Bad Honnef, December 14th 2004

Final State Interactions in Hadronic D decays

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- **Introduction**
- **FSI in the $D^+ \to \pi^-\pi^+\pi^+$ decay**
- **FSI in the $D_s^+ \to \pi^-\pi^+\pi^+$ decay**
- **FSI in the $D^+ \to K^-\pi^+\pi^+$ decay**
- **Summary**

Introduction

1. Some decays of D mesons offer experiments with high statistics where the meson-meson S-waves are dominant. This is very intersting and new.

2. This has given rise to observe experimentally with large statistical significance the $f_0(600)$ or σ E791 Collaboration PRL 86, 770 (2001) $\mathbf{D}^+ \to \pi^-\pi^+\pi^+$, and the $K^*_0(800)$ or κ mesons E719 Collaboration PRL 89, 121801 (2002) $\mathbf{D}^+ \to K^-\pi^+\pi^+$.

3. Clear observation of the σ resonance has been also reported by the Collaborations CLEO, Belle and BaBar.

4. There are no Adler zeroes that destroy the bumps of the σ and κ, contrary to scattering.

However in the E791 Analyses:

- The phases of the Breit-Wigner's used for the σ and κ do not follow the I=0 $\pi\pi$ S-wave and I=1/2 $K\pi$ S-wave phase shifts, respectively. Despite that at low two-body energies one expects that the spectator hypothesis should work and then it should occur by Watson's theorem.

- Furthermore, the $f_0(980)$ resonance has non standard couplings, e.g. it couples just to pions while having a coupling to kaons compatible with zero.

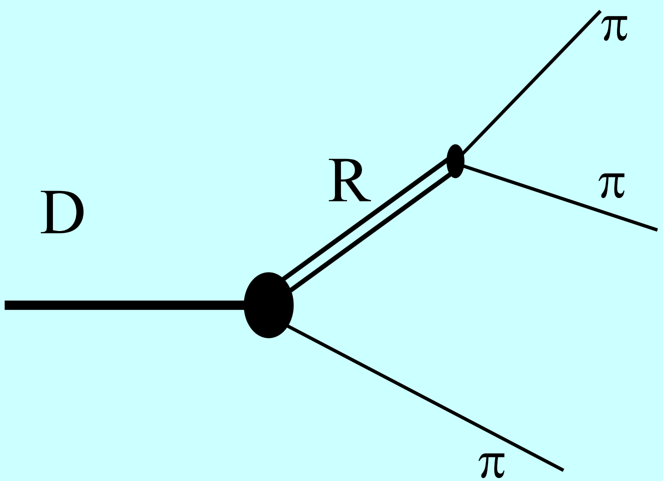
- The width of the $K^*_0(1430)$ is a factor of 2 smaller than PDG value, typical from scattering studies.

FSI in the $D^+ \rightarrow \pi^-\pi^+\pi^+$ decay

$D^+ \rightarrow \pi^-\pi^+\pi^+$ E791 Col. PRL 86, 770 (2002) 1686 candidates,
Signal:background 2:1 $\rightarrow$ 1124 events.

E791 Analysis follows the Isobar Model to study the Dalitz plot

It is based on two assumption: Third pion is an spectator, and one sums over intermmediate two body resonances



INTERMEDIATE RESONANCES R ARE SUMMED

FSI in the $D^+ \to \pi^-\pi^+\pi^+$ decay

$D^+ \to \pi^-\pi^+\pi^+$ E791 Col. PRL 86, 770 (2002) 1686 candidates, Signal:background 2:1 $\to$ 1124 events.

E791 Analysis follows the Isobar Model to study the Dalitz plot

It is based on two assumption: Third pion is an spectator, and one sums over intermmediate two body resonances

$$\mathcal{A} = a_0 e^{i\delta_0}\mathcal{N}_0 + \sum_{n=1}^{N} a_n e^{i\delta_n}\mathcal{A}_n(s_{12}, s_{13})\mathcal{N}_n$$

$\pi^-(1)$, $\pi^+(2)$, $\pi^+(3)$

$s_{12}=(p_1+p_2)^2$, $s_{13}=(p_1+p_3)^2$

$a_0\, e^{i\delta 0}$: Non-Resonant Term

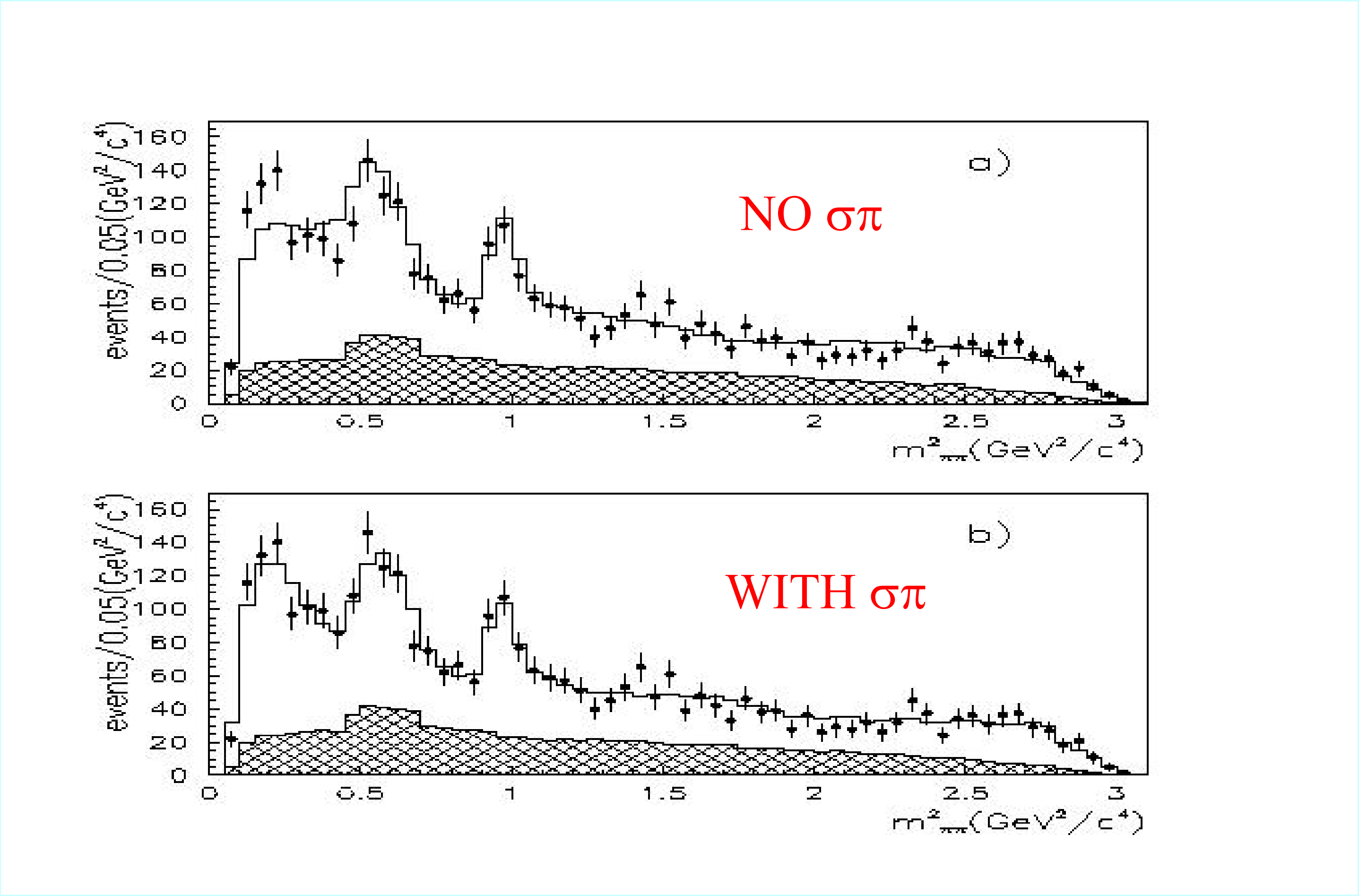
A_n : Breit-Wigner (BW) term corresponding to a (12) or (13) resonance

$$\mathcal{A}_n[(12)3] = BW_n(s_{12})\mathcal{M}_n^{(J)}[(12)3]F_D^{(J)}(s_{12})F_n^{(J)}(s_{12})$$

