Chiral effective field theory for nuclear matter

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In collaboration with A. Lacour and U.-G. Meißner

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Outline

Lacour, Oller, Meißner,
J.Phys.G 37(2010)015106 [1];
To appear in Annals of Physics, arXiv:0906.2349 [2];
arXiv:1007.2574 [3].

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Introduction

EFT with short- and long range few-nucleon interactions is quite advanced in vacuum

Pion-nucleon interactions in nuclear matter are already largely exploited, considering chiral Lagrangians

For a recent review: Epelbaum,Hammer,Meißner, Rev.Mod.Phys.**81**(2009)1773

Commonly, free parameters are fixed to nuclear matter properties

Many important results and studies of nuclear processes have been accomplished.

Nonetheless it would be desirable to develop a Chiral EFT in nuclear matter.

1. Need of a in-medium power counting to include both short- and long-range multi-N interactions
2. The power counting has to take into account
   - any number of nucleon loops can be arranged in any way
   - in reducible diagrams for NN-interactions N-propagators are enhanced, $\frac{1}{k^0 - E_{\mathbf{k}} + i\epsilon} \sim \mathcal{O}(p^{-2})$

     S. Weinberg, Nucl.Phys.B**363**(1991)3
   - in-medium multi-N interactions must be taken into account consistent with the vacuum
3. Pion-nucleon interactions have to be included with the same requirements

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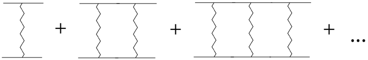
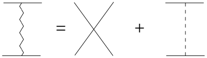
Chiral power counting

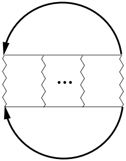
$$G_0(k)_{i_3} = \frac{\theta(k-\xi_{i_3})}{k_0 - E_k + i\epsilon} + \frac{\theta(\xi_{i_3}-k)}{k_0 - E_k - i\epsilon} = \frac{1}{k_0 - E_k + i\epsilon} + 2\pi i\,\delta(k_0 - E_k)\,\theta(\xi_{i_3} - k)$$



where

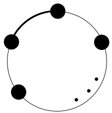


The NN irreducible diagrams are very abundant in the nuclear medium



Every nucleon propagator $G_0(k)_{i_3} \sim \mathcal{O}(p^{-2})$
Despite this the chiral power counting is still bounded from below

Concept of in-medium generalized vertex (IGV):
thick line: Fermi sea insertion, thin lines: full in-medium nucleon propagator, filled circles: bilinear nucleon vertices



Oller,Phys.Rev.C65(2002)025204; Meißner,Oller,Wirzba,Ann.Phys.297(2002)27

Let **H** be an auxiliary field for heavy mesons responsible for short range $2N$, $3N, \ldots$ interactions, the **H**-"propagator" counts as $\mathcal{O}(p^0)$[1]

Short-range interactions enter the counting via bilinear vertices of $\mathcal{O}(\geq p^0)$

$$\mathcal{L}_{eff} = \sum_{n=1}^{\infty} \mathcal{L}_{\pi\pi}^{(2n)} + \sum_{n=1}^{\infty} \mathcal{L}_{\pi N}^{(n)} + \sum_{n=0}^{\infty} \mathcal{L}_{NN}^{(2n)}$$

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$$

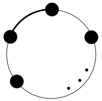
$$\xi_F \sim M_\pi \sim q_\pi \sim \mathcal{O}(p)$$

Order p^ν of a diagram:[1]

$$\nu = 4 - E_\pi + \sum_{i=1}^{V_\pi}(n_i + l_i - 4) + \sum_{i=1}^{V_B}(d_i + v_i + w_i - 2) + V_\rho$$

ν is bound from below (modulo external sources):

1. adding pions to pionic vertices: $n_i \geq 2$, $l_i \geq 2$
2. nucleon mass renormalization terms: $d_i \geq 2$, $w_i = 0$, $v_i \geq 0$
3. adding pions to pion-nucleon vertices: $d_i \geq 1$, $w_i = 0$, $v_i \geq 1$
4. adding heavy mesons to bilinear vertices: $d_i \geq 0$, $w_i \geq 1$, $v_i \geq 1$
5. $V_\rho \Rightarrow$ adding **1** IGV rises counting at least by **1**



IGV

The power counting equation is applied increasing step by step V_ρ.

