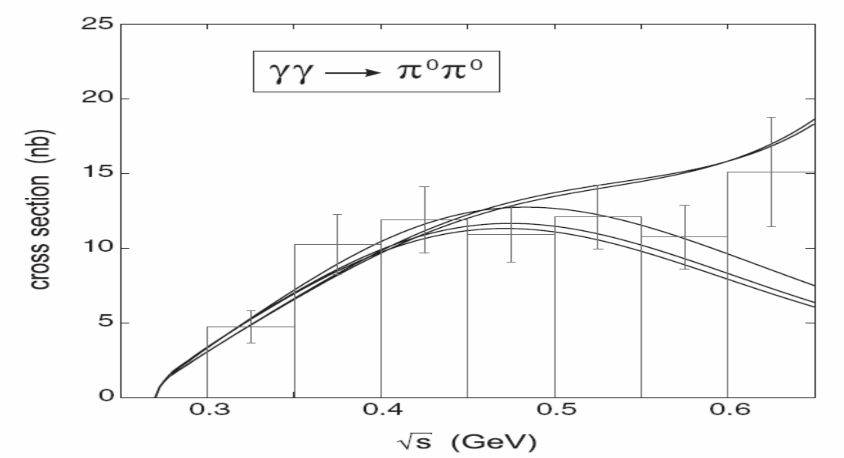
For the coupled channel case, this theorem can also be applied if $\eta \simeq 1$

Corolary: $\phi(s) = \delta(s)$ for $s \leq 4m_K^2$

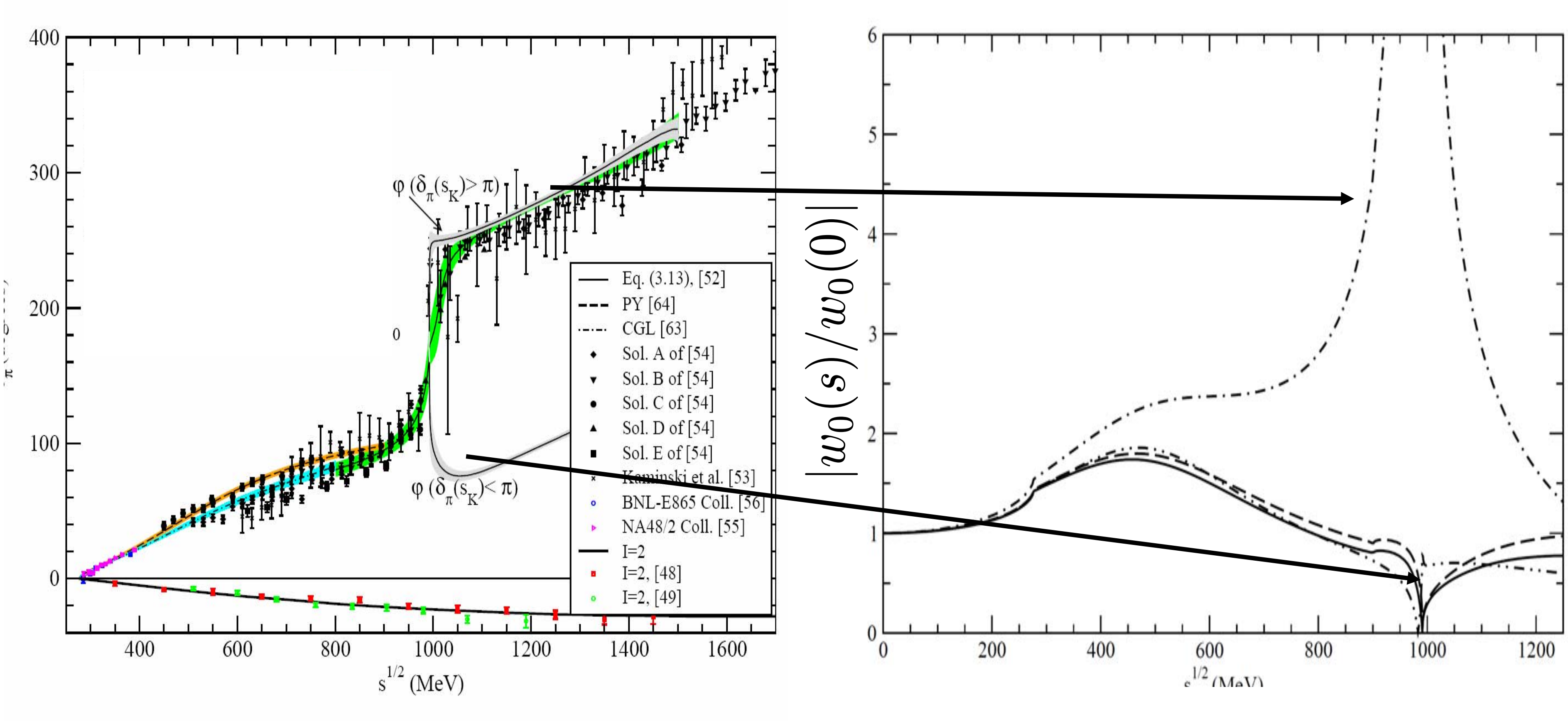
$t_{\pi\pi} = \rho\, T_{\pi\pi} = \sin\delta_\pi\, e^{i\delta_\pi}$, $\delta_\pi(4m_\pi^2) = 0$, δ_π is continuous and at most differs by modulo π from the phase of $T_{\pi\pi}$

This happens when δ_π crosses π ($\sin\delta_\pi < 0$)

Above $K\bar{K}$ threshold $\eta \neq 1$, it suddenly decreases



The rapid increase for $\sqrt{s} \gtrsim 0.5$ GeV is due to the phase used to calculate $\omega_0(s)$



A rapid **increase** in the phase used in the Omnès implies a **maximum**, while a rapid **decrease** implies a **minimum**.

This makes $|w_0(s)|$ to pass from 0 to $+\infty$ when evolving continuously from one scenario to the other.

This can be achieved e.g. by employing $\phi_0(s) = \varphi(s)$ in the transition $\delta_\pi(s_K) \to \pi$ from below to above π.

s_K is the Kaon threshold

How to build a continuous $\omega(s)$ was shown in L. Roca and J.A.O. PLB651 (2007)139

For a rapid increasing $\phi_0(s)$ above s_K use:

$$\Omega(s) = \left(1 - \frac{s}{s_1}\right) \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\phi_0(s')}{s'(s'-s)} ds'\right]$$

This factor gives a zero at s_1

With $s_1 \simeq s_K$ the point at which $\phi_0(s_1) = \pi$

For $I = 2$ Watson's final state theorem applies to good approximation in the whole energy range and $w_2(s)$ is smooth and well behaved.
$\phi_2(s) = \delta_\pi(s)_2$

Our approach to $\gamma\gamma \to (\pi\pi)_I$

We perform a twice subtracted dispersion relation for $(F_0(s) - L_0(s))/\Omega_0(s)$

$$F_0(s) = L_0(s) + c_0 s \Omega_0(s) + \frac{s^2}{\pi}\Omega_0(s) \int_{4m_\pi^2}^{\infty} \frac{L_0(s') \sin \overline{\phi}_0(s')}{s'^2 (s' - s)|\Omega_0(s')|} ds'$$
$$+ \theta(z) \frac{\omega_0(s)}{\omega_0(s_1)} \frac{s^2}{s_1^2} (F_0(s_1) - L_0(s_1)) \; .$$