Then is Bose-symmetrized because of the two identical π^+

When in the sum over resonances the $\sigma\pi^+$ state was not included, then

χ^2/dof=1.5 → CL=10^{-5} WHEN included χ^2/dof=0.9 → CL=76 %

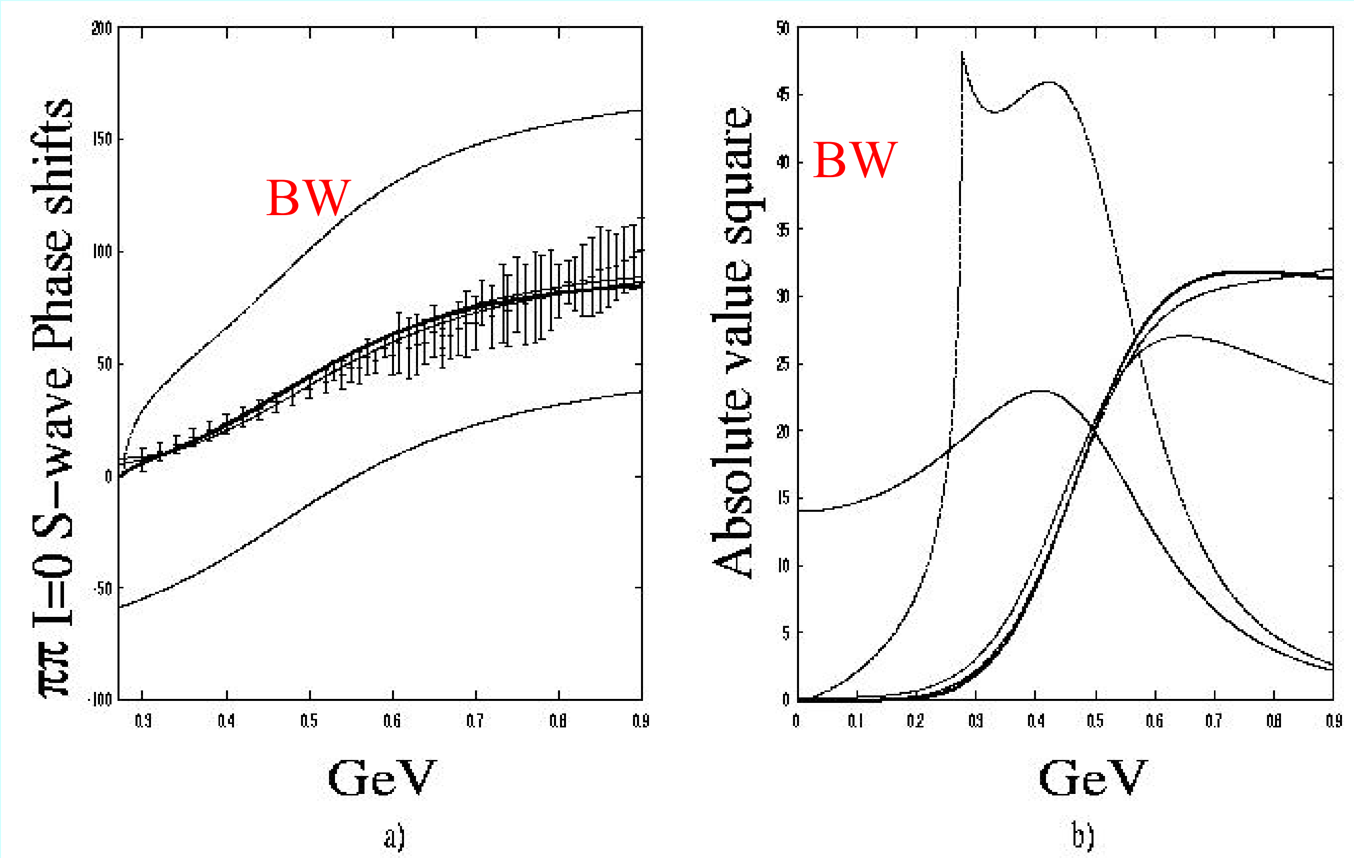


Exchanged resonances:

$\rho^0(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$, $\rho^0(1450)$, σ

$BW_\sigma(s)=(s-m_\sigma^2+i\, m_\sigma\, \Gamma(s))^{-1}$ energy dependent width !

$$\Gamma(s)=\Gamma_\sigma \frac{m_\sigma}{\sqrt{s}} \frac{p(s)}{p_0} \frac{F_n^{(0)}(s)^2}{F_n^{(0)}(m_\sigma^2)^2}$$



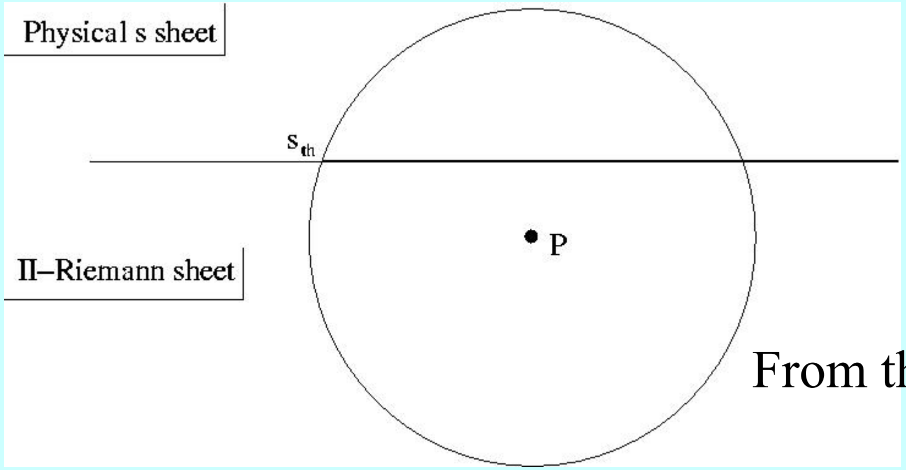
BW does not follow the experimental S-wave I=0 ππ phase shifts

M_σ=478±24 MeV

Γ_σ= 324±42 MeV

Laurent series around the σ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\sigma} + \gamma_1 + \gamma_2 (s - s_\sigma) + \ldots$$



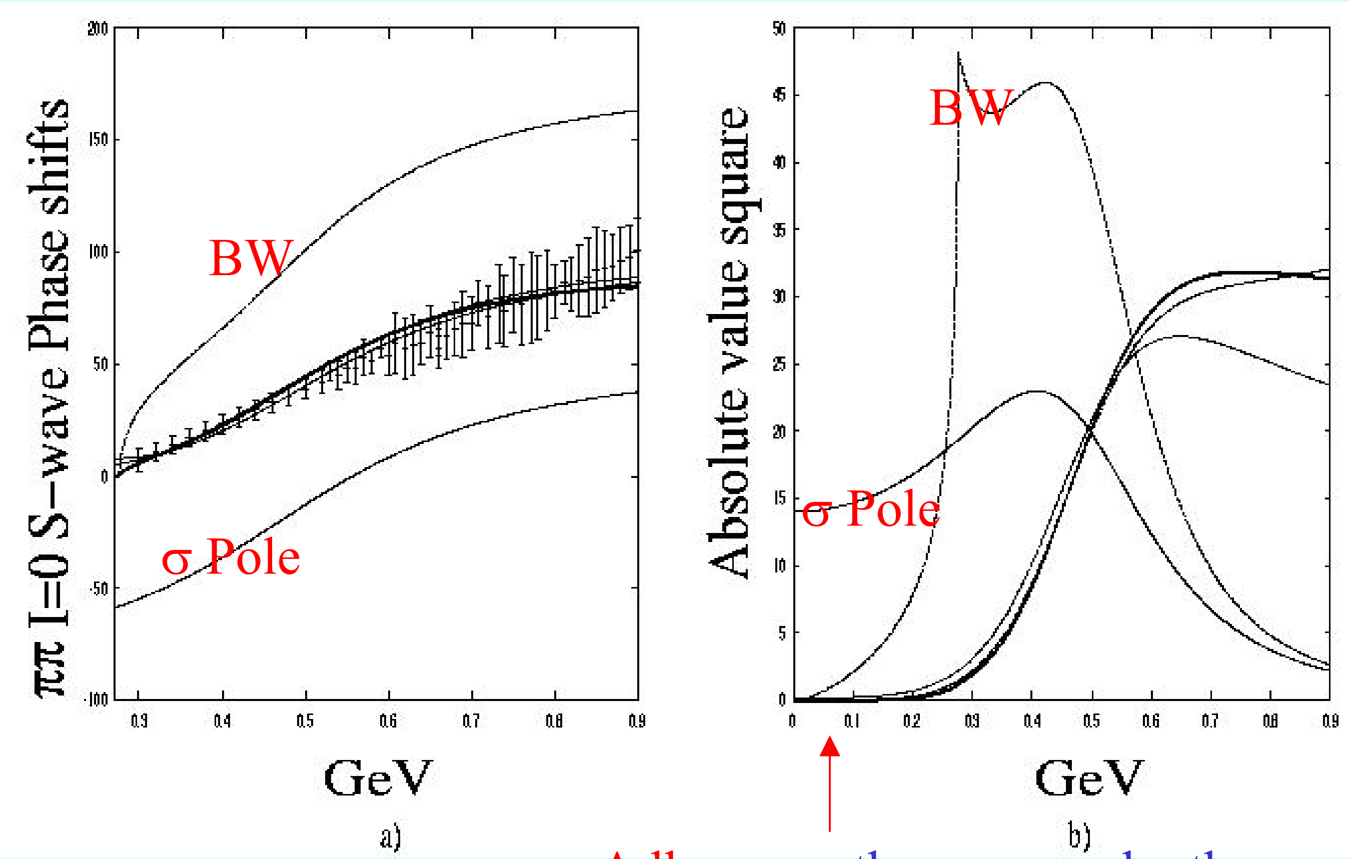
$$s_\sigma = (0.47 - i0.22)^2 GeV^2 \quad \gamma_0^2 = 5.3 + i7.7 GeV^2$$

$$\gamma_1 = -8.1 + i36.9 \quad \gamma_2 = 1.1 + i0.1 GeV^{-2}$$

From the T-matrix of Oset,J.A.O.,NPA620(1997)435

Laurent series around the σ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\sigma} + \underbrace{\gamma_1 + \gamma_2(s - s_\sigma) + \ldots}_{\text{background}}$$