Augmenting the number of lines in a diagram **without increasing** the chiral power by adding:

1. Pionic lines attached to lowest order mesonic vertices, $\ell_i = n_i = 2$
2. Pionic lines attached to lowest order meson-baryon vertices, $d_i = v_i = 1$
3. Heavy mesonic lines attached to lowest order bilinear vertices, $d_i = 0$, $\omega_i = 1$.

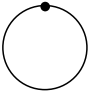
Source of **non-perturbative** physics. These rule give rise to infinite resummations.

Nuclear matter energy density - Contributions

$V_\rho = 1$

$\mathcal{O}(p^5)$

Leading Order

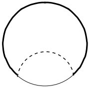


1

$V_\rho = 1$

$\mathcal{O}(p^6)$

Next-to-leading Order

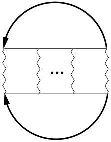


2

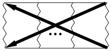
$V_\rho = 2$

$\mathcal{O}(p^6)$

Next-to-leading Order

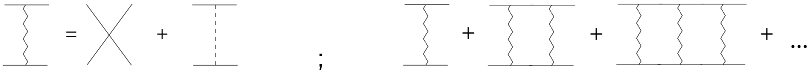


3.1



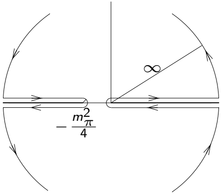
3.2

Non-perturbative methods



Unitary χPT: resummations are performed partial wave by partial wave

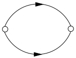
Oller,Oset,PRD60(1999)074023; Oller,Meißner,PLB500(2001)263.



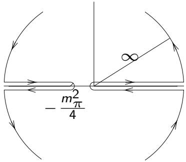
Once subtracted dispersion relation of the inverse of a partial wave amplitude

$$T_{JI}(\ell',\ell,S) = \left[I + N_{JI}(\ell',\ell,S)\cdot g\right]^{-1}\cdot N_{JI}(\ell',\ell,S)$$

Interaction kernel: $N_{JI}(\ell',\ell,S)$ Unitarity loop g:



$$g(A) = g(D) - \frac{m(A-D)}{4\pi^2}\int_0^\infty dk^2 \frac{k}{(k^2-A-i\epsilon)(k^2-D-i\epsilon)}$$
$$\equiv g_0 - i\frac{m\sqrt{A}}{4\pi}\ , \qquad D=0$$



$$T_{JI}^{-1}(A) = \left[N_{JI}^{-1}+g\right]^{-1} = T_{JI}^{-1}(D) - \frac{m(A-D)}{4\pi^2}\int_0^\infty dk^2 \frac{k}{(k^2-A-i\epsilon)(k^2-D)}$$
$$-\frac{(A-D)}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\mathrm{Im}\, T_{JI}/|T_{JI}|^2}{(k^2-A-\epsilon)(k^2-D)}$$

Interaction kernel:

$$\mathrm{Im} N_{JI} = \frac{|N_{JI}|^2}{|T_{JI}|^2}\mathrm{Im}\, T_{JI} = |1+gN_{JI}|^2 \mathrm{Im}\, T_{JI} \quad , \quad |\mathbf{p}|^2 < -\frac{m_\pi^2}{4}\ .$$

$$N_{JI}(A) = N_{JI}(D) + \frac{A-D}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\mathrm{Im}\, T_{JI}(k^2)\,|1+g(k^2)N_{JI}(k^2)|^2}{(k^2-A-i\epsilon)(k^2-D)}$$