$\Omega_0(s) = \left(1 - \theta(z)\frac{s}{s_1}\right) \omega_0(s)$

c_0, c_2 and $F_0(s_1) - L_0(s_1)$ have to be fixed

$\overline{\phi}_0(s)$ is the phase of $\Omega_0(s)$.
For $s > s_1$ and $z = +1$ $\overline{\phi}_0(s) = \phi_0(s) - \pi$

Our equation is equivalent to a *three times* subtracted dispersion relation for $(F_0(s) - L_0(s))/\omega_0(s)$

We have taken two subtractions at $s = 0$ and one at s_1

We could have taken them also at $s = 0$

$$F_0(s) = L_0(s) + c_0 s w_0(s) + d_0 s^2 w_0(s) + \frac{s^3 w_0(s)}{\pi} \int_{4m_\pi^2}^{\infty} \frac{L_0(s') \sin \phi_0(s')}{s'^3 (s' - s)|\omega_0(s')|} ds' \, ,$$

In the other form the more physical (continuous) Omnès function $\Omega_0(s)$ is used

In this way the the $f_0(980)$ can be also very easily handled because $\Omega_0(s_1) = 0$, it is isolated.

More on the phase $\phi_0(s)$

- Watons's theorem: $\phi_0(s) = \delta_\pi(s)$ for $s \leq s_K$

- For $1.1 \lesssim \sqrt{s} \lesssim 1.5$ GeV, $\eta \simeq 1$ and Watson's final state theorem approximately applies.
Then $\phi_0(s) \simeq \delta^{(+)}(s)$ modulo π

F.J. Ynduráin, PLB578 (2004) 99

In order to fix the integer factor in front of π one needs to follow the track of $\phi_0(s)$ in the narrow region $1 \lesssim \sqrt{s} \lesssim 1.1$ GeV (so that continuity can be invoked)

Two main physical effects:

- The appearance of the $f_0(980)$ on top of the $K\bar{K}$ threshold.
- The cusp effect of the latter

For $1.05 < \sqrt{s} < 1.1$ GeV there are no further narrow structures. Observables evolve smoothly with energy.

Two Options:

A.- $\phi_0(s)$ keeps increasing with energy for $s > s_K$.
Then matches smoothly with $\simeq \delta_\pi(s)_0$ for $\sqrt{s} \gtrsim 1.05$ GeV.

$$z = +1$$

E.g. $\varphi(s)$ for $\delta_\pi(s_K)_0 \geq \pi$

B.- The cusp effect makes the deriative of $\phi_0(s)$ discontinuous.
$\phi_0(s)$ decreases rapidly with energy for $s > s_K$.
Then matches smoothly with $\simeq \delta_\pi(s)_0 - \pi$ for $\sqrt{s} \gtrsim 1.05$ GeV

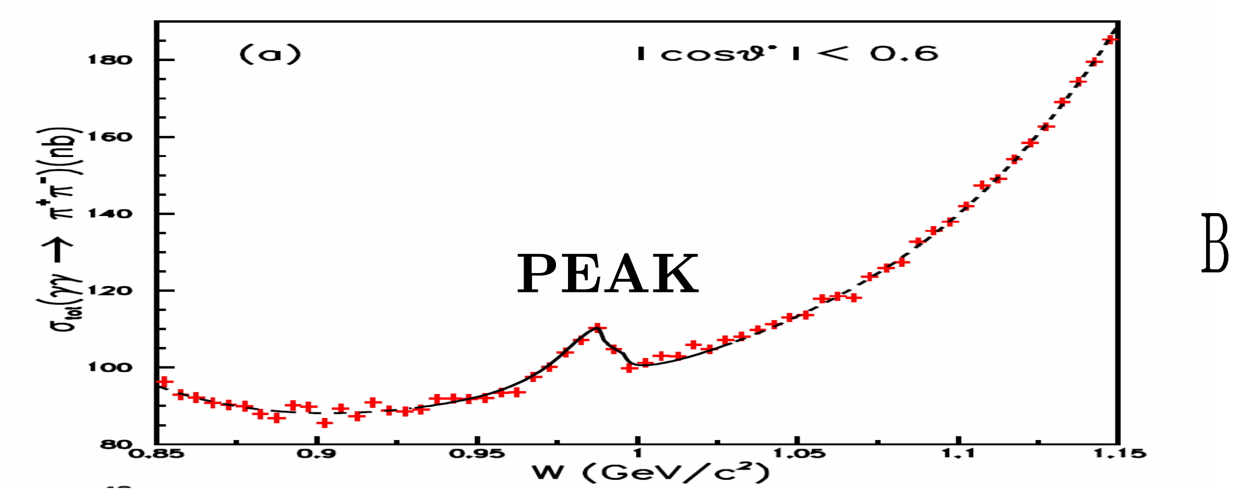
$$z = -1$$

E.g. $\varphi(s)$ for $\delta_\pi(s_K)_0 < \pi$

An ambiguity of π is left for $\phi_0(s)$ and $s > s_K$

$$F_0(s) = L_0(s) + c_0 s\Omega_0(s) + \frac{s^2}{\pi}\Omega_0(s)\int_{4m_\pi^2}^{\infty}\frac{L_0(s')\sin\overline{\phi}_0(s')}{s'^2(s'-s)|\Omega_0(s')|}ds'$$
$$+ \quad \theta(z)\frac{\omega_0(s)}{\omega_0(s_1)}\frac{s^2}{s_1^2}(F_0(s_1) - L_0(s_1)) \ .$$

A) If $z = -1$ (ϕ_0 decreases for $s \geq s_K$) then $|F_0(s)|$ has a minimum
B) If $z = +1$ (ϕ_0 increases for $s \geq s_K$) then $|F_0(s)|$ has a maximum



B) is the physical case ($z = +1$)

T. Mori *et al.* [Belle Coll.] PRD75, 051101 (2007)

The size of the peak is controlled by $F_0(s_1) - L_0(s_1)$

This constant is fixed such that $\Gamma(f_0(980) \to \gamma\gamma)$ is compatible with

$$\Gamma(f_0(980) \to \gamma\gamma) = 205^{+95}_{-83}(stat)^{+147}_{-117}(sys)\ \mathrm{eV}$$

From T. Mori *et al.* [Belle Coll.] PRD75, 051101 (2007)

c_0 and c_2 are fixed by Low's theorem and χPT:

$$\begin{aligned} \gamma\gamma \longrightarrow \pi^0\pi^0 \qquad & F_N(s) = -\frac{1}{\sqrt{3}}F_0 + \sqrt{\frac{2}{3}}F_2 \\ \gamma\gamma \longrightarrow \pi^+\pi^- \qquad & F_C(s) = -\frac{1}{\sqrt{3}}F_0 - \sqrt{\frac{1}{6}}F_2 \end{aligned}$$

c_0 and c_2 are fixed by Low's theorem and χPT:

1. $F_C(s) - B_C(s)$ vanishes linearly in s for $s \to 0$
2. $F_N(s)$ vanishes linearly in s for $s \to 0$