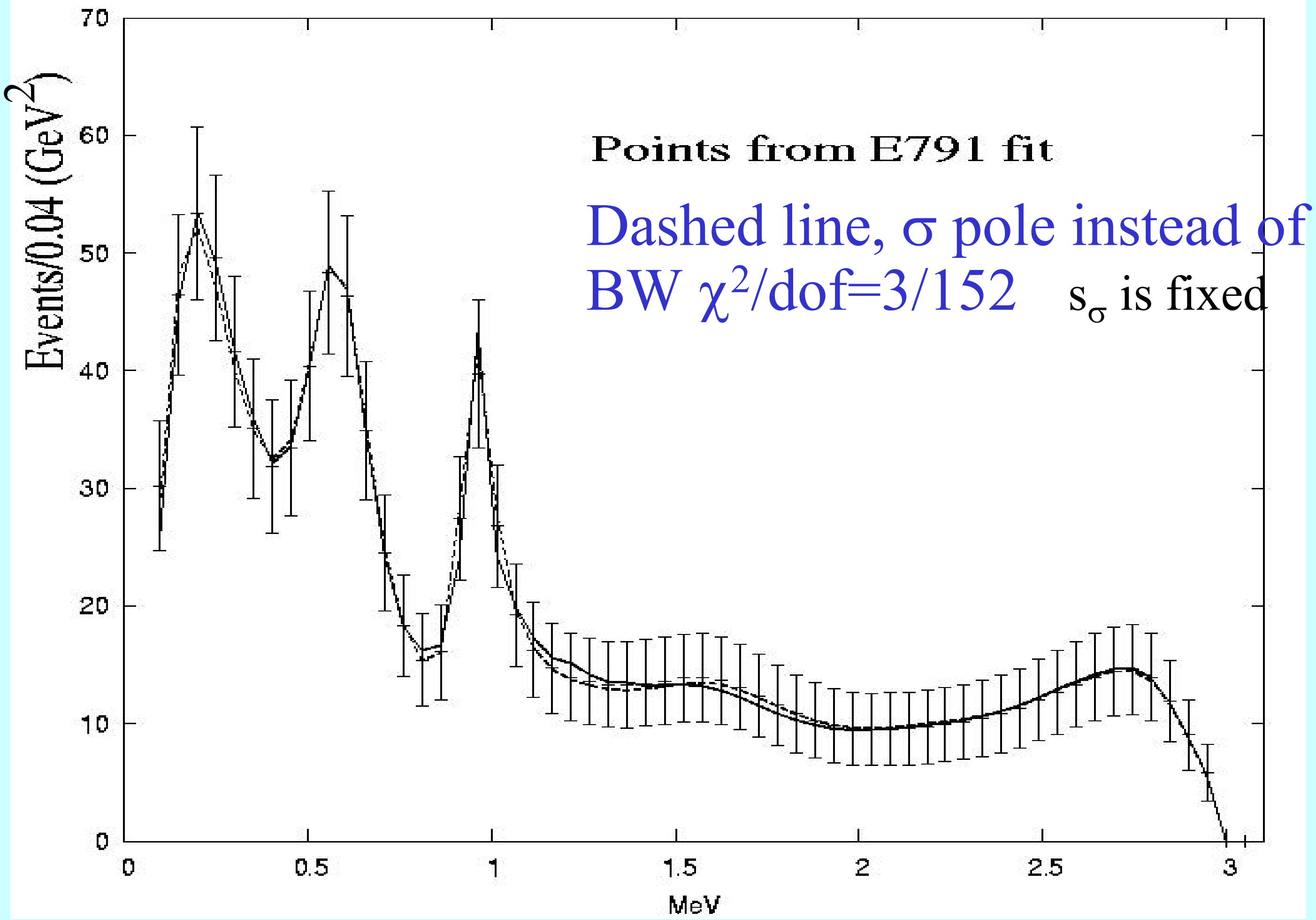


Adler zero: the reason why the background is so large in order to cancel the σ pole for low energies

The movement of the phase of the pole follows the experimental phase shifts

We substitue the σ BW by

$$\frac{a_1 e^{i\delta_1}}{s - s_\sigma}$$



Dashed line, σ pole instead of σ BW χ^2/dof=3/152 s_σ is fixed

At the same time the phase motion of the σ contribution follows the experimental phase shifts.

Full Final State Interactions (FSI) from the T-matrix of Oset,J.A.O.,NPA620(1997)435

SCATTERING:

Primary Vertex: $N_{\ell k}$

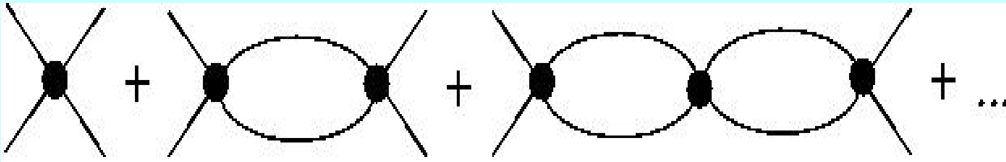
$$T_{\ell k} = \sum_j [I + N \cdot g]^{-1}_{\ell j} N_{jk}$$

D-decays:

Primary Vertex: $\xi_{\ell D}$

$$A_{\ell k} = \sum_j [I + N \cdot g]^{-1}_{\ell j} \xi_{jD}$$

In Unitarized Chiral Perturbation Theory the matrix of scattering amplitudes N is fixed by matching at a given chiral order with the full amplitude T calculated in CHPT



g is the scalar loop function or unitarity bubble

$$D = [I + N \cdot g]$$

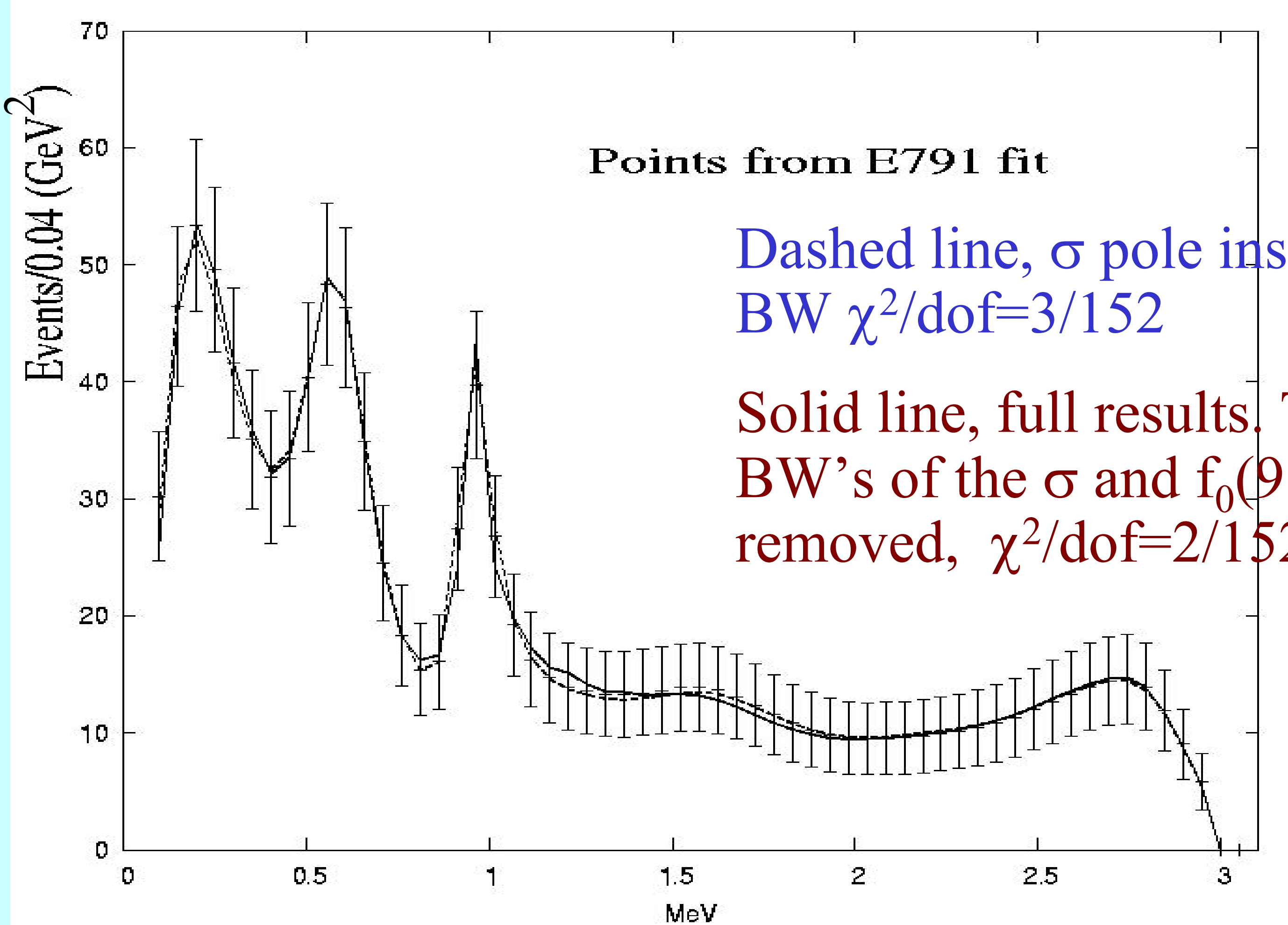
$$\left[D^{-1}_{11}(s_{12}) + D^{-1}_{11}(s_{13})\right] a_{\pi\pi} e^{i\delta_{\pi\pi}} +$$
$$\left[D^{-1}_{12}(s_{12}) + D^{-1}_{12}(s_{13})\right] a_{K\bar{K}} e^{i\delta_{K\bar{K}}}$$

We now have meson-meson intermediate states

The σ and $f_0(980)$ resonances appear as poles in the D matrix (they are dynamically generated)

Thus, the σ and $f_0(980)$ BW's are removed and substituted by the previuos expression.

χ^2/dof=2/152 , solid line



Dashed line, σ pole instead of σ BW χ^2/dof=3/152

Solid line, full results. The BW's of the σ and $f_0(980)$ are removed, χ^2/dof=2/152

FSI are driven by the fixed scattering amplitudes from UCHPT in agreement with scattering experimental data

No background, in the Laurent expansion of D_{11} the background accompanying the σ pole is negligible (No Adler zero)

FSI in the $D_s^+ \to \pi^-\pi^+\pi^+$ decay

E791 Collaboration PRL86,765 (2001)   625 events

Isobar Model: $f_0(980)$, $\rho^0(770)$, $f_2(1270)$, $f_0(1370)$ with a $f_0(980)$ dominant contribution.

$$D_s^+ \longrightarrow \begin{matrix} \text{non-resonant} \\ \rho^0(770)\pi^+ \\ f_0(980)\pi^+ \\ f_2(1270)\pi^+ \\ f_0(1370)\pi^+ \\ \rho^0(1450)\pi^+ \end{matrix} \longrightarrow \pi^-\pi^+\pi^+$$