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If $g(k^2)$ is small in the low-energy part of the left-hand cut, $|\mathbf{k}| \sim i\, m_\pi$

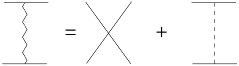
$$g(k^2) = g_0 - i\frac{m|\mathbf{k}|}{4\pi} \approx 0 \quad \longrightarrow \quad \text{natural value:} \;\; g_0 \simeq -\frac{m m_\pi}{4\pi} = -0.54\; m_\pi^2$$

g is treated as small $\sim \mathcal{O}(p)$ in order to fix N_{JI}

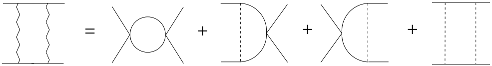
An approximate algebraic solution for N_{JI} results

Chiral expansion in powers of g : $\quad N_{JI} = N_{JI}^{(0)} + N_{JI}^{(1)} + \mathcal{O}(p^2)$

$$\begin{aligned} T_{JI} &= N_{JI} - N_{JI}\cdot g \cdot N_{JI} + \ldots \\ &= N_{JI}^{(0)} + N_{JI}^{(1)} - N_{JI}^{(0)} \cdot g \cdot N_{JI}^{(0)} + \mathcal{O}(p^2) + \ldots \\ T_{JI} &\doteq L_{JI}^{(0)} + L_{JI}^{(1)} + \mathcal{O}(p^2) \end{aligned}$$



$\rightarrow \; N_{JI}^{(0)} = L_{JI}^{(0)}$



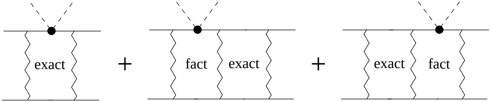
$\rightarrow \; N_{JI}^{(1)} = L_{JI}^{(1)} + N_{JI}^{(0)} \cdot g \cdot N_{JI}^{(0)}$

UχPT can also be applied to production diagrams

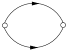
Oller, Oset NPA629(1998)739 , Oller PRD71(2005)054030

$$F_{JI} = D_{JI}^{-1} \cdot \xi_{JI} \quad , \quad D_{JI} = 1 + N_{JI} g \ ,$$

$$\xi_{JI} = \sum_{k=0}^{n} \xi_{JI}^{(k)}$$



$$\xi_{JI}^{(0)} + \xi_{JI}^{(1)} = DL_{JI}^{(1)} - \left\{ L_{JI}^{(1)} + {N_{JI}^{(0)}}^2 \cdot L_{10}, N_{JI}^{(0)} \right\} \cdot DL_{10}$$

In-medium unitarity loop  :

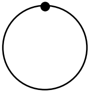
$$g \doteq L_{10,f} \longrightarrow L_{10} = L_{10,f} + L_{10,m}(\xi_1) + L_{10,m}(\xi_2) + L_{10,d}(\xi_1, \xi_2)$$
$$= L_{10,pp}(\xi_1, \xi_2) + L_{10,hh}(\xi_1, \xi_2)$$