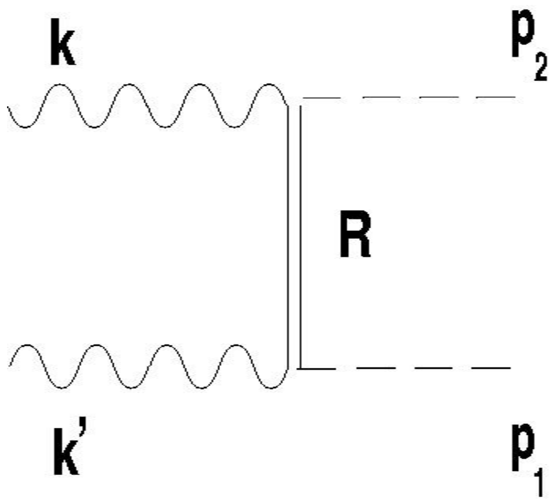
The coefficients are calculated from one loop χPT
We use either f_π^2 or f^2 in the expressions $\propto 1/f^2$
Estimate for higher orders, $\sim 12\%$ of uncertainty
(taken into account in the error analysis)

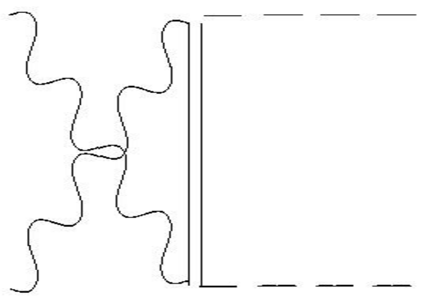
$L_I(s)$ is due to the $\gamma\pi \to \gamma\pi$ dynamics

At $s = 0$ it is given by the Born Term by Low's theorem
This exchange of pions gives rise to the left hand cut for $s < 0$
The main contribution for low energies.

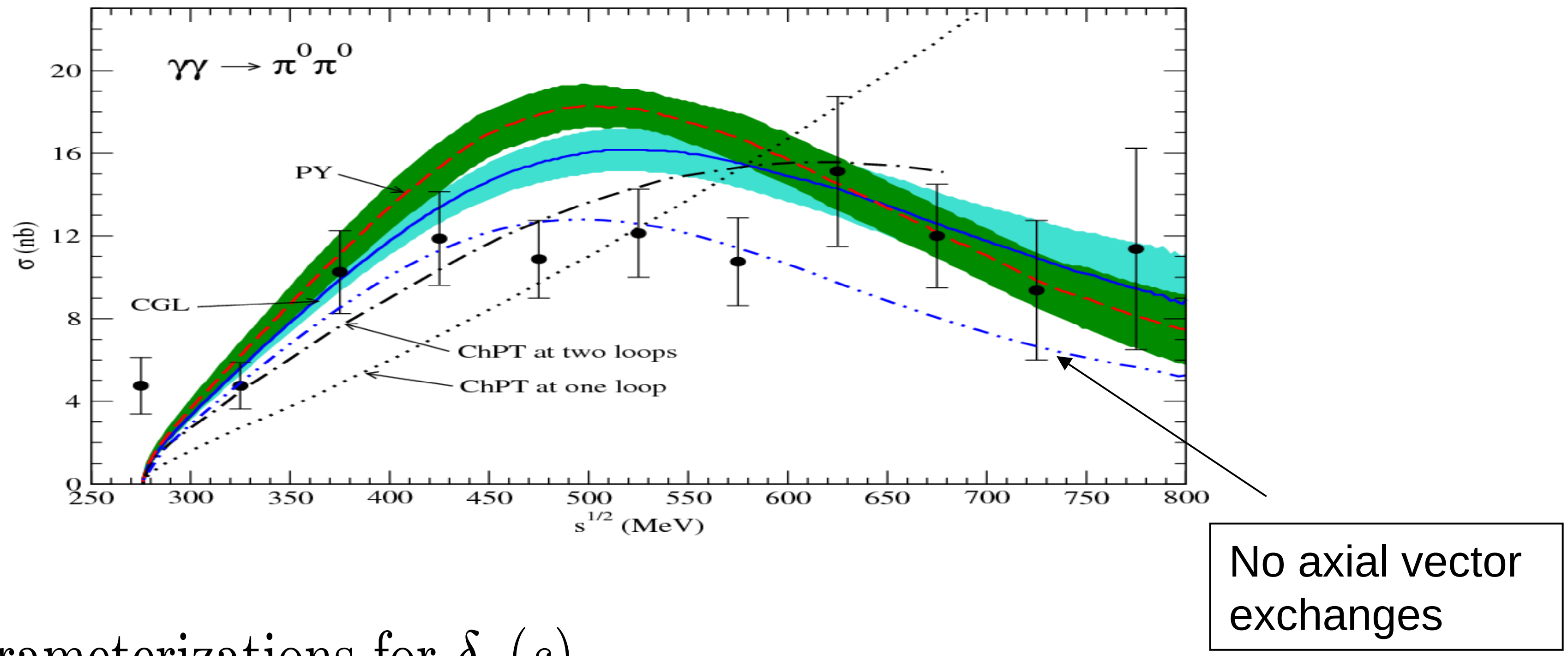
Vector $J^{PC} = 1^{--}$ and Axial-Vector 1^{++}, 1^{+-} exchanges