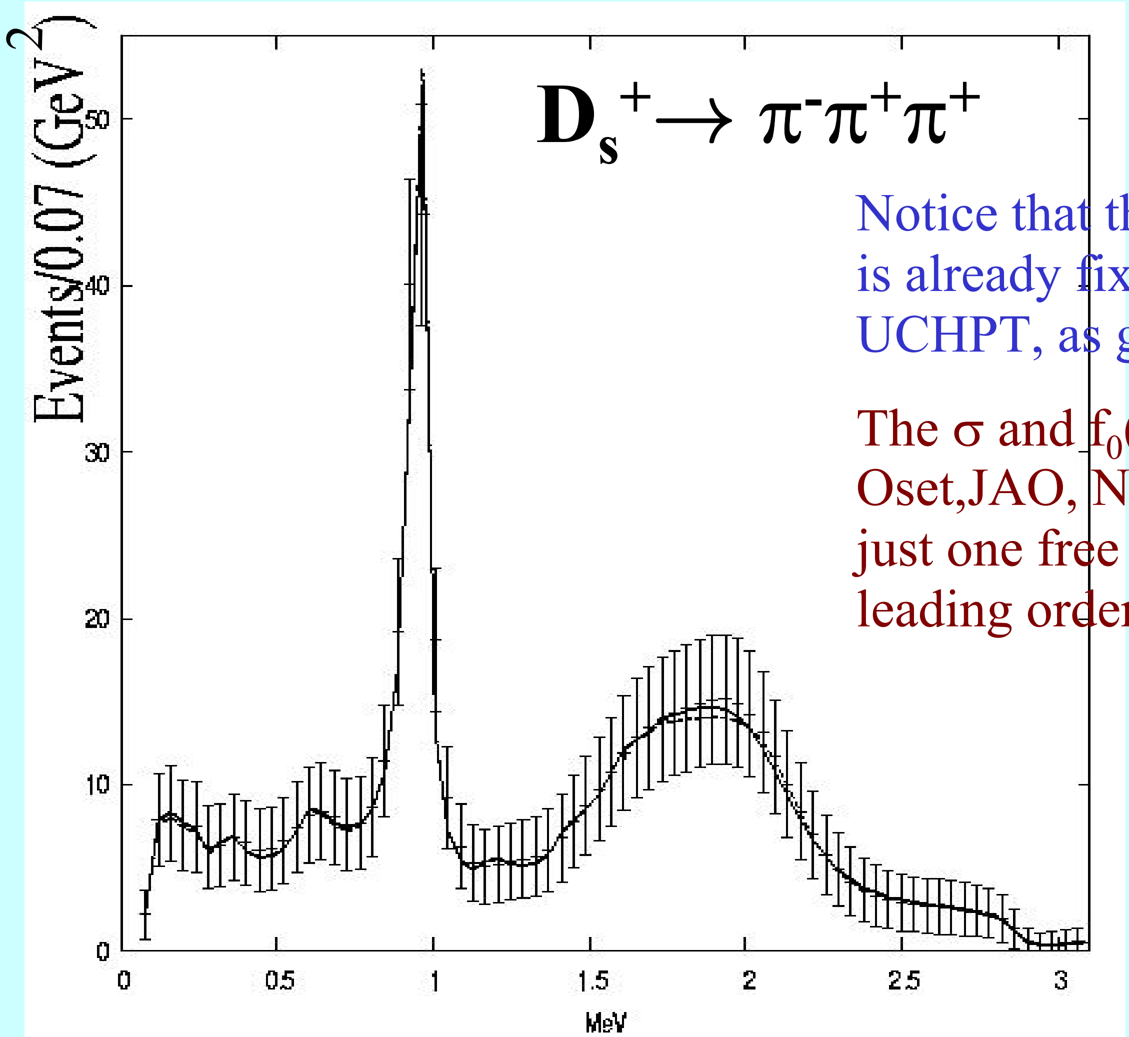
$M_{f_0(980)}=(977 \pm 4)$ MeV, $g_K= 0.02 \pm 0.05$, $g_\pi=0.09 \pm 0.01$

$g_K >> g_\pi$ and g_K compatible with zero !! In constrast with its proved affinity to couple with strangeness sources, SU(3) analysis, etc

We employ our formalism to take into account FSI from UCHPT

$$\left[D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13})\right] a_{\pi\pi} e^{i\delta_{\pi\pi}} +$$
$$\left[D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13})\right] a_{K\bar{K}} e^{i\delta_{K\bar{K}}}$$

Resonance	a_n Fraction	δ_n (radians)	a_n Fraction(%)	δ_n (radians)
NR	0.40 13%	0.16	0.40 14%	-0.24
$(\pi\pi)\pi^+$	0.28 6%	2.36	0.25 5%	2.23
$(K\bar{K})\pi^+$	1(fixed) 78%	0(fixed)	1(fixed) 84%	0(fixed)
$\rho^0(770)\pi^+$	0.24 4%	0.14	0.22 4%	0.19
$f_0(1370)\pi^+$	0.60 28%	1.68	0.57 28%	2.01
$f_2(1270)\pi^+$	0.50 20%	0.39	0.48 19%	0.41
$\rho^0(1450)\pi^+$	0.25 5%	0.67	0.24 5%	0.87
χ^2/ν	11/142		8.5/140	



Notice that the $f_0(980)$ resonant pole position is already fixed from scattering from UCHPT, as given in the D matrix

The σ and $f_0(980)$ poles were fixed in Oset,JAO, NPA620,435 (1997) in terms of just one free parameters plus CHPT at leading order.

FSI in the $D^+ \to K^- \pi^+ \pi^-$ decay

E791 Collaboration PRL 89,121801 (2002) 28400 events only
6% background

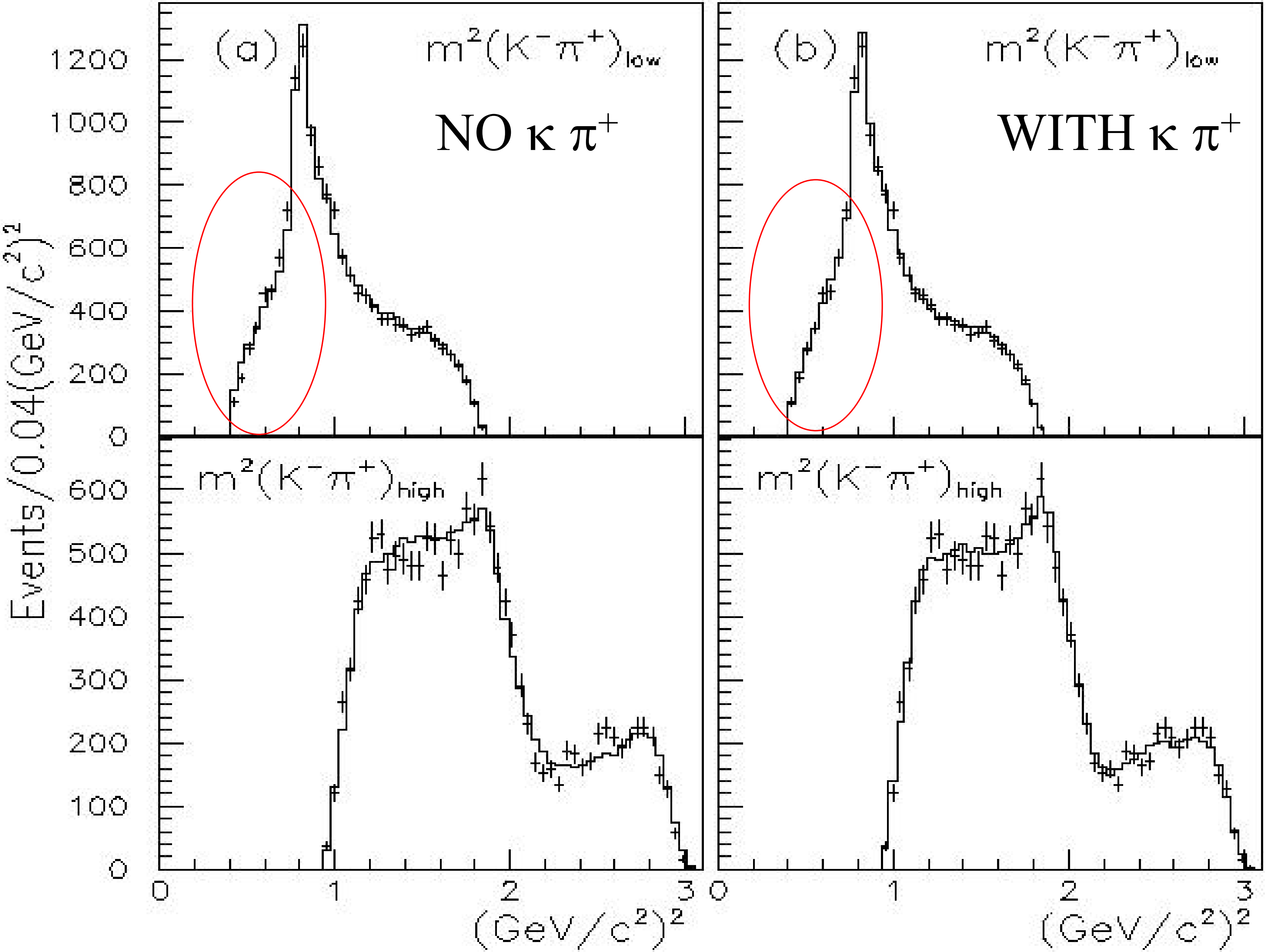
Similar situation to the $\mathbf{D}^+ \to \pi^- \pi^+ \pi^+$ case $\sigma \leftrightarrow \kappa$

Isobar Model:

$$D^+ \longrightarrow \begin{array}{c} \text{non-resonant} \\ \bar{K}^*(892)\pi^+ \\ \bar{K}_0^*(1430)\pi^+ \\ \bar{K}_2^*(1430)\pi^+ \\ \bar{K}^*(1680)\pi^+ \\ \left[\kappa\pi^+\right] \end{array} \longrightarrow K^-\pi^+\pi^+$$