Nuclear matter energy density - Contributions

$V_\rho = 1$

$\mathcal{O}(p^5)$

Leading Order

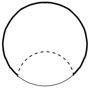


1

$V_\rho = 1$

$\mathcal{O}(p^6)$

Next-to-leading Order

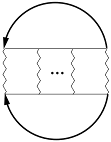


2

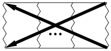
$V_\rho = 2$

$\mathcal{O}(p^6)$

Next-to-leading Order



3.1



3.2

Nuclear matter energy per particle

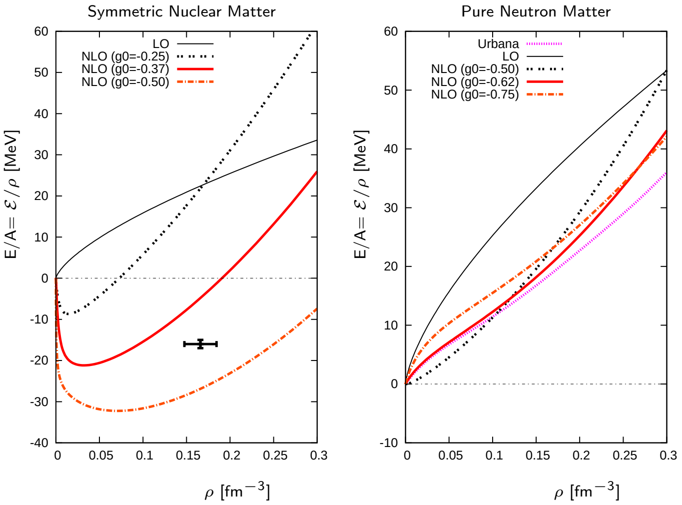
$$\mathcal{E}_3 = -4 \sum_{I,J,\ell,S} \sum_{i_3=\alpha_1+\alpha_2} (2J+1)\chi(S\ell I)^2 \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{q}|)$$
$$\times\, \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{q}|)\Bigg[T^{i_3}_{JI}\Big|_{(\mathbf{q}^2,\mathbf{P}^2,\mathbf{q}^2)}$$
$$+\, m\int \frac{d^3p}{(2\pi)^3}\frac{1-\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{p}|) - \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{p}|)}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\Big|T^{i_3}_{JI}\Big|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)}$$

It is real. $m\int \frac{d^3p}{(2\pi)^3}\frac{1}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\Big|T^{i_3}_{JI}\Big|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)}$ is divergent. Expansion

$$T^{i_3}_{JI\,(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} = T^{i_3}_{JI\,(+\infty,\mathbf{P}^2,\mathbf{q}^2)} + \mathcal{O}(|\mathbf{p}|^{-2})$$

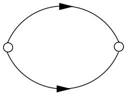
$$m\int \frac{d^3p}{(2\pi)^3}\frac{1}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\Bigg\{\Big|T^{i_3}_{JI}\Big|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} - \Big|T^{i_3}_{JI}\Big|^2_{(+\infty,\mathbf{P}^2,\mathbf{q}^2)}\Bigg\}$$
$$- g(\mathbf{q}^2)\Big|T^{i_3}_{JI}\Big|^2_{(+\infty,\mathbf{P}^2,\mathbf{q}^2)}$$

Nuclear matter energy per particle

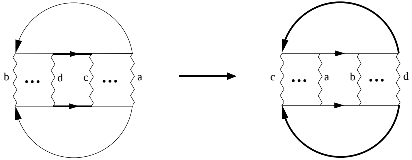


More stable for pure neutron matter (less dependent on g_0).

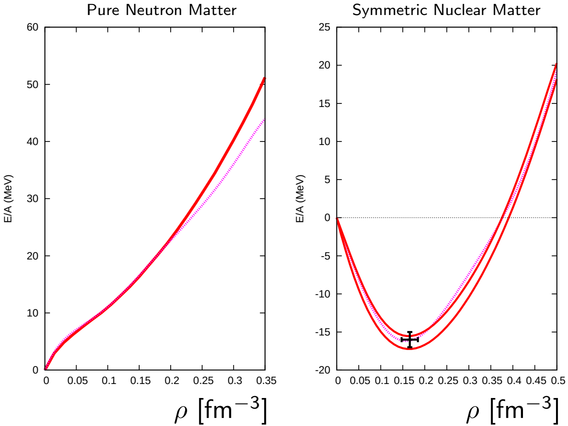
$$
\begin{aligned}
\mathcal{E}_3 = 4 \sum_{I,J,\ell,S} \sum_{i_3=\alpha_1+\alpha_2} (2J+1)\chi(S\ell I)^2 \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{q}|) \\
\times\, \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{q}|)\Bigg[-T^{i_3}_{JI}\Big|_{(\mathbf{q}^2,\mathbf{P}^2,\mathbf{q}^2)} \\
+ m \int \frac{d^3p}{(2\pi)^3} \frac{\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{p}|) + \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{p}|)}{\mathbf{p}^2 - \mathbf{q}^2 - i\epsilon} \Big| T^{i_3}_{JI}\Big|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} \\
- m \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\mathbf{p}^2 - \mathbf{q}^2 - i\epsilon} \Big| T^{i_3}_{JI}\Big|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} - \frac{1}{\mathbf{p}^2}\Big| T^{i_3}_{JI}\Big|^2_{\mathbf{p}^2\to\infty} \right\} + \tilde{g}_0 \Big| T^{i_3}_{JI}\Big|^2_{\mathbf{p}^2\to\infty} \Bigg]
\end{aligned}
$$



