Vienna, February 13th 2004

Order p^6 Chiral Couplings from the Scalar $K\pi$ Form Factor

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hep-ph/0401080 JHEP

- **Introduction**
- **Strangeness Changing Scalar $K\pi$, $K\eta'$ Form Factors**
- **$O(p^6)$ Counterterms, $F_+(0)$, F_K/F_π**
- **Conclusions**

1. Introduction

We focus our attention on the Scalar $K\pi$ Form Factors:

$$<P|\partial^\mu \, \bar{s}\gamma_\mu u|K> \;= i(m_s - m_u)<P|\bar{s}u|K> \;= -i\frac{\Delta_{K\pi}}{\sqrt{2}}C_{KP}F_{KP}(t)$$

$P = \pi, \eta_1, \eta_8$	C_{KP} Clebsch-Gordan coefficients such
$\Delta_{K\pi} = M_K^2 - M_\pi^2$	that $F_{KP}(t)=1$ at lowest order in U(3) CHPT

We respect isospin symmetry

Due to the absence of scalar probes the scalar form factors seem to be rather theoretical observables....

Nevertheless, they have important implications for the knowledge of input parameters for the Standard Model like m_s or V_{us}

The previous scalar form factors were calculated by solving the corresponding coupled channel (K**π** and Kη') integral equations in Jamin, Pich, J.A.O., NPB622('02)279, NPB587('00)331 for the first, and still unique, time.

- In Jamin, Pich, J.A.O. EPJC24('02)237 they were applied to calculate $\mathbf{m_s}$

$$\mathbf{m_s(2\ GeV)=(99 \pm 16)\ \ MeV} \qquad \mathbf{in\ \overline{MS}}$$

Making use of Scalar Sum Rules, by studying the correlator

$$\psi(p^2)=i\int dx e^{ipx} <\Omega|T[J(x)\,J(0)]|\,\Omega>\,, \quad J(x)=\partial^\mu(\bar{s}\gamma_\mu q)(x)=i(m_s-\hat{m})(\bar{s}q)(x)$$

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- **Now we are interested in calculating $F_{K\pi}(0)$, necessary to determine V_{us} from K_{l3} decays**

Charge Changing currents in the SM

$$L_{CC} = \frac{g}{2\sqrt{2}} W^{+}_{\mu} \left[\bar{U}_i \gamma^{\mu}(1-\gamma_5) V_{ij} D_j \quad + \quad \bar{\nu}_k \gamma^{\mu}(1-\gamma_5) l_k \right] \; + \; h.c.$$

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \upsilon = \begin{pmatrix} \upsilon_e \\ \upsilon_\mu \\ \upsilon_\tau \end{pmatrix} \qquad l = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ut} \\ \mathbf{V}_{cd} & \mathbf{V}_{us} & \mathbf{V}_{ut} \\ \mathbf{V}_{td} & \mathbf{V}_{us} & \mathbf{V}_{ut} \end{pmatrix}$$

CKM matrix

Unitariy Matrix

$$\mathbf{V}\mathbf{V}^{\dagger} = \mathbf{V}^{\dagger}\mathbf{V} = \mathbf{I}$$

$$\sum_{\beta} \mathrm{V}_{\alpha\beta} \mathrm{V}^{*}_{\lambda\beta} = \delta_{\alpha\lambda}$$

The most accurate test of CKM unitarity

$|V_{ud}|=0.9739 \pm 0.0005$ (Superallowed FT, $n \to p\, e\, \nu$ nβ-decay)

$|V_{us}|=0.2196 \pm 0.0026$ (K_{e3})

$|V_{ub}|_{pdg}=0.0036 \pm 0.0010$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \Rightarrow \quad |V_{us}|=0.2270 \pm 0.0021$$

- There is around a 2.5σ deviation among both values of V_{us}

(this disappears with the high-stadistics **K^{+}_{e3} E865** Collaboration hep-ex/0305042 experiment)

- V_{ub} is negligible (1% of the uncertainty from V_{us} or V_{ud})
- There is a problem between unitarity of CKM and presently accepted values of V_{ij}
- Close relationship between $V_{us} \leftrightarrow F_{+}(0)$ from K_{e3}