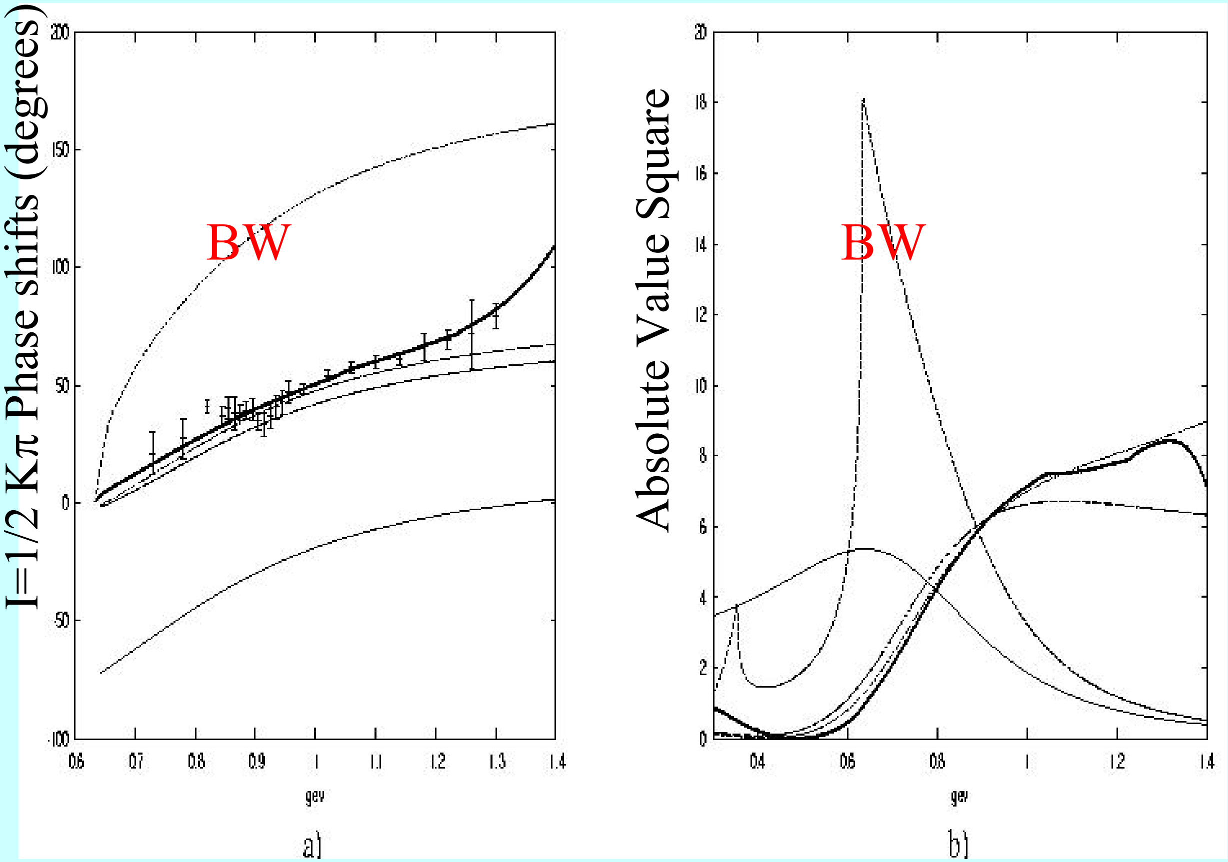
Without $\kappa\,\pi^+$: $\chi^2/\text{dof}=2.7 \rightarrow \text{CL}=10^{-11}$

With $\kappa\,\pi^+$: $\chi^2/\text{dof}=0.73 \rightarrow \text{CL}=95\%$


(a)
m²(K⁻π⁺)low
NO κ π⁺
(b)
WITH κ π⁺
m²(K⁻π⁺)high
Events/0.04(GeV/c²)²
(GeV/c²)²
0
200
400
600
800
1000
1200
100
300
500
1
2
3

$BW_\kappa(s)=(s-m_\kappa^2+i\, m_\kappa\, \Gamma(s))^{-1}$ energy dependent width

$$\Gamma(s) = \Gamma_\kappa \frac{m_\sigma}{\sqrt{s}} \frac{p(s)}{p_0} \frac{F_n^{(0)}(s)^2}{F_n^{(0)}(m_\kappa^2)^2}$$



BW does not follow the experimental S-wave I=1/2 Kπ phase shifts

M_κ=797±47 MeV

Γ_κ=410 ± 97 MeV

Laurent series around the σ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\kappa} + \gamma_1 + \gamma_2(s - s_\kappa) + \ldots$$

$$s_\kappa = (0.71 - i0.31)^2 GeV^2 \quad \gamma_0^2 = 19.0 + i5.9 GeV^2$$

$$\gamma_1 = 12.5 + i43.1 \quad \gamma_2 = 0.8 + i7.3 GeV^{-2}$$

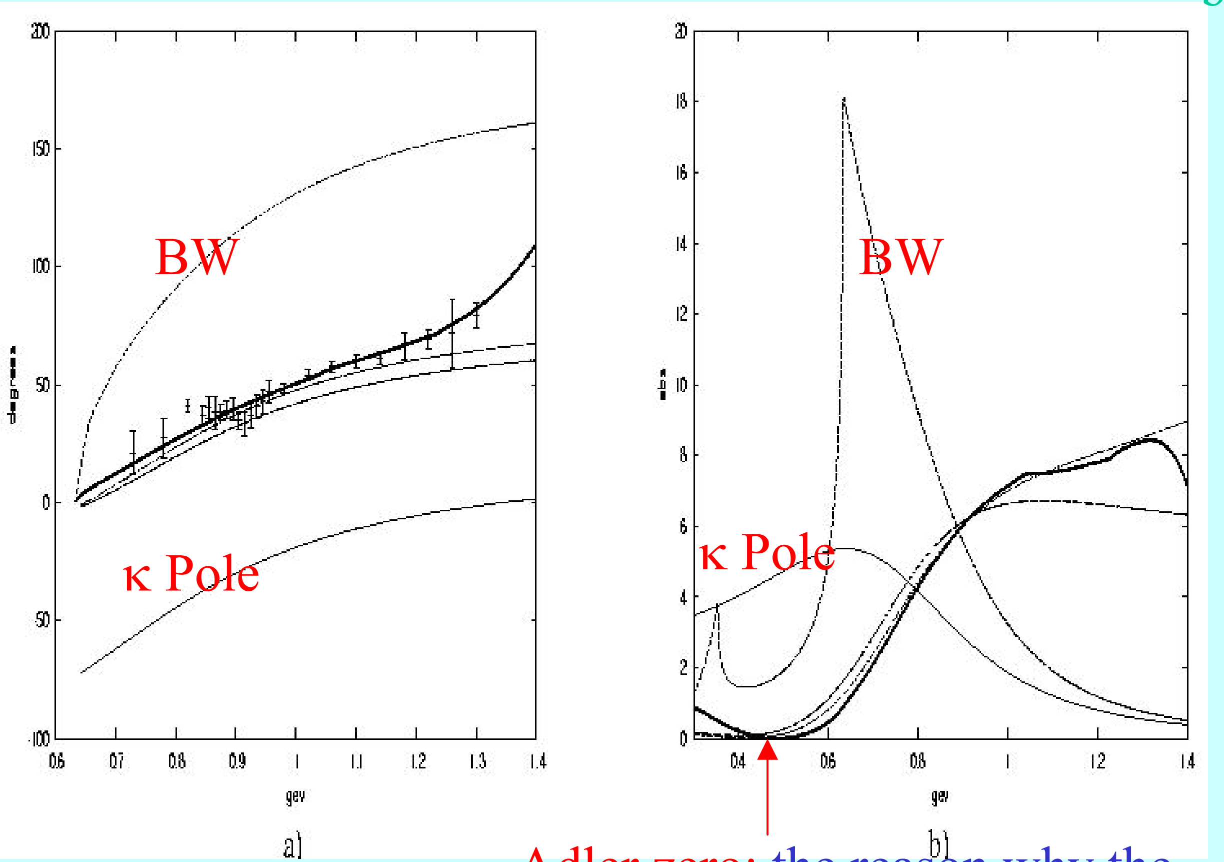
From the T-matrix of M.Jamin, A. Pich, J.A.O., NPB587,331 (2000)

UCHPT matching with U(3) CHPT +Resonances+large N_c constraints (vanishing of scalar form factors for $s \to \infty$)

Kπ,Kη,Kη' channels are included

Laurent series around the κ pole position at the second Riemann sheet:

$$t_{11}^{II}(s) = \frac{\gamma_0^2}{s - s_\kappa} + \underbrace{\gamma_1 + \gamma_2 (s - s_\kappa) + \ldots}_{\text{background}}$$

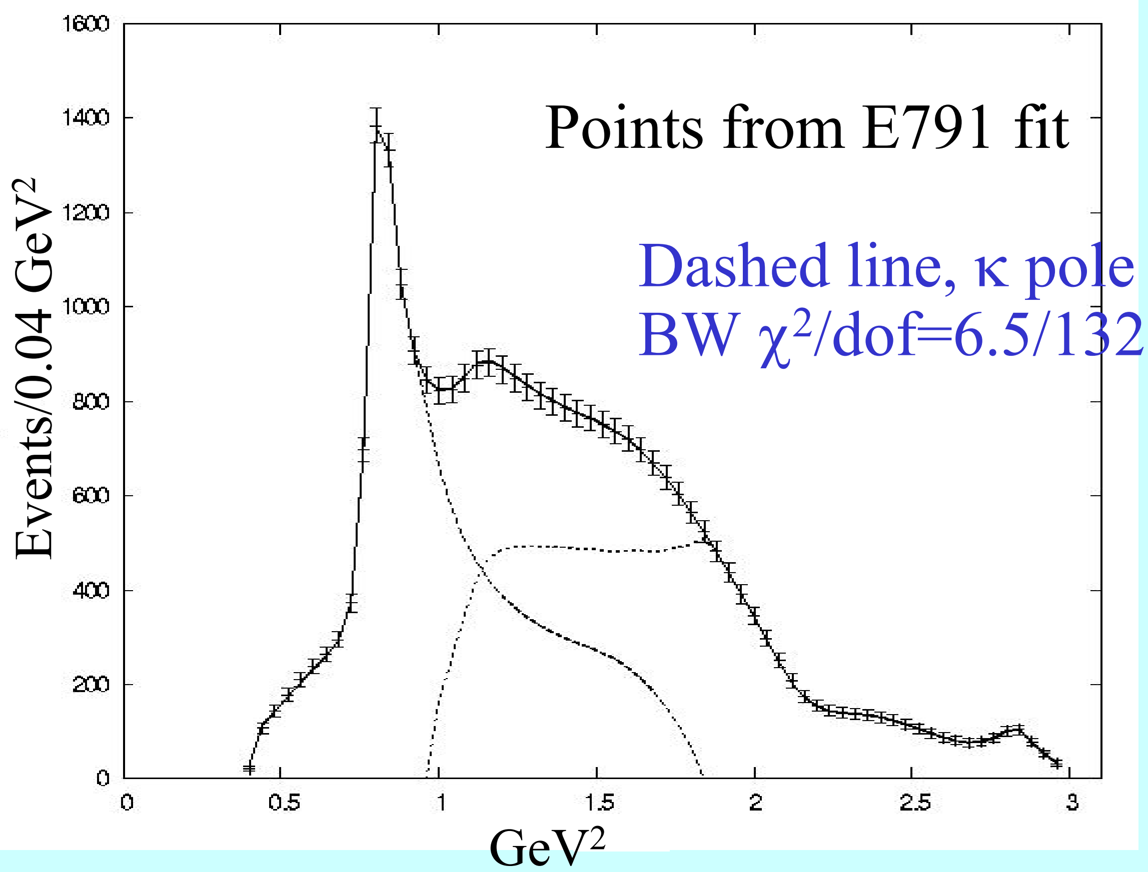


Adler zero: the reason why the background is so large in order to cancel the κ pole for low energies

The movement of the phase of the pole follows the experimental phase shifts

We substitue the κ BW by

$$\frac{a_1 e^{i\delta_1}}{s - s_\kappa}$$



Dashed line, κ pole instead of κ BW χ^2/dof=6.5/132 s_κ is fixed

At the same time the phase motion of the κ pole contribution follows the experimental phase shifts.