g_0 in NN scattering



$\tilde{g}_0$, particle-particle intermediate state



Akmal,Pandharipande,Ravenhall, PRC **58**(1998)1804

PNM: $g_0 = \tilde{g}_0 \simeq -0.6\ m_\pi^2$
SNM: $(g_0, \tilde{g}_0) \simeq (-1.0, -0.5)\ m_\pi^2$

$$K = \xi^2 \frac{\partial^2 \mathcal{E}/\rho}{\partial \xi^2}\bigg|_{\xi_0} = 240 - 260 \text{ MeV}$$

exp.250 ± 25 MeV

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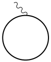
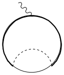
In-medium chiral quark condensate - Contributions

$V_\rho = 1$
$\mathcal{O}(p^5)$
Leading Order



1

$V_\rho = 1$
$\mathcal{O}(p^6)$
Next-to-leading Order



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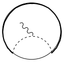
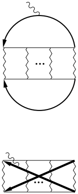
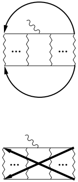


3

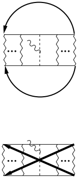
$V_\rho = 2$
$\mathcal{O}(p^6)$
Next-to-leading Order



4



5



6

$$m_q \langle \Omega | \bar{q}_i q_j | \Omega \rangle = m_q \langle 0 | \bar{q}_i q_j | 0 \rangle - m_q (\Xi_1 + \Xi_6)$$

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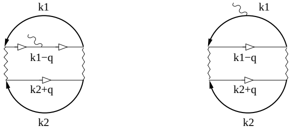
In-medium chiral quark condensate



$$\Xi_5^L = -\frac{1}{2}\sum_{\alpha_1,\alpha_2}\sum_{\sigma_1,\sigma_2}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}e^{ik_1^0\eta}e^{ik_2^0\eta}G_0(k_1)_{\alpha_1}G_0(k_2)_{\alpha_2}\frac{\partial}{\partial k_1^0}\Big[i\sum_{\alpha_1',\alpha_2'}\int\frac{d^4k}{(2\pi)^4}$$

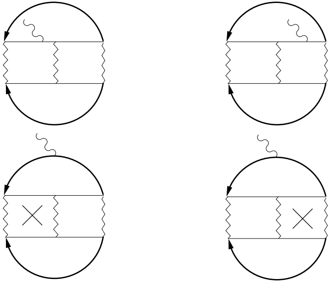
$$\times V_{\alpha_1\alpha_2;\alpha_1'\alpha_2'}(k)2B\left[2c_1\delta_{ij}+c_5\tau^3_{ij}\tau^3_{\alpha_1'\alpha_1'}\right]G_0(k_1-k)_{\alpha_1'}G_0(k_2+k)_{\alpha_2'}V_{\alpha_1'\alpha_2';\alpha_1\alpha_2}(-k)\Big]$$

$$\Xi_4^L = \sum_{\alpha_1,\alpha_2}\sum_{\sigma_1,\sigma_2}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}e^{ik_1^0\eta}e^{ik_2^0\eta}G_0(k_1)_{\alpha_1}2B\left[2c_1\delta_{ij}+c_5\tau^3_{ij}\tau^3_{\alpha_1\alpha_1}\right]G_0(k_2)_{\alpha_2}$$

$$\times\frac{\partial}{\partial k_1^0}\Big[\frac{i}{2}\sum_{\alpha_1',\alpha_2'}\int\frac{d^4k}{(2\pi)^4}V_{\alpha_1\alpha_2;\alpha_1'\alpha_2'}(k)G_0(k_1-k)_{\alpha_1'}G_0(k_2+k)_{\alpha_2'}V_{\alpha_1'\alpha_2';\alpha_1\alpha_2}(-k)\Big]$$