$s < -M_R^2 + m_\pi^2/2$





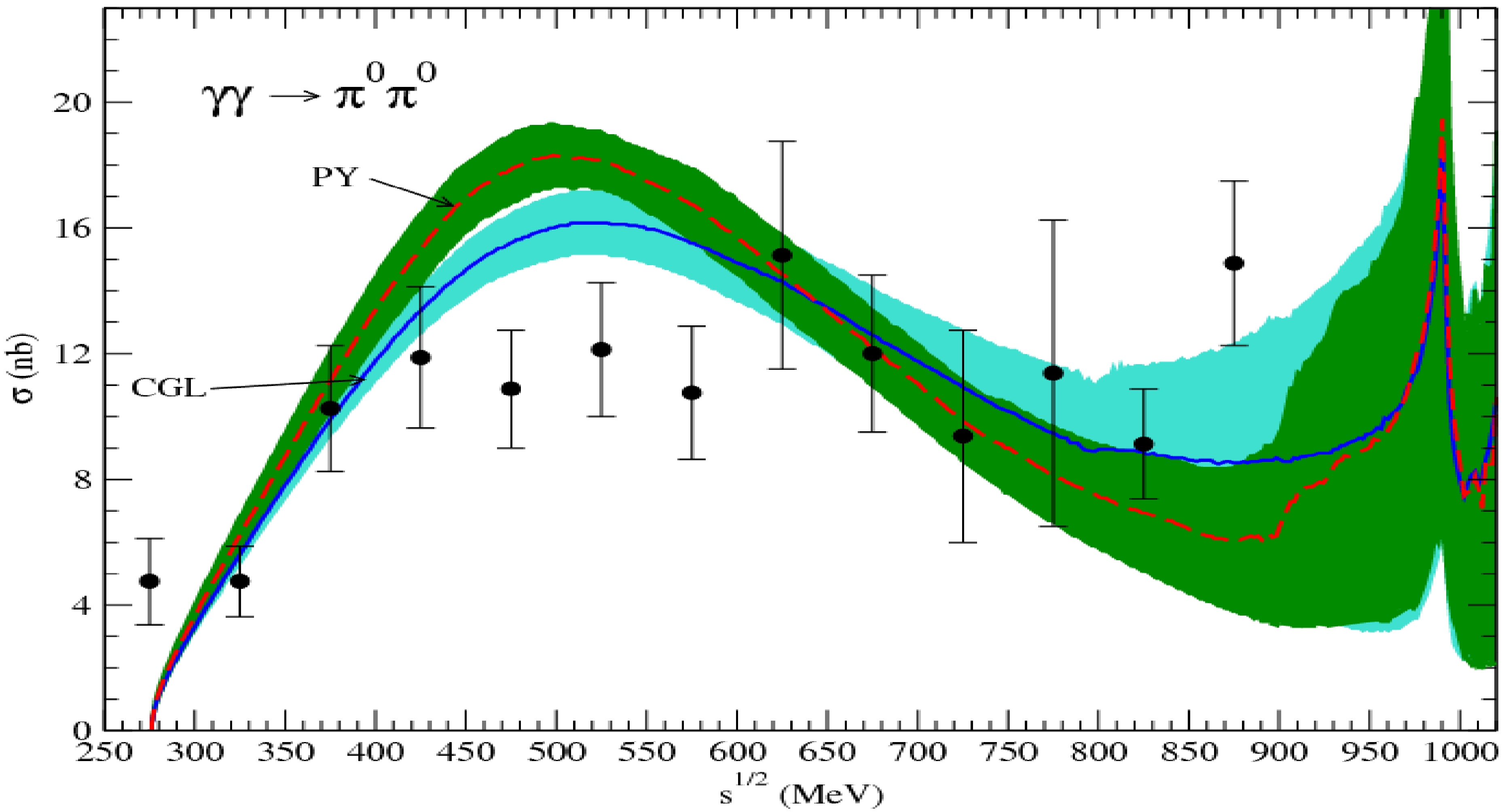
The axial-vector 1^{++} exchanges are the most important
They give rise to $L_9 + L_{10}$ the $\mathcal{O}(p^4)$ counterterm in $\gamma\gamma \to \pi^+\pi^-$



Parameterizations for $\delta_\pi(s)_0$

Light Blue: Colangelo, Gasser, Leutwyler, NPB603,125 (2001)
$\sqrt{s} < 0.8$ GeV (Parameterization I)
Dark Green: Yndurain, Pelaez, PRD68,074005 (2003)
Similar phase shifts to Unitary χPT results
$\sqrt{s} < 0.9$ GeV (Parameterization II)

Above those energies up to $\sqrt{s} = 1.5$ GeV
Energy dependent analysis (K-matrix) of data Hyams *et al.* NPB64,134 (1973).



To improve the precision in the $f_0(980)$ region we need to work out $\gamma\gamma \to \pi^+\pi^-$ as well

Error bands include:
Uncertainties in the parameterizations CGL, PY and Hyams et al.
The uncertainties in $\phi_0(s)$ above s_K
The bound $\sigma(\gamma\gamma \to \pi^0\pi^0) \leq 40$ nb at s_1
c_0 and c_2 calculation employing either f_π^2 or f^2.

4. $\Gamma(\sigma \longrightarrow \gamma\gamma)$

Calculation of the coupling:

Analytical extrapolation to the second Riemann sheet where the σ pole locates, s_σ

Unitarity $4m_\pi^2 \leq s \leq 4m_K^2$

$$F_0(s+i\epsilon) - F_0(s-i\epsilon) = -2iF_0(s+i\epsilon)\rho(s+i\epsilon)T^0_{II}(s-i\epsilon)$$

Continuity in the change of sheets: $F_0(s-i\epsilon) = \widetilde{F}_0(s+i\epsilon)$, $T_I(s-i\epsilon) = T_{II}(s+i\epsilon)$

$\widetilde{F}_0(s) = F_0(s)\left(1 + 2i\rho(s)T^{I=0}_{II}(s)\right)$. On the second sheet: $\widetilde{F}_0(s)$ and $T^{I=0}_{II}(s)$

Around s_σ

$$T_{II}^{I=0} = -\frac{g^2_{\sigma\pi\pi}}{s_\sigma - s} \quad \widetilde{F}_0(s) = \sqrt{2}\frac{g_{\sigma\gamma\gamma}g_{\sigma\pi\pi}}{s_\sigma - s}$$

$$\boxed{\frac{g^2_{\sigma\gamma\gamma}}{g^2_{\sigma\pi\pi}} = -\frac{1}{2}F_0(s_\sigma)^2\left(\frac{\beta(s_\sigma)^2}{8\pi}\right)^2}$$

Ratio independent of the **strong** coupling used

This is not used in our dispersive approach.

M.Albaladejo, J.A.O. arXiv:0801.4929 [hep-ph] (**talk of M. Albaladejo, Friday 15.05h**)

$s_\sigma = (456 \pm 6 - i\,241 \pm 7)$ MeV^2 (AO)

Caprini,Colangelo,Leutwyler PRL96,132001 (2006)

$s_\sigma = (441^{+16}_{-8} - i\,272^{+9}_{-13})^2$ MeV^2 (CCL)

$|\frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}}| = (2.02 \pm 0.15)\cdot 10^{-3}$ CCL

$|\frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}}| = (1.85 \pm 0.13)\cdot 10^{-3}$ AO

$|\frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}}| = (2.53 \pm 0.09)\cdot 10^{-3}$ CCL

M. Pennington, PRL97 (2006) 011601, it is a 20% Bigger

Pennington does not include axial-vector exchanges (1^{++} and 1^{+-})$\to$ 10%

The other 10% comes from the improvement in our approach.

$$\Gamma(\sigma \to \gamma\gamma) = \frac{|g_{\sigma\gamma\gamma}|^2}{16\pi M_\sigma}$$

It requires to know $|g_{\sigma\pi\pi}|$

AO: $|g_{\sigma\pi\pi}| = (3.17 \pm 0.03)$ GeV

Albaladejo, Piqueras and J.A.O., forthcoming

$$g^2_{\sigma\pi\pi} = -(s_\sigma - m_\pi^2/2)^2/f^2/\left(1 - \frac{dg}{ds}\frac{(s_\sigma - m_\pi^2/2)^2}{f^2}\right)$$

All is given in terms of s_σ. dg/ds is a known and fixed function.

$|g^{CCL}_{\sigma\pi\pi}| = |g^{AO}_{\sigma\pi\pi}|1.003$, they are the "same"

$\Gamma(\sigma \to \gamma\gamma)^{AO} = (1.50 \pm 0.21)$ KeV

$\Gamma(\sigma \to \gamma\gamma)^{CCL} = 1.85 \pm 0.28$ KeV

Pennington for CCL: $\Gamma(\sigma \to \gamma\gamma) = 4.09 \pm 0.3$ KeV
40% difference due to the difference in the ratio $|g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|^2$
Different value for $|g_{\sigma\pi\pi}| = 3.86$ GeV when squared $\to$ 50%.

Around a factor of 2 too large

$$\boxed{\Gamma(\sigma \to \gamma\gamma) = 1.6 \pm 0.2 \text{ KeV}}$$

Recently Mennessier,Minkowski,Narison,Ochs arXiv:0707.4511 [hep-ph]

$\Gamma(\sigma \to \gamma\gamma) \simeq (1.4 - 3.2)$ KeV

5. Conclusions I

- We have introduced *three* subtractions constants (more precisision) instead of the *two* used previously in the literature.