Full Final State Interactions (FSI) from the T-matrix of Jamin, Pich, J.A.O. NPB587,331 (2000)

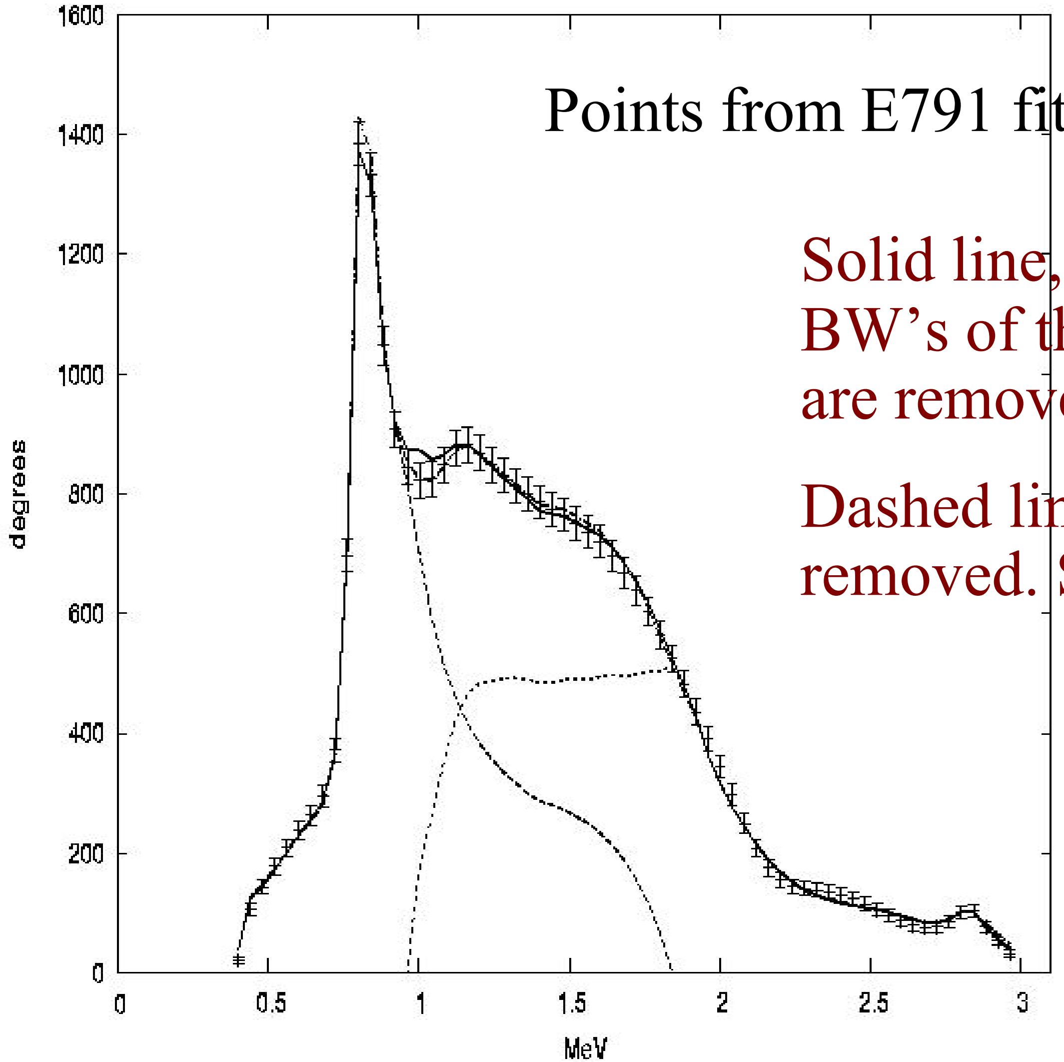
$$\left[D_{11}^{-1}(s_{12}) + D_{11}^{-1}(s_{13})\right] a_{K\pi} e^{i\delta_{K\pi}} +$$
$$\left[D_{12}^{-1}(s_{12}) + D_{12}^{-1}(s_{13})\right] a_{K\eta} e^{i\delta_{K\eta}} +$$
$$\left[D_{13}^{-1}(s_{12}) + D_{13}^{-1}(s_{13})\right] a_{K\eta'} e^{i\delta_{K\eta'}}$$

We now have meson-meson intermediate states

The κ and $K^*_0(1430)$ resonances appear as poles in the D matrix

Thus, the κ and $K^*_0(1430)$ BW's are removed and substituted by the previuos expression.

$\chi^2/dof=127/128$, solid and dashed lines



Solid line, full results. The BW's of the κ and $K^*_0(1430)$ are removed

Dashed line: the Kη channel is removed. Stability.

FSI are driven by the fixed scattering amplitudes from UCHPT in agreement with scattering experimental data

No background, in the Laurent expansion of D_{11} the background accompanying the κ pole is negligible (No Adler zero)

Resonance	a_n	δ_n (radians)	Fraction
NR	1.60	0.10	29.6%
$(K\pi)\pi^+$	1.66	4.10	31.8%
$(K\eta)\pi^+$	0.86	2.63	2.0%
$(K\eta')\pi^+$	2.33	-1.54	9.8%
$K_1^*(892)\pi^+$	1 (fixed)	0 (fixed)	11.6%
$K_2^*(1430)\pi^+$	0.11	-0.62	0.2%
$K_1^*(1680)\pi^+$	0.72	0.80	5.9%
χ^2/ν	127/128		

$K^*_0(1430)$ E791... $M_{K^*_0(1430)} = 1459 \pm 9$ MeV

$\Gamma_{K^*_0(1430)} = 175 \pm 17$ MeV

PDG... $M_{K^*_0(1430)} = 1412 \pm 6$ MeV

$\Gamma_{K^*_0(1430)} = 294 \pm 23$ MeV

Jamin,Pich,JAO (1430-1450,140-160) MeV $\simeq (M,\Gamma/2)$

In their fit (6.10) (1450,142) MeV

employed here

Summary

- We have considered simultaneously the FSI driven by the S-waves in the decays: $D^+ \to \pi^-\pi^+\pi^+$, $D_s^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+$.

- **We have reproduced the E791 Collaboration signal distribution functions in terms of new parameterizations.**

- In E791 analyses the disagreement between the phase motions of the σ and κ and the elastic S-wave I=0,1/2 phase shifts is due to the employment of BW's.

- **Once these BW's are substituted by the pole contributions of these resonances the agreement is restored.**

- These poles are fixed from T-matrices already determined from CHPT, unitarity, analiticity plus fitting scattering data.

- We have also reproduced the results of E791 making use of the full results of I=0,1/2 S-wave T-matrices in agreement with scattering and from Uunitarized CHPT.

- **The reason why the σ and κ pole contributions are not distorted in contrast with scattering is the absence of Adler zeroes.**

- These poles dominate the D matrix in the low energy region. No significative background.

- **The $f_0(980)$ from D decays turns out also with standard properties regarding its coupling to kaon.**

- The width of the $K^*_0(1430)$ from D decays is then in agreement with that from scattering data and reported in the PDG. That from E791 analysis was a factor 2 smaller.

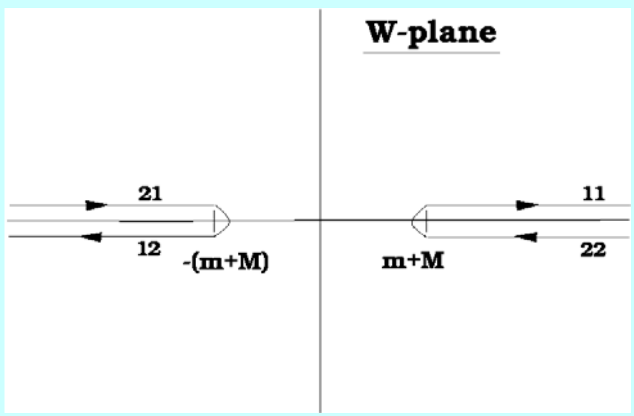
General Expression for a Partial Wave Amplitude

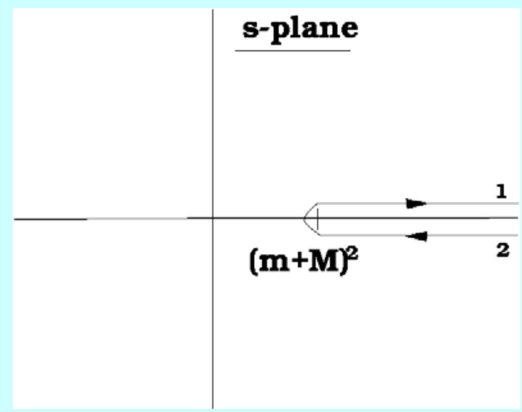
- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$Im\,T_{ij} = \sum_k T_{ik}\,\rho_k\,T^*_{kj} \quad \rightarrow \quad Im\,T^{-1}_{ij} = -\rho_i\,\delta_{ij}$$

Unitarity Cut

$W=\sqrt{s}$



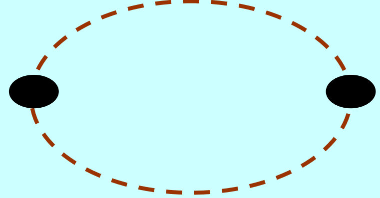


We perform a dispersion relation for the inverse of the partial wave (the unitarity cut is known)

$$T^{-1}_{ij} = \boxed{R^{-1}_{ij}} + \underbrace{\delta_{ij}\left(g(s_0)_i - \frac{s-s_0}{\pi}\int \frac{\rho(s')_i\,ds'}{(s'-s-i0^+)(s'-s_0)} \right)}$$