K_{l3} Decays: $K \to \pi\, l^+ \nu$

$$M = \frac{G_F}{\sqrt{2}} V^*_{us} L^\mu \underbrace{C_K \left[F_+(t)\,(p_K + p_\pi)_\mu + F_-(t)\,(p_k - p_\pi)_\mu \right]}_{<\pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)>}$$

$$t = (p_K - p_\pi)^2 \quad ; \quad L^\mu = \bar{u}(p_\nu)\,\gamma^\mu\,(1 - \gamma_5)\,v(p_l)$$

$$C_K = \left(\frac{1}{\sqrt{2}} \; , \; 1 \right) \quad for \;\; (K^+ \;\; , K^0)$$

$F_+(t)$ is the **vector K π form factor**

$F_0(t)$ is the **scalar K π form factor**

$$F_0(t) = F_+(t) + \frac{t}{M_K^2 - M_\pi^2} F_-(t)$$

$F_+(0) = F_0(0)$

$|F_0(t)|^2$ is always multiplied by m_l^2/M_K^2 and can be disregarded in the calculation of the width for K_{e3}.

Decay Rate

$$\Gamma_K = C_K^2 \frac{G_F^2 M_K^5}{192\pi^3} S_{ew} \, |V_{us}|^2 \, |F_+(0)|^2 \, I_K$$

$$F_+(t) = F_+(0)\left(1 + \lambda_+ \frac{t}{M_\pi^2} + c_+ \frac{t^2}{M_\pi^4}\right)$$

$$F_0(t) = F_+(0)\left(1 + \lambda_0 \frac{t}{M_\pi^2} + c_0 \frac{t^2}{M_\pi^4}\right)$$

$$m_l^2 \leq t \leq (M_K - M_\pi)^2$$

I_K results from integrating over phase space the form factor dependence on t

- From Γ_K one can obtain $|V_{us}|$, once $F_+(0)$ is known Leutwyler, Roos Z.Phys C25('84)91
- New high precission experiments on K_{e3} are ongoing or prepared (CMD2, NA48, KTEV, KLOE) to improve Γ_K

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These facts, together with the possible unitarity violation of the CKM matrix, has triggered important efforts within (G)CHPT

- $O(p^4)$ Gasser, Letwyler NPB250('85)517

- + estimated $O(p^6)$ analytical contribution Leutwyler, Roos Z.Phys C25('84)91

- $O(p^4)$ within GCHPT Fuchs, Knecht, Stern PRD62('00)033003

- $O(p^4,(m_d-m_u)p^2,e^2p^2)$ Cirigliano, Knecht, Neufeld, Rupertsberger and Talavera EPJC23('02)121

- $O(p^6)$ isospin limit Post, Schilcher, EPJC25('02)427;
  Bijnens, Talavera NPB699('03)341 (BT)

$O(p^4) \sim O(p^6)$; $O(p^4)$ suppressed by large Nc

- From BT, $\mathbf{F_+(0)}$ only depends on two new order p^6 chiral counterterms: $\mathbf{C^r_{12}}$, $\mathbf{C^r_{34}}$

- The same countererms are also the only ones that appear in the slope and the curvature of $\mathbf{F_0(t)}$, **and hence by a knowledge of $\mathbf{F_0(t)}$ one can determine those counterterms.**

This is our aim

So that an order p^6 calculation of $F_+(0)$ becames feassible

2. Strangeness Changing Scalar K π , K η' Form Factors

$$<P|\,\partial^\mu\, \bar{s}\gamma_\mu u\,|K> \; = i(m_s - m_u)<P|\,\bar{s}\,u|K> \; = -i\frac{\Delta_{K\pi}}{\sqrt{2}} C_{KP} F_{KP}(t)$$

P= π, η', η

$F_0(t) \equiv F_{K\pi}(t)$

•$F_{KP}(t)$ are analytical functions in t, except for a cut from the lightest threshold, $(M_K+M_\pi)^2$ to ∞ (the unitarity cut).