- Drastic reduction in the uncertainty of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ for $\sqrt{s} \gtrsim 0.5$ GeV due to the uncertainty in $\phi_0(s)$ above s_K.

- One can discern between different $I = 0$ S-wave $\pi\pi$ parameterizations when new and more precise data become available.

- The method is also adequate to study the $f_0(980)$ resonance. L. Roca, C. Schat and J.A.O. , to appear soon.

- $\Gamma(\sigma \to \gamma\gamma) = 1.6 \pm 0.2$ KeV

6. The ratio $f_0(980) \to \pi\pi / f_0(980) \to K\bar{K}$

Maini, Polosa in PRL93,212002 (2004) have a four-quark model where the OZI rule requires that the $f_0(980)$ has an almost vanishing coupling to $\pi\pi$.

Is there really a suppression of the coupling of the $f_0(980)$ to $\pi\pi$ due to the OZI rule (large N_c)?

In E. Oset and J.A.O. NPA620,438('97) by unitarizing CHPT one obtains simultaneously the σ, $f_0(980)$ and $a_0(980)$. The $f_0(980)$ appears as a pole below the $K\bar{K}$ threshold that develops an imaginary part because of the coupling to $\pi\pi$

Chiral Symmetry+ Unitarity+Analyticity

With only one free parameter
the $a_0(980)$, $f_0(980)$, σ are generated
and the S-wave scattering data
in I=0 and I=1 are reproduced

At the $f_0(980)$ pole position:

$$\frac{T(\pi\pi\to K\bar{K})}{T(K\bar{K}\to K\bar{K})} = \frac{\gamma(f_0\to\pi\pi)}{\gamma(f_0\to K\bar{K})} = \frac{1/\sqrt{3}}{1+g_1 3s/4f^2}$$

$$\Gamma_i = \frac{|\gamma_i|^2\beta_i}{16\pi M_{f_0}}$$

$$g_i = \frac{1}{16\pi^2}\left(\alpha_i + \beta_i(s)\log\frac{\beta_i(s)-1}{\beta_i(s)+1}\right)$$

$$\alpha_i = -\log(1+\sqrt{1+m_i^2/\Lambda_\chi^2})^2 - \log\frac{m_i^2}{\Lambda_\chi^2},$$

$$m_1 = m_\pi,\ m_2 = m_K,\ \Lambda_\chi \simeq M_\rho \simeq 0.8\ \text{GeV}$$

G is $\mathcal{O}(N_c^0)$ by its definition

$T = V + VGT$ implies that at the pole position s_R

$V \propto s_R/f^2$ scales as N_c^0,

Otherwise there is a mismatch between the running in N_c of T on the left and right

The ratio $\frac{\gamma(f_0(980)\to\pi\pi}{\gamma(f_0(980)\to K\bar{K}} = \frac{1/\sqrt{3}}{1+3s_R g_1/4f^2} = \mathcal{O}(N_c^0)$
does not run with the number of colours,
This ratio $\simeq 1/3$

The ratio of couplings is $\mathcal{O}(N_c^0)$

No OZI rule is involved

(This rule is a requirement of the large N_c limit)

Thus, Unitary CHPT and its phenomenologycal success
(strong interactions, J/Ψ decays, ϕ decays, D decays, etc...)
show that the semiquantitative four quark picture
(one of the many four quark pictures)
of Maiani et al PRL93,212002 (2004) is not adequate.

7. Scalar Glueball

M. Albaladejo, J.A.O., arXiv: 0801.4929 [hep-ph]

Talk by M. Albaladejo (much nicer format!), Friday, 15.05 h

I=0, 1/2 S-wave

I=0: $\pi\pi$, $K\bar{K}$, $\sigma\sigma$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\rho\rho$, $\omega\omega$, $K^*\bar{K}^*$, $\omega\phi$, $\phi\phi$, $a_1\pi$, $\pi^*\pi$: 13−channels!

I=1/2: $K\pi$, $K\eta$, $K\eta'$

Data are fitted up to $\lesssim$ 2 GeV. (370 data points for 12 free parameters).

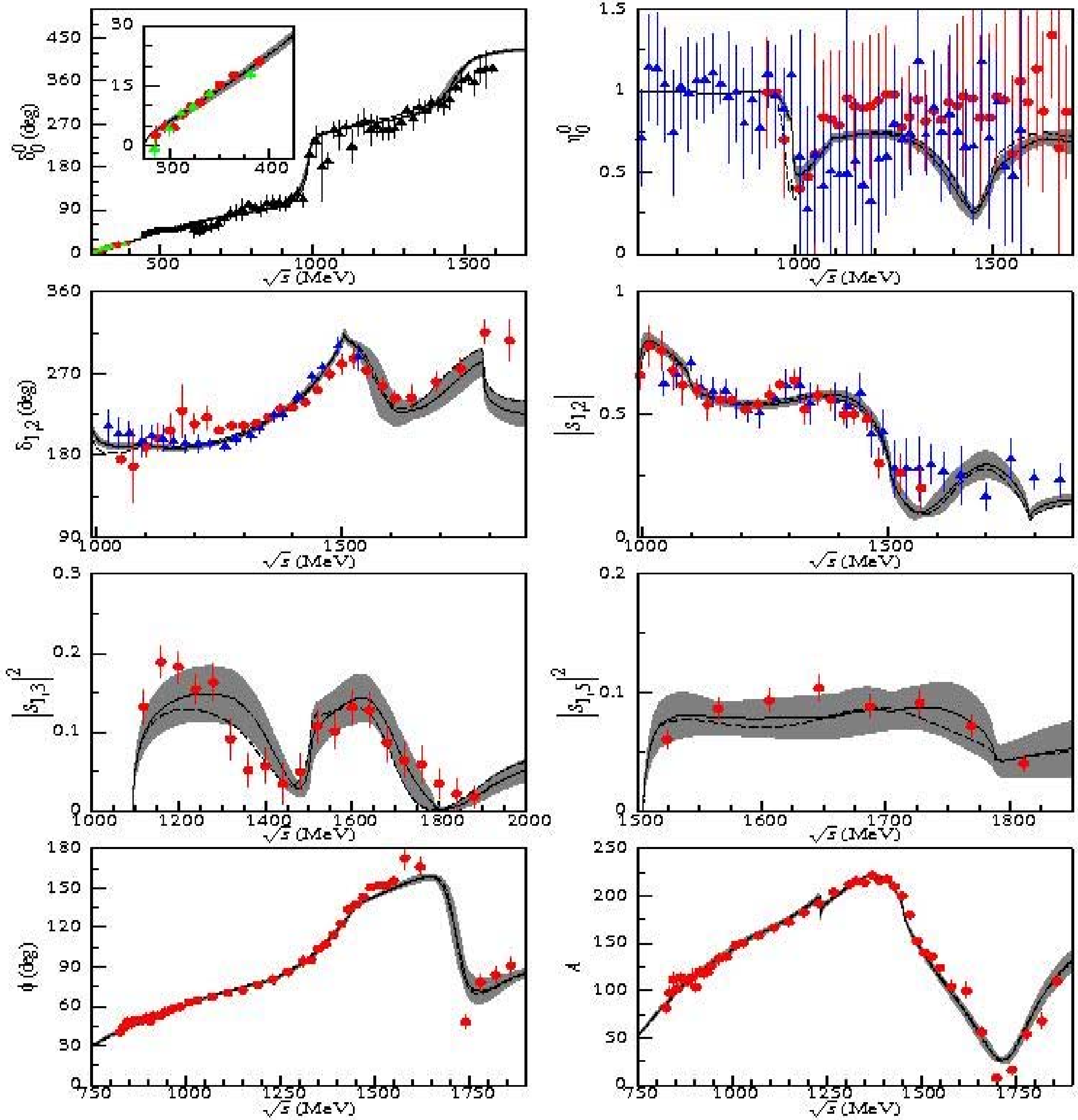
One tree level octet at 1.3 GeV. Fixed from the previous study to $K^-\pi^+ \to K^-\pi^+$ of Pich, Jamin and J.A.O. NPB587,331 (00). Another octet at 1.9 GeV, the mass is fixed from the same ref.