The rest

g(s): Single unitarity bubble

g(s)= 

$$g(s)=\frac{1}{4\pi^2}\left(a_{SL}+\sigma(s)\log\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)$$

$$\sigma(s)=\frac{2q}{\sqrt{s}}$$

$$T=\left(R^{-1}+g(s)\right)^{-1}$$

T obeys a CHPT/alike expansion

R is fixed by matching algebraically with the CHPT/alike
CHPT/alike+Resonances
expressions of T

In doing that, one makes use of the CHPT/alike counting for g(s)

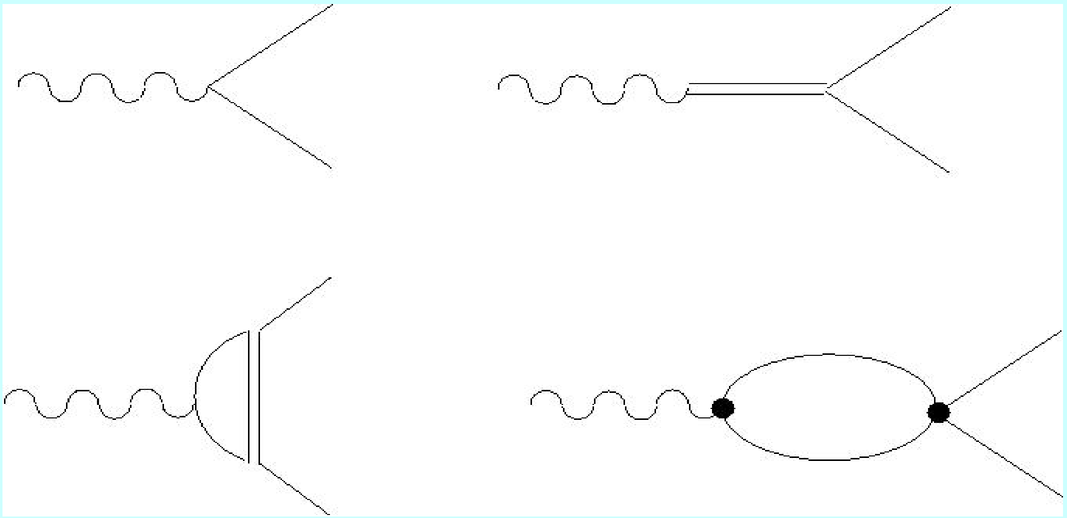
The counting/expressions of R(s) are consequences of the known ones of g(s) and T(s)

The CHPT/alike expansion is done to R(s). Crossed channel dynamics is included perturbatively.

The final expressions fulfill unitarity to all orders since R is real in the physical region (T from CHPT fulfills unitarity pertubatively as employed in the matching).

Production Processes

The re-scattering is due to the strong „final“ state interactions from some „weak“ production mechanism.



$$Im F_i = \sum_k F_k \, \rho_k \, T^*_{ki}$$

We first consider the case with only the right hand cut for the strong interacting amplitude, R^{-1} is then a sum of poles (CDD) and a constant. It can be easily shown then:

$$F = (I + R\,g(s))^{-1}\,\xi$$

Finally, ξ is also expanded pertubatively (in the same way as R) by the matching process with CHPT/alike expressions for F, order by order. The crossed dynamics, as well for the production mechanism, are then included pertubatively.

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LET US SEE SOME APPLICATIONS

Meson-Meson Scalar Sector

1) **The mesonic scalar sector has the vacuum quantum numbers 0^{++}. Essencial for the study of Chiral Symmetry Breaking: Spontaneous and Explicit m_u, m_d, m_s.**

2) **In this sector the mesons really interact strongly.**

   1) **Large unitarity loops.**
   2) **Channels coupled very strongly, e.g. π π- $K\bar{K}$, π η- $K, \bar{K}$ ...**
   3) **Dynamically generated resonances, ~~Breit-Wigner formulae~~, ~~VMD~~, ...**

3) **OZI rule has large corrections.**

   1) **No ideal mixing multiplets.**
   2) **~~Simple quark model.~~**

Points 2) and 3) imply large deviations with respect to Large Nc QCD.

4) A **precise knowledge** of the scalar interactions of the lightest hadronic thresholds, $\pi\ \pi$ and so on, is often required.

- Final State Interactions (FSI) in ε'/ε , Pich, Palante, Scimemi, Buras, Martinelli,...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- Fluctuations in order parameters of S♍SB.

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- Final State Interactions (FSI) in ε′/ε , Pich, Palante, Scimemi, Buras, Martinelli,...
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- Fluctuations in order parameters of S♍SB.

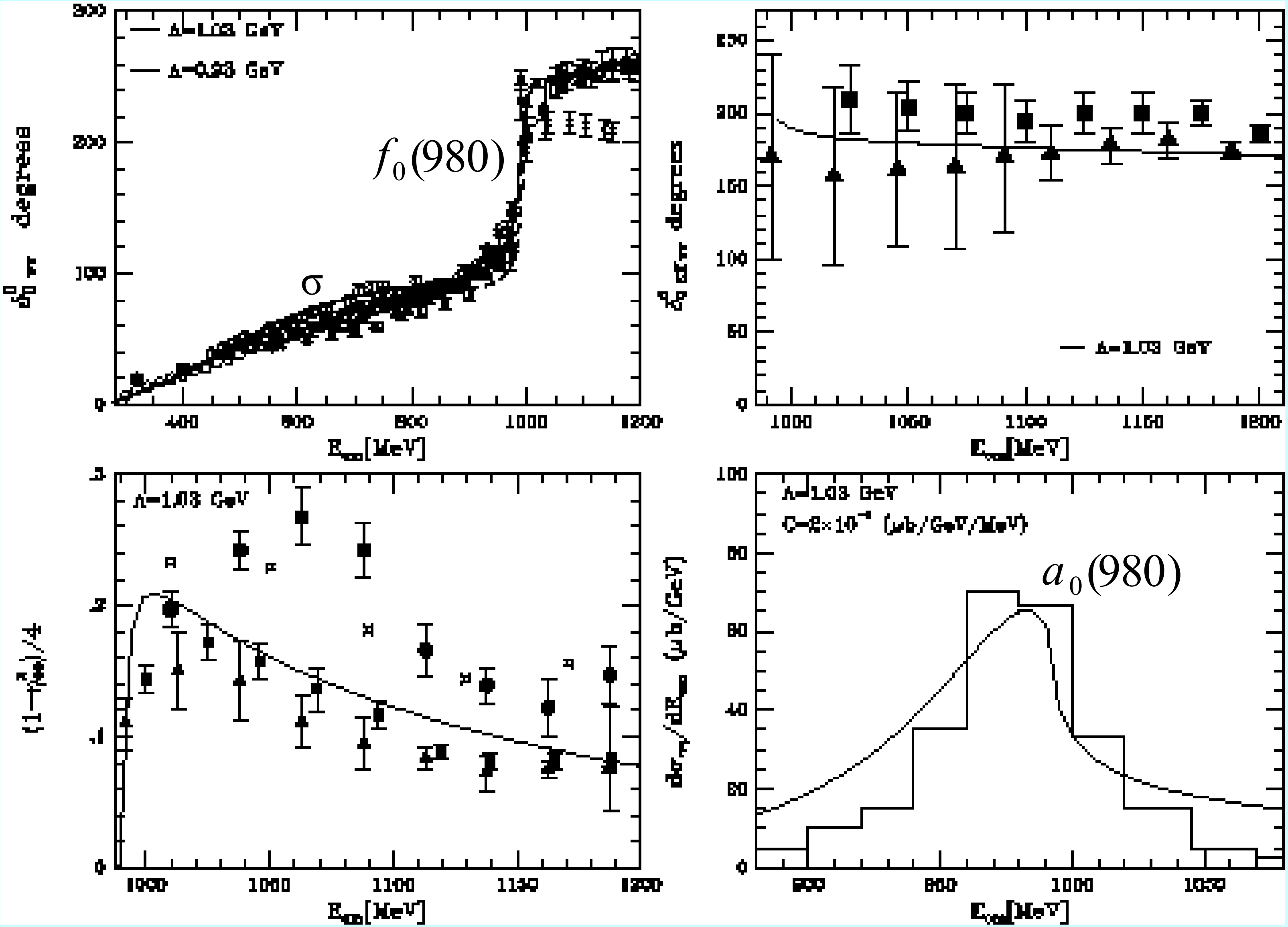
Let us apply the chiral unitary approach $T = \left(R^{-1} + g(s) \right)^{-1}$

- LEADING ORDER:

g is order 1 in CHPT $T = T_2 = R_2 - R_2 g R_2 + ..$ $R = R_2 = T_2$

Oset, Oller, NPA620,438(97)

$a_{SL} \cong -0.5$ only free parameter, equivalently a three-momentum cut-off $\Lambda \cong 0.9$ GeV


Λ=1.03 GeV
Λ=0.93 GeV
σ
f0(980)
a0(980)
C=2×10^-4 (μb/GeV/MeV)
E_cm [MeV]

Pole positions and couplings

$f_0(980)$ (GeV)	$a_0(980)$ (GeV)
$0.993 - i\, 0.012$	$1.009 - i\, 0.056$
$\lvert g^{f}_{\pi\pi} \rvert = 1.90$	$\lvert g^{a}_{\pi\eta} \rvert = 3.54$
$\lvert g^{f}_{KK} \rvert = 3.80$	$\lvert g^{a}_{KK} \rvert = 5.20$

$Br(f_0(980) \rightarrow \pi\pi)=0.70$ **$Br(a_0(980) \rightarrow \pi\eta)=0.63$**

All these resonances were dynamically generated from the lowest order CHPT amplitudes due to the enhancement of the unitarity loops.