For Ξ_5 the derivative acts directly in the scattering amplitude.
For Ξ_4 there is an integration by parts (extra sign).

This is a general argument following from the power counting



Cancellations happen explicitly for all orders in UχPT[3].

Feynman-Hellman theorem:

$$m_q\langle\Omega|\bar{q}_i q_j|\Omega\rangle - m_q\langle 0|\bar{q}_i q_j|0\rangle = \frac{m_q}{2}\left(\delta_{ij}\frac{d}{d\hat{m}} + (\tau_3)_{ij}\frac{d}{d\bar{m}}\right)(\rho\, m + \mathcal{E})\ ,$$

$$\frac{\langle\Omega|\bar{q}_i q_j|\Omega\rangle}{\langle 0|\bar{q}_i q_j|0\rangle} = 1 - \frac{\rho\,\sigma_N}{m_\pi^2 f_\pi^2} + \frac{2c_5(\tau_3)_{ij}}{f_\pi^2}(\rho_p - \rho_n) - \frac{1}{f_\pi^2}\frac{\partial\mathcal{E}(\rho, m_\pi)}{\partial m_\pi^2}$$

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Long-range NN interactions dominate in the quark condensate calculation.

Kaiser, Homont, Weise PRC77(2008)025204; Plohl, Fuchs NPA798(2008)75

We offered an explanation for this observed fact:

- The quark mass dependence of nucleon propagators cancels

$$\Xi_4 + \Xi_5 = 0$$

- The short distance part $|\mathbf{p}|^2 \to \infty$ cancels when taking the derivative

$$\frac{\partial \mathcal{E}(\rho, m_\pi)}{\partial m_\pi^2}$$

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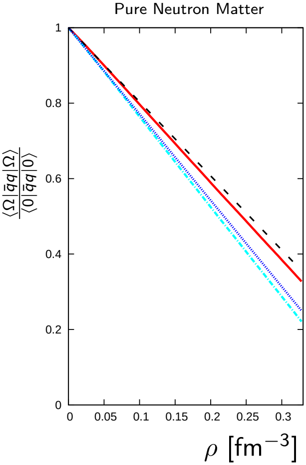
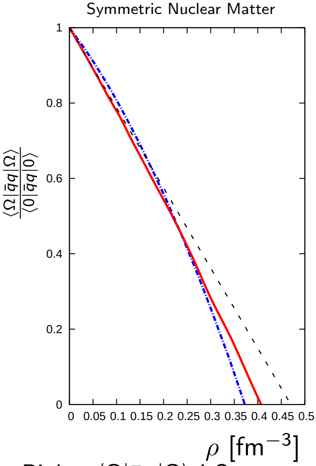
In-medium chiral quark condensate





Left: $\langle\Omega|\bar{u}u|\Omega\rangle$ LO,
NLO$(g_0 = -0.6 m_\pi^2)$;
$\langle\Omega|\bar{d}d|\Omega\rangle$ LO, NLO$(g_0 = -0.6 m_\pi^2)$.