$\pi\pi$ I=0 S-wave

$\pi\pi \to K\bar{K}$

$\pi\pi \to \eta\eta,\ \eta\eta'$

$K^-\pi^+ \to K^-\pi^+$



Poles	MeV
σ	$(456 \pm 6 - i\,241 \pm 7)$
$f_0(980)$	$(983 \pm 4 - i\,25 \pm 4)$
$f_0(1370)$	$(1466 \pm 15 - i\,158 \pm 12)$
$f_0(1500)$	$(1602 \pm 15 - i\,44)$
$f_0(1710)$	$(1690 \pm 20 - i\,110 \pm 20)$
$f_0(1790)$	$(1810 \pm 15 - i\,190 \pm 20)$
κ	$(708 \pm 6 - i\,142 \pm 8)$
$K_0^*(1430)$	$(1435 \pm 6 - i\,142 \pm 8)$
$K_0^*(1950)$	$(1750 \pm 20 - i\,150 \pm 20)$

$f_0(1370)$, $K_0^*(1430)$ are pure octet members

The first octet ($K_0^*(1430)$, $f_0(1370)$, $a_0(1450)$) is not mixed, pure octet.

$f_0(1370)$: Physical(bare) couplings

$|\gamma_{\pi^+\pi^-}| = 3.6(3.9)$, $|\gamma_{K^0\bar{K}^0}| = 2.2(2.3)$, $|\gamma_{\eta\eta}| = 1.7(1.4)$, $|\gamma_{\eta\eta'}| = 4.0(3.7)$, $|\gamma_{\eta'\eta'}| = 3.7(3.8)$

$K_0^*(1430)$: Physical(bare) couplings

$|\gamma_{K\pi}| = 4.8(5.0)$, $|\gamma_{K\eta}| = 0.9(0.7)$, $|\gamma_{\eta\eta}| = 3.8(3.4)$

$a_0(1450)$: with a less developed chiral approach see Oset, J.A.O. PRD60,074023(1999)

GeV	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$
$\|g_{\pi^+\pi^-}\|$	3.59 ± 0.16	1.31 ± 0.22	1.24 ± 0.16
$\|g_{K^0\bar{K}^0}\|$	2.23 ± 0.18	2.06 ± 0.17	2.0 ± 0.3
$\|g_{\eta\eta}\|$	1.7 ± 0.3	3.78 ± 0.26	3.3 ± 0.8
$\|g_{\eta\eta'}\|$	4.0 ± 0.3	4.99 ± 0.24	5.1 ± 0.8
$\|g_{\eta'\eta'}\|$	3.7 ± 0.4	8.3 ± 0.6	11.7 ± 1.6

$f_0(1500)$ and $f_0(1710)$ have similar couplings
They are the same pole but seen on different Riemann sheets
These poles connect continuously

The $f_0(1500)$ appears at 1.5 GeV because of the opening of the $\eta\eta'$ treshold that cuts the 1.6 GeV pole. The sheet that connects with the physical one is another.
Because of this its effective width is larger than the one from the pole position $\rightarrow$ 105 MeV.

Their couplings to pseudoscalar-pseudoscalar nicely match with the predicted supression of $G_0 \rightarrow \bar{q}q \propto m_q$
Chanowitz PRL95,172001 (05)
$(G_0 \rightarrow \bar{s}s)/(G_0 \rightarrow \bar{n}n) \propto m_s/2\hat{m} \simeq m_K^2/m_\pi^2$

With a pseudoscalar mixing angle $\sin\beta = -1/3$

$\eta = -\eta_s/\sqrt{3} + \eta_u\sqrt{2/3}$

$\eta' = \eta_s\sqrt{2/3} + \eta_u/\sqrt{3}$ $\qquad$ $\eta_s = \bar{s}s$ and $\eta_u = (\bar{u}u + \bar{d}d)/\sqrt{2}$.

$\eta_s = \bar{s}s$ and $\eta_u = (\bar{u}u + \bar{d}d)/\sqrt{2}$.

g_{ss} is the production of $\eta_s\eta_s$, g_{sn} that of $\eta_s\eta_u$ and g_{nn} for $\eta_u\eta_u$,

$$
\begin{aligned}
g_{\eta'\eta'} &= \frac{2}{3}g_{ss} + \frac{1}{3}g_{nn} + \frac{2\sqrt{2}}{3}g_{ns} \ , \\
g_{\eta\eta'} &= -\frac{\sqrt{2}}{3}g_{ss} + \frac{\sqrt{2}}{3}g_{nn} + \frac{1}{3}g_{ns} \ , \\
g_{\eta\eta} &= \frac{1}{3}g_{ss} + \frac{2}{3}g_{nn} - \frac{2\sqrt{2}}{3}g_{ns} \ .
\end{aligned}
$$

If the chiral suppression of M. Chanowitz, PRL95,172001(2005) operates one expects that $|g_{ss}| >> |g_{nn}|$. On the other hand, the OZI rule suppresses the coupling g_{ns}. Taking the couplings of the $f_0(1500)$ pole one obtains $g_{ss} = 11.5 \pm 0.5$, $g_{ns} = -0.2$ and $g_{nn} = -1.4$ GeV. For the $f_0(1710)$ pole one has $g_{ss} = 13.0 \pm 1.0$, $g_{ns} = 2.1$ and $g_{nn} = 1.2$ GeV.