•Its discontinuity through the cut is fixed by unitarity:

$$Im F_{KP}(t) = \sum_Q F_{KQ}\, \rho_{KQ}\, T^*_{KP,KQ} \qquad \rho_{KQ}(t) = \frac{p_{KQ}}{8\pi\sqrt{s}} \times \theta\left(t - (M_K + M_Q)^2\right)$$

$T_{KP,KQ}$ is the I=1/2 S-wave T-matrix coupling toghether $K\pi$, $K\eta$, $K\,\eta'$ and calculated in Jamin, Pich, J.A.O. NPB587('00)331 (JOP00)

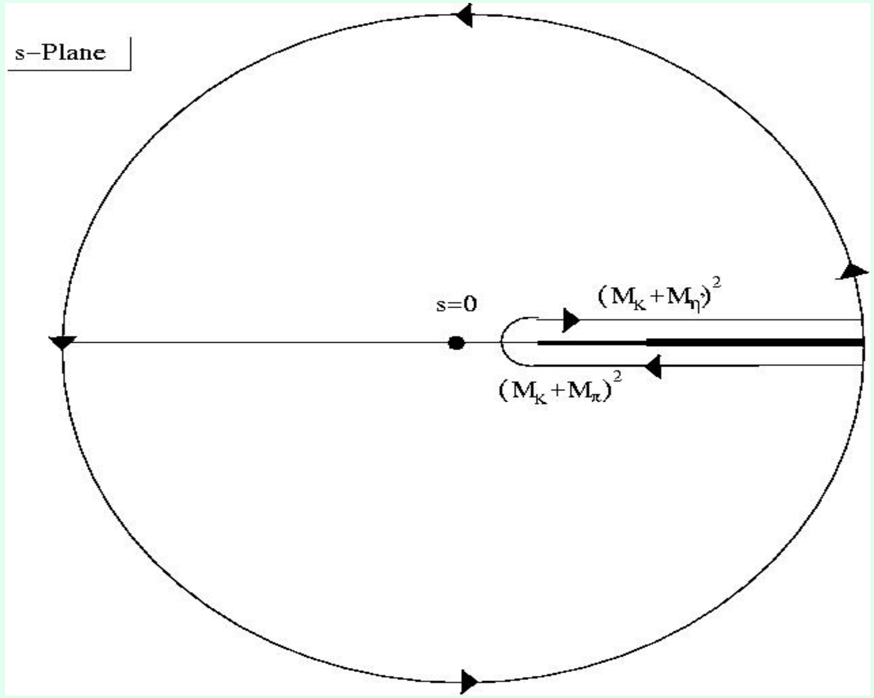
The $K\eta$ turns out to be negligible, both experimentally as well as theoretically, and we neglect it in the following.

The form factors satisfy the dispersion relations:

$$F_1(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\rho_1(s')\,F_1(s')\,T_{11}^*(s')}{s'-s-i\varepsilon}+\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\rho_2(s')\,F_2(s')\,T_{12}^*(s')}{s'-s-i\varepsilon}$$

$$F_2(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\rho_1(s')\,F_1(s')\,T_{12}^*(s')}{s'-s-i\varepsilon}+\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\rho_2(s')\,F_2(s')\,T_{22}^*(s')}{s'-s-i\varepsilon}$$

This is the so called **Muskhelishvili-Omnès** problem in coupled channels solved in **Jamin, Pich, J.A.O. NPB622('02)279 (JOP02)**



For only one channel (elastic case), namely $K\pi$, one has the Omnès solution:

$$F_1(s)=F_1(0)\exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s-i\varepsilon)}\right)$$

According to Muskhelishvili, *Singular Integral Equations* (Dover,1992) ; Babelon, Basdevant, Mennessier NPB113('76)445

The solutions of the Muskhelishvili-Omnès problem for two coupled channels can be expressed in terms of two linearly indepedent form factors

$G_1(s) \equiv \{G_{11}(s) , G_{12}(s)\}$ and $G_2(s) \equiv \{G_{21}(s) , G_{22}(s))$

$F(s)=\alpha_1 G_1(s) + \alpha_2 G_2(s)$ with $F(s)=\{F_1(s),F_2(s)\} \equiv \{F_{K\pi}(s), F_{K\eta'}(s)\}$

α_1 and α_2 are, in general, polynomials.