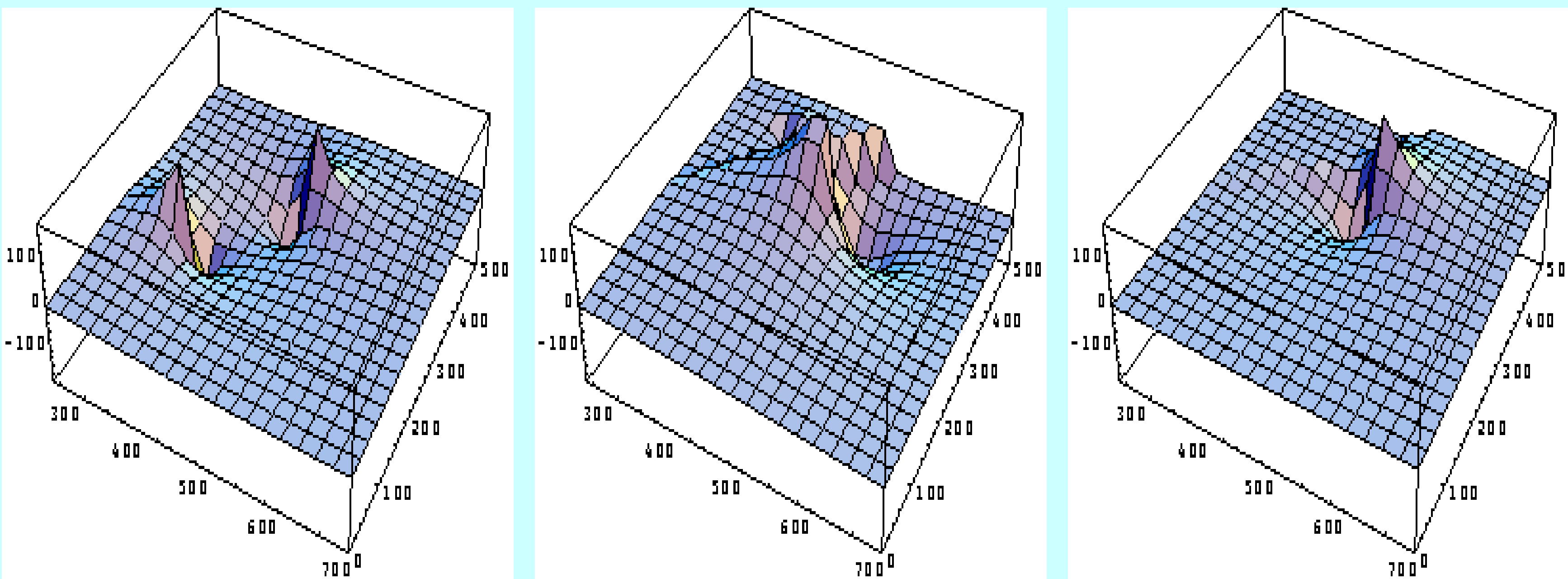
In Oset,Oller PRD60,074023(99) we studied the I=0,1,1/2 S-waves.

The input included next-to-leading order CHPT plus resonances:

1. Cancellation between the crossed channel loops and crossed channel resonance exchanges. (Large Nc violation).

2. Dynamically generated renances. The tree level or preexisting resonances move higher in energy (octet around 1.4 GeV). Pole positions were very stable under the improvement of the kernel R (convergence).

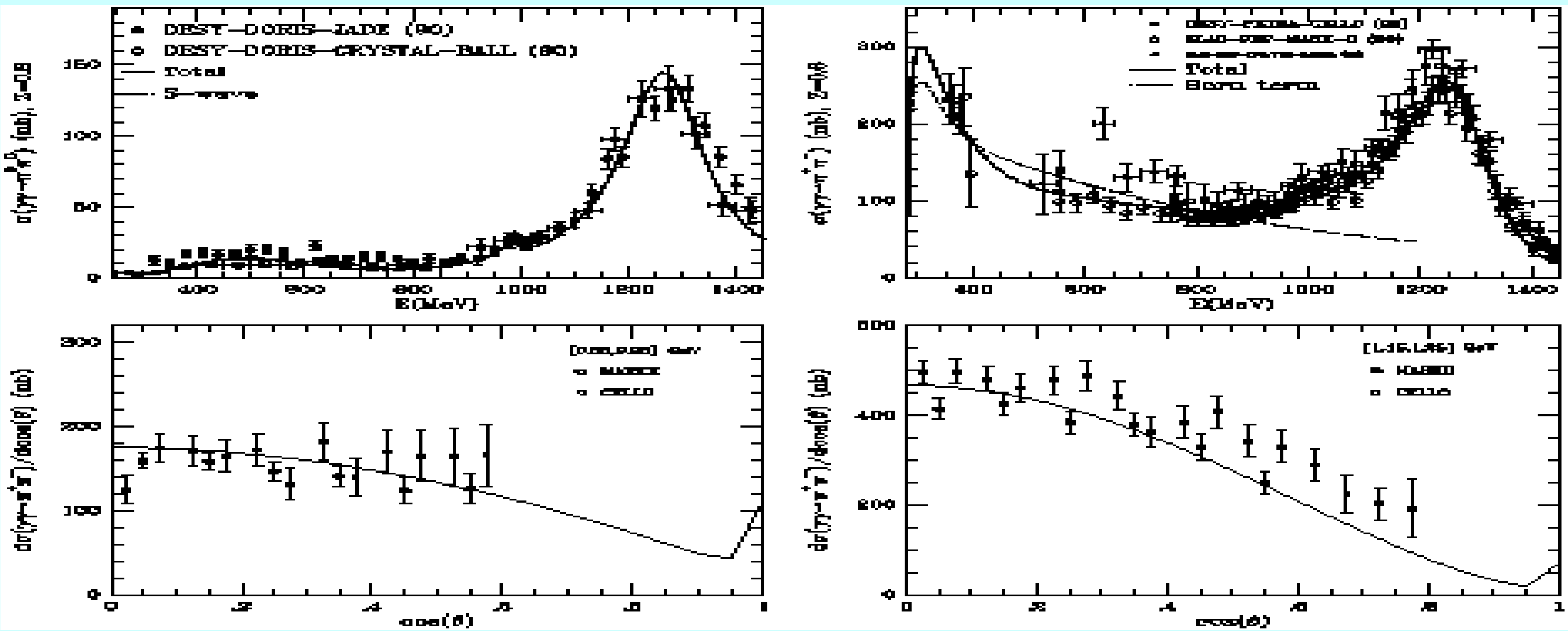
3. In the SU(3) limit we have a degenerate octet plus a singlet of dynamically generated resonances σ, $f_0(980)$, $a_0(980)$, $\kappa(700)$



Using these T-matrices we also corrected by Final State Interactions the processes $\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0, \pi\eta, K^+K^-, K^0\bar{K}^0$

Where the input comes from CHPT at one loop, plus resonances. There were some couplings and counterterms but were taken from the literature. No fit parameters.

Oset, Oller NPA629,739(98).

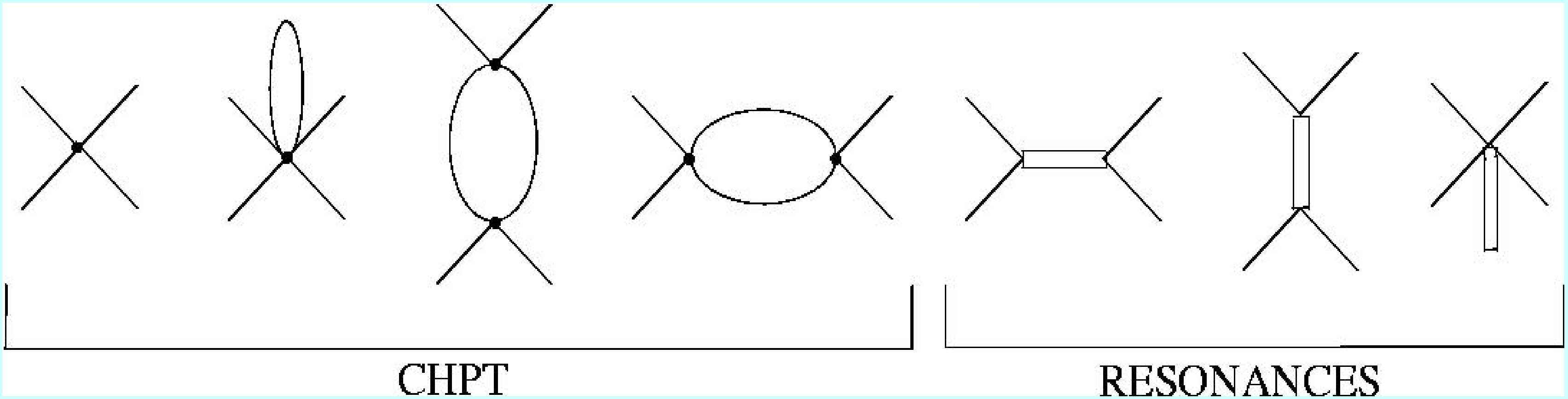


CHPT+Resonances

Ecker, Gasser, Pich and de Rafael, NPB321, 311 (´98)

Resonances give rise to a resummation of the chiral series at the tree level (local counterterms beyond O(p^4).

$$\frac{1}{M^2-q^2}=\frac{1}{q^2}+\frac{q^2}{M^4}+\frac{q^4}{M^6}+\ldots \quad q^2<<M^2$$



The counting used to perform the matching is a simultaneous one in the number of loops calculated at a given order in CHPT (that increases order by order). E.g:

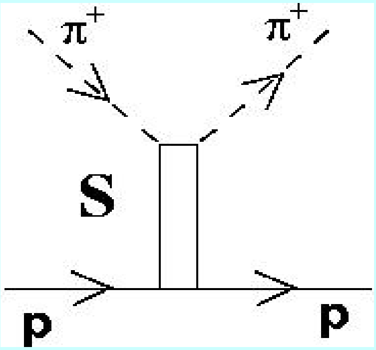
– Meissner, J.A.O, NPA673,311 (´00) the πN scattering was studied up to one loop calculated at O(p^3) in HBCHPT+Resonances.

– Jamin, Pich, J.A.O, NPB587, 331 (´00), $K\pi, K\eta, K\eta'$ scattering.

- The inclusion of the resonances require the knowlodge of their bare masses and couplings, that were fitted to experiment

A theoretical input for their values would be very welcome:

  - The CHUA would reduce its freedom and would increase its predictive power.
  - For the microscopic models, one can then include the so important final state interactions that appear in some channels, particularly in the scalar ones. Also it would be possible to identify the final physical poles originated by such bare resonances and to work simultaneously with those resonances dynamically generated.



$$L_{SNN} = -g_{sNN}\bar{\Psi}\Psi S$$

$$L_{S\pi\pi} = S\left(\bar{c}_m \mathrm{Tr}(X^+) + \bar{c}_d \mathrm{Tr}\left(u_\mu u^\mu\right)\right)$$