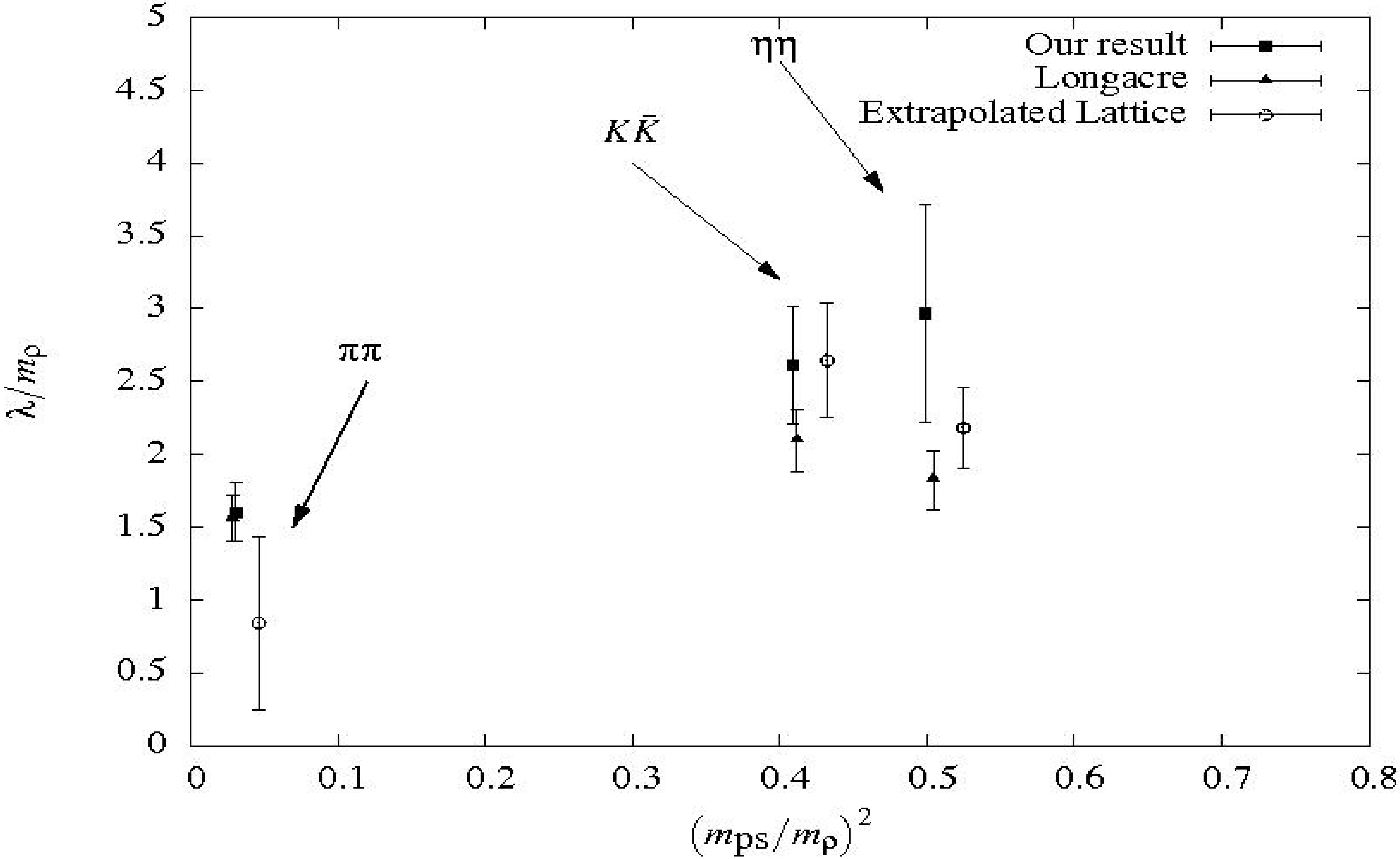
$K\bar{K}$. From the colour wave function of a kaon $\bar{s}_i u^i/\sqrt{3}$
in order to get a colour singlet $\bar{s}s\,\bar{u}u$ a factor 1/3 of suppression appears.
There is an additional 1/2 factor because $\eta_s\eta_s$ is $\bar{s}s\bar{s}s$ with two $\bar{s}s$
We then have $g_{K^0\bar{K}^0} = g_{ss}/6 \simeq 2.0$ GeV in very good agreement with the table

The chiral suppression mechanism is also seen in quenched lattice QCD prediction for the pseudoscalar-pseudoscalar couplings of Sexton, Vaccarino, Weingarten PRL75, 4563 (1995) for the scalar glueball around 1.7 GeV

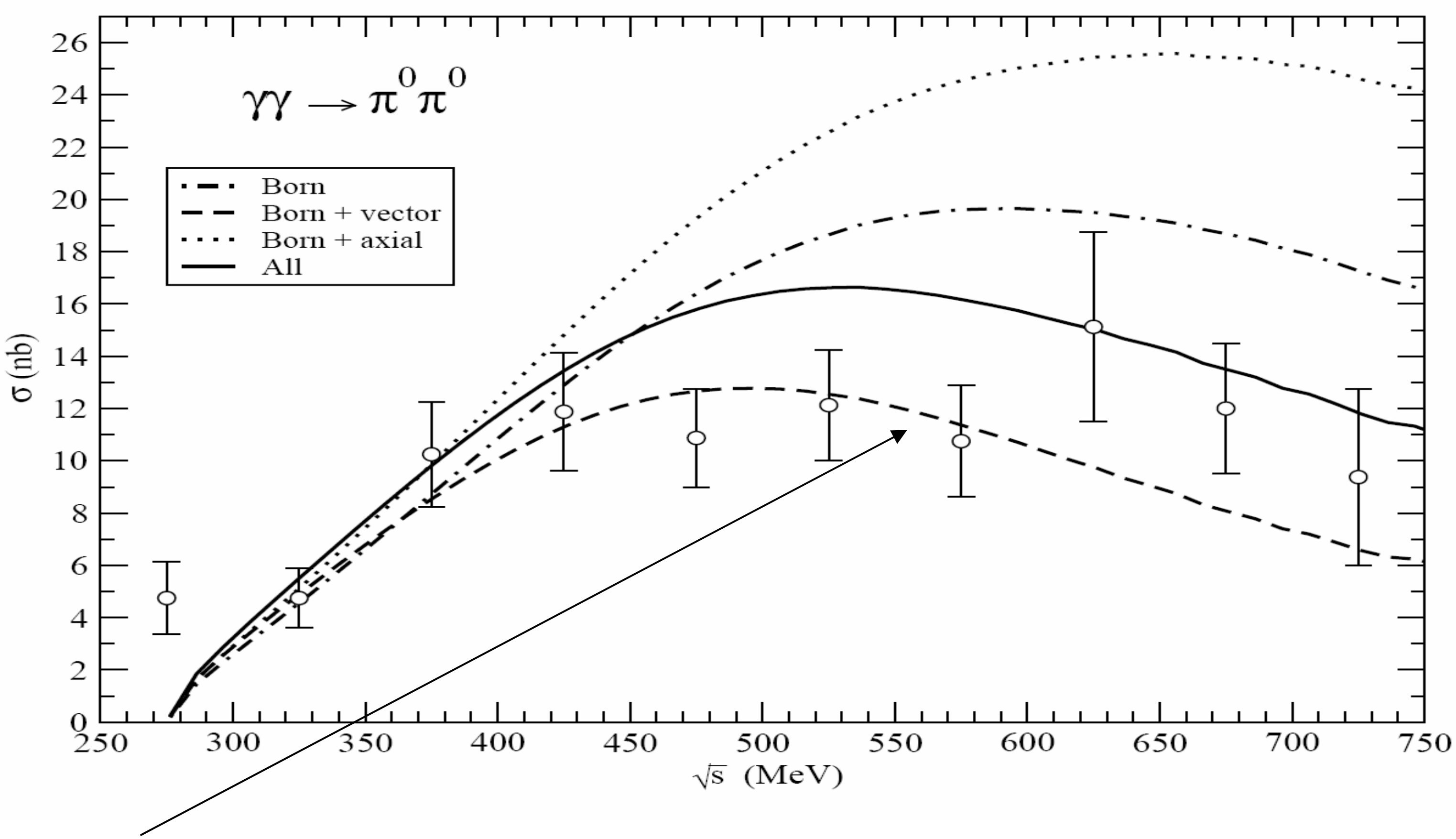
- $f_0(1370)$ is not mixed, pure octet
- $f_0(1500)$, $f_0(1710)$ are scalar glueballs

The same glueball but seen on different Riemann sheets

- σ, $f_0(980)$, $a_0(980)$ and κ constitute the lightest scalar nonet.