The behaviour at inifinity of the basic solutions $G_{ij}(s)$ is determined by the times the argument of the determinant of the *S-matrix (L=0, I=1/2)* rounds to 2π, *m*

$$s \to \infty \quad G_i(s) \Rightarrow \frac{1}{s^{\chi_i}} \quad such \quad that \quad \chi_1 + \chi_2 = m$$

For the T-matrices of Jamin, Pich, J.A.O. NPB587('00)331 ***m = 2 or 1***

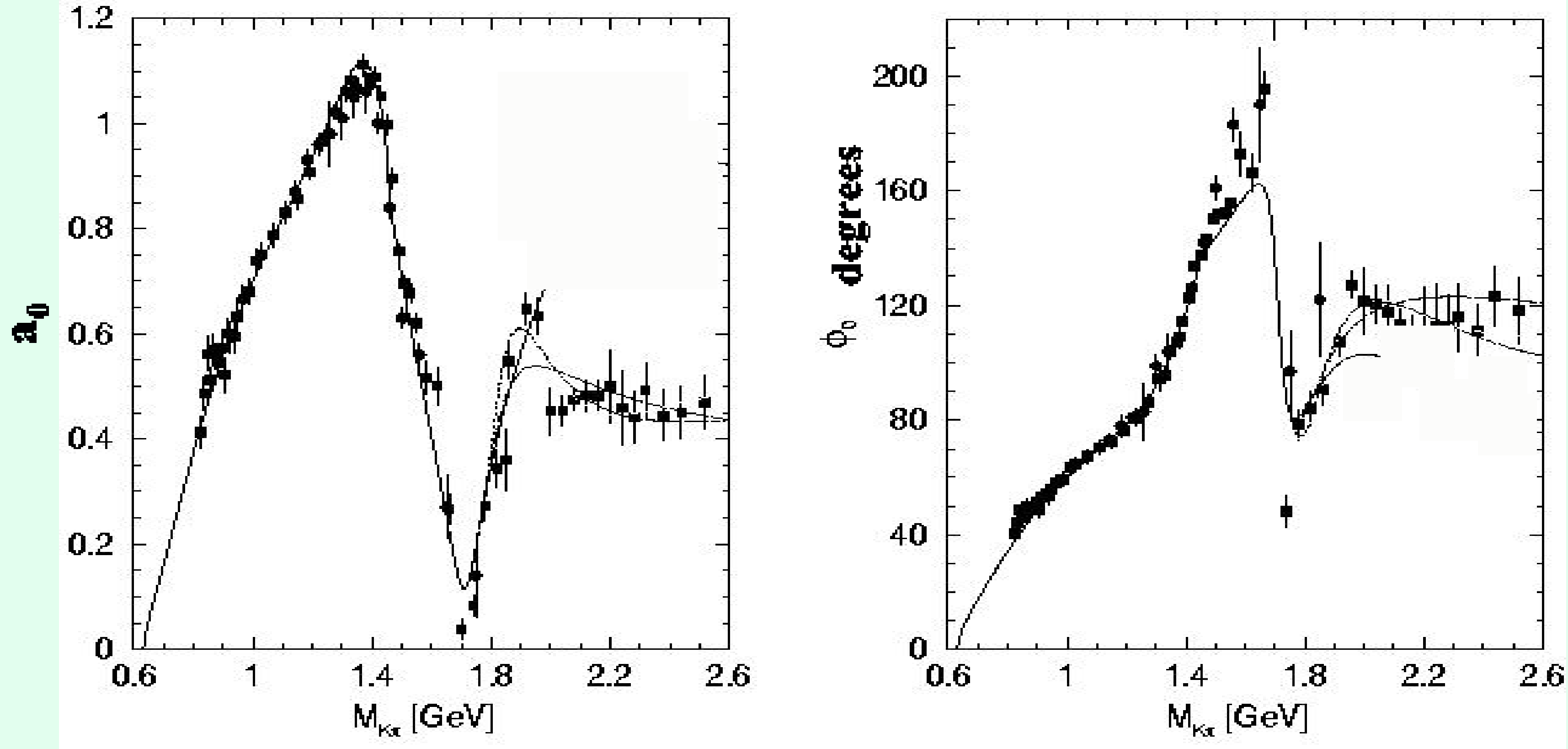
then for vanishing form factors at inifinite ***$\chi_1 = 1$ (1) , $\chi_2 = 1$ (0)***