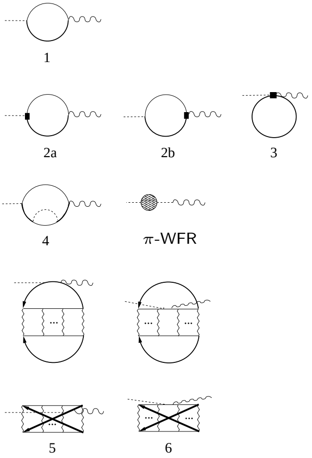
Right: $\langle\Omega|\bar{q}q|\Omega\rangle$ LO,
NLO$(g_0 = -1.0 m_\pi^2)$,
NLO$(g_0 = -0.5 m_\pi^2)$.
The quark condensate is independent of $\tilde{g}_0$

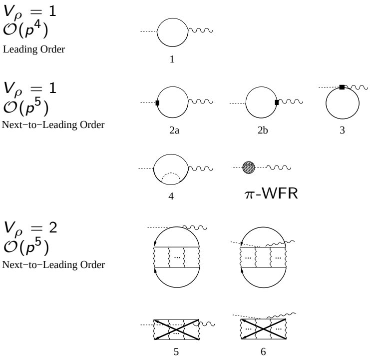
Contributions to the in-medium pion axial couplings

$V_\rho = 1$
$\mathcal{O}(p^4)$
Leading Order

$V_\rho = 1$
$\mathcal{O}(p^5)$
Next−to−Leading Order

$V_\rho = 2$
$\mathcal{O}(p^5)$
Next−to−Leading Order





Diagrams 1–3 discussed in Meißner, Oller, Wirzba ANP297(2002)27

Diagram with π-WFR also discussed there

Diagram 4 is one order too high

Diagrams 5–6 mutually cancel

$$f_t = f_\pi \left\{ 1 - \frac{\rho}{\rho_0}(0.26 \pm 0.04) \right\}$$

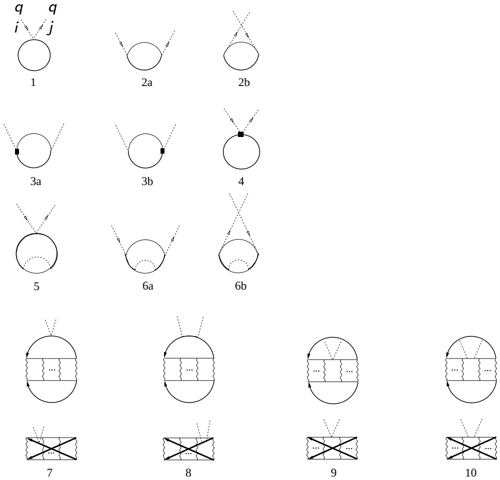
$$f_s = f_\pi \left\{ 1 - \frac{\rho}{\rho_0}(1.23 \pm 0.07) \right\}$$

Contributions to the in-medium pion self-energy

$V_\rho = 1$
$\mathcal{O}(p^4)$
Leading Order

$V_\rho = 1$
$\mathcal{O}(p^5)$
Next−to−Leading Order

$V_\rho = 2$
$\mathcal{O}(p^5)$
Next−to−Leading Order



NN-interactions cancel at $\mathcal{O}(p^5)$. Linear density approximation holds up to NLO

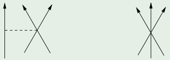
Summary & Outlook

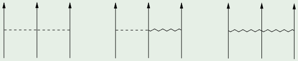
Summary

- Developed a power counting scheme for nmEFT combining short- and long-range multi-N interactions
- Nuclear matter energy density (up to NLO)
- In-medium chiral quark condensate (up to NLO)
- In-medium f_t, f_s (up to NLO)
- In-medium pion self-energy (up to NLO)
- Quite good results at just NLO by applying non-perturbative methods of UχPT to NN-interactions

Outlook

- Exact solution of $N_{JI}(A)$
- $V_\rho = 3$ contributions, 3 nucleon force (N^{2}LO)
- Two-pion exchange (N^{3}LO)
- "Genuine" 3-nucleon force (N^{4}LO)





$d_2 + v_2 + w_2 - 2 = 2$ and $V_\rho = 3$ (instead of 0 and 1, respectively)

- Clarify the dependence on $\widetilde{g}_0$
- Neutron stars, finite temperature, other N-point Green functions, adding strangeness...