Our result
Longacre
Extrapolated Lattice
ηη
$K\bar{K}$
ππ
λ/m_ρ
$(m_{ps}/m_\rho)^2$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Different $L_I(s)$ contributions



Pennington overlooked the 1^{++} and 1^{+-} axial vector exchanges altogether

Regarding the $f_0(980)$

With the original approach of Pennington and Morgan it is a metter of fine tuning

$$F_0(s) = L_0(s) + c_0 s\omega_0(s) + \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{L_0(s')\sin\phi_0(s')}{s'^2(s'-s)|\omega_0(s')|} ds'$$

c_0 is fixed from the position of the Adler zero in $F_N(s)$ at m_π^2, $m_\pi^2/2$ or $2M_\pi^2$

$$c_0 + \frac{s_1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{L_0(s')\sin\phi_0(s')}{s'^2(s'-s_1)|\omega_0(s')|} \simeq 0 \ .$$

Then $\phi_0(s)$ must be precisely given such that this cancellation occurs
But $\phi_0(s)$ is not precisely known for $s > s_K$

In our approach one does not need to impose such specific knowledge of $\phi_0(s)$ for $s > s_K$
The $f_0(980)$ is isolated in the last term and its size controlled by $F_0(s_1) - L_0(s_1)$

We follow here Y2 and diagonalize the 2×2 S-matrix.
We also apply it to calculate inelasticity errors.
We give the expressions directly in terms of observables.

$$T = \begin{pmatrix} \frac{1}{2i}(\eta e^{2i\delta_\pi} - 1) & \frac{1}{2}\sqrt{1-\eta^2}e^{i(\delta_\pi+\delta_K)} \\ \frac{1}{2}\sqrt{1-\eta^2}e^{i(\delta_\pi+\delta_K)} & \frac{1}{2i}(\eta e^{2i\delta_K} - 1) \end{pmatrix}$$ Diagonalization

Orthogonal Matrix C

$$C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\cos\theta = \frac{\left[(1-\eta^2)/2\right]^{1/2}}{\left[1-\eta^2\cos^2\Delta - \eta|\sin\Delta|\sqrt{1-\eta^2\cos^2\Delta}\right]^{1/2}},$$

$$\sin\theta = -\frac{\sin\Delta}{\sqrt{2}} \frac{\eta - \sqrt{1+(1-\eta^2)\cot^2\Delta}}{\left[1-\eta^2\cos^2\Delta - \eta|\sin\Delta|\sqrt{1-\eta^2\cos^2\Delta}\right]^{1/2}},$$

$\sin\theta \to 0$ as $\sqrt{(1-\eta)/2}$ for $\eta \to 1$

Eigenvalues

$\delta_{(+)}$ follows rather closely $\varphi(s)$

$$e^{2i\delta_{(+)}} = S_{11}\frac{1+e^{2i\Delta}}{2}\left[1-\frac{i}{\eta}\tan\Delta\sqrt{1+(1-\eta^2)\cot^2\Delta}\right]$$

$$e^{2i\delta_{(-)}} = S_{22}\frac{1+e^{-2i\Delta}}{2}\left[1+\frac{i}{\eta}\tan\Delta\sqrt{1+(1-\eta^2)\cot^2\Delta}\right]$$

One has then two channels diagonalized that are ***elastic***

$$\Gamma' \equiv \begin{pmatrix} \Gamma'_1 \\ \Gamma'_2 \end{pmatrix} = C^T Q^{1/2} F = C^T Q^{1/2} \begin{pmatrix} F_\pi \\ F_K \end{pmatrix}$$

$$F_\pi = q_\pi^{-1/2}\left(\lambda\cos\theta\,|\Gamma'_1|e^{i\delta_{(+)}} \pm \sin\theta\,|\Gamma'_2|e^{i\delta_{(-)}}\right)$$

$$F_K = q_K^{-1/2}\left(\pm\cos\theta\,|\Gamma'_2|e^{i\delta_{(-)}} - \lambda\sin\theta\,|\Gamma'_1|e^{i\delta_{(+)}}\right)$$

If $\delta_\pi(s_K) \geq \pi$ one has the zero at $s_1^{1/2} < 2m_K$, this introduces a minus sign due to the prefactor $s_1 - t$. $\lambda = (-1)^{\theta(\delta_\pi(s_K)-\pi)}$

Notice that Γ'_2 is 0 at s_K, this is why we cannot fix the $\pm$ in front of $|\Gamma'_2|$

Shift in $\delta_{(+)}$ because of inelasticity

$$F_\pi = \lambda \cos\theta \, |\Gamma_1'| e^{i\delta_{(+)}} (1 + \epsilon \cos\theta) \left(1 + i \tfrac{\epsilon \sin\rho}{1+\epsilon \cos\rho}\right) \quad \rho = \delta_- - \delta_+$$

With $\epsilon = \pm \tan\theta \left|\frac{\Gamma_2'}{\Gamma_1'}\right|$ $\qquad |\Gamma_2'/\Gamma_1'| \lesssim |\widetilde{t}_{11} \tan\theta / \widetilde{t}_{22}| \simeq |\tan\theta| < 1$

$\tan\theta \to 0$ when $\eta \to 1$. First order correction to $\delta_{(+)}$

$$1 + i \tfrac{\epsilon \sin\rho}{1+\epsilon \cos\rho} = \exp\left(i \tfrac{\epsilon \sin\rho}{1+\epsilon \cos\rho}\right) + \mathcal{O}(\epsilon^2) \qquad \delta_{(+)} \to \delta_{(+)} + \tfrac{\epsilon \sin\rho}{1+\epsilon\cos\rho}$$

- $1.1 \le s^{1/2} \le 1.5$ GeV, $\eta \simeq 1$ experimentally (Hyams, Grayer).

Typically $\eta \gtrsim 0.8$ Then $\epsilon \simeq 0.3$.

$\delta_\pi(s_K) < \pi$	$\delta_\pi(s_K) \ge \pi$
$\delta_{(+)} \ge 3\pi/4$	$\delta_{(+)} \ge 3\pi/2$
Correction: $6\% \times 2 \to 12\%$	$12\% \times 2 \to 25\%$

Fixing s_1

$$\omega(t) = \omega(0) + \omega(0)' t + \frac{t^2}{\pi} \int_{4m_\pi^2}^{+\infty} \frac{\mathrm{Im}[\omega(s)]}{s^2(s-t-i\epsilon)} ds$$

Both $\omega(0)$ and $\omega(0)'$ are real
The only points where $\omega(t)$ can vanish are those for which $\phi_0(s) = 0$. Otherwise the integral develops an imaginary part that cannot be cancelled.

Around s_K the only point is s_1