$m=2 \Rightarrow \alpha_1, \alpha_2$ constants ; $m=1 \Rightarrow \alpha_1 \neq 0$ constant , $\alpha_2 = 0$

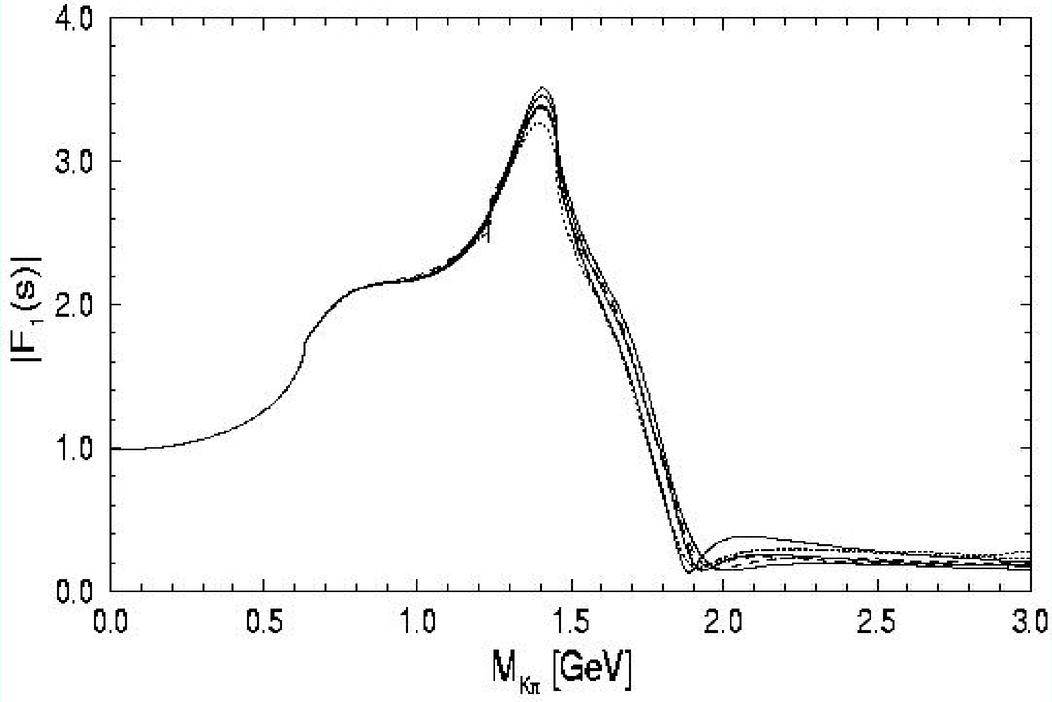
In JOP00 we presented several fits:

m=2	m=1
Fits: 6.10K2, 6.10K3 6.11K2, 6.11K3	Fits: 6.10K1, 6.11K1

All of them are equally acceptable, tiny differences above 2 GeV



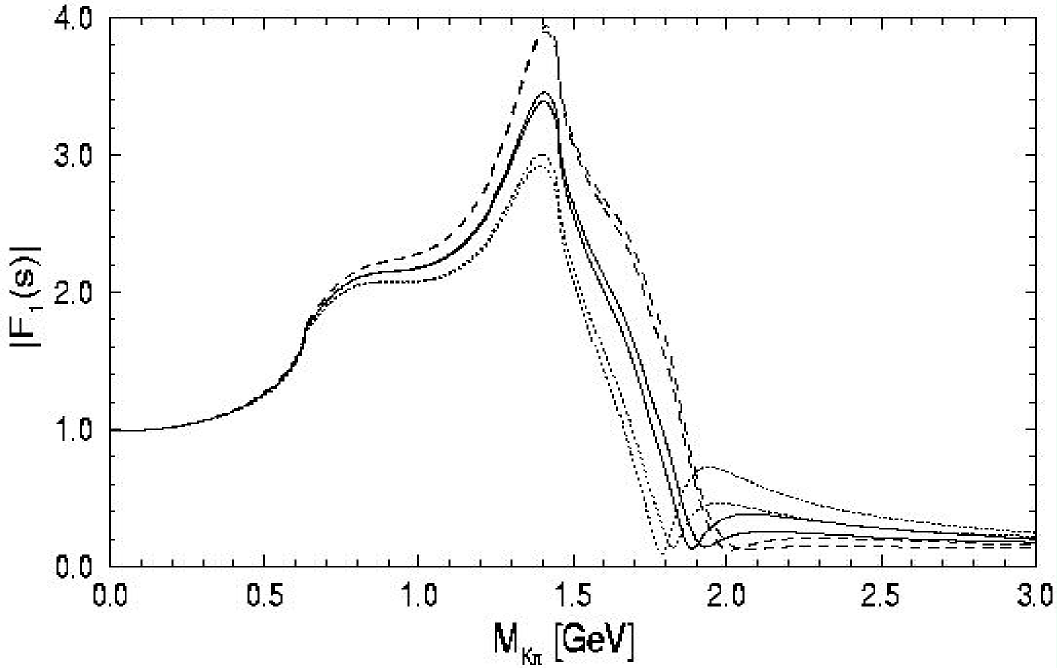
Data: Sol. A of Aston et al. NPB296('88)493 (We showed in JOP00 that Sol. B of Aston et al. is unphysical)



m=1 and m=2 fits

$F_0(0)=0.981$

$F_K/F_\pi=1.22$ JOP02



m=2 fits

$F_0(0)=0.981$

$F_K/F_\pi=1.22\pm0.01$ JOP02

3. $O(p^6)$ Counterterms (C^r_i), $F_+(0)$, F_K / F_π

$F_+(t)$, $F_0(t)$ calculated recently at order p^6 in CHPT .

Post,Silcher EPJC25('02)427, Bijnens, Talavera NPB669('03)341 (BT03)

Some differences were found in both calculations for $t \neq 0$.

We follow BT03 that employs the $O(p^6)$ CHPT Lagrangian of Bijnens,Colangelo,Ecker NPB508('97)263

$$F_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C^r_{12} \overset{?}{+} C^r_{34} \right)\Delta^2_{K\pi}$$

$\Delta(0)$ → L_i^r $O(p^4)$; **No C^r_i**

L_i^r from Amoros,Bijnens,Talavera NPB602(01)87

$$\Delta(0) = -0.0080 \pm 0.0057\,[loops] + 0.0028\,[L^r_i]$$

$$= -\mathbf{0.0080 \pm 0.0064}$$

Momentum dependence

$$\Delta_{K\pi} = M_K^2 - M_\pi^2 \qquad \Sigma_{K\pi} = M_K^2 + M_\pi^2$$

$$F_0(t) = F_+(0) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} t + \frac{8}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\Sigma_{K\pi} t - \frac{8}{F_\pi^4} C_{12}^r t^2 + \bar{\Delta}(t)$$

$$\bar{\Delta}(t) = \bar{\Delta}_1(t) t + \bar{\Delta}_2(t) t^2 + \bar{\Delta}_3(t) t^3 + O(t^4)$$

$$= -0.259(9) t + 0.840(31) t^2 + 1.291(170) t^3$$

Dependence on L^r_i not on C^r_i

C_{12}^r and C_{34}^r — Can be fixed from the t-dependence, ***slope and curvature***, of the scalar form factor $F_0(t) = F_{K\pi}(t)$

$$C_{12}^r = \left[2\bar{\Delta}_2 - F_0''(0)\right]\frac{F_\pi^4}{16}$$

$$C_{12}^r + C_{34}^r = \left[F_0'(0) - \bar{\Delta}_1 - \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} - \frac{8\Sigma_{K\pi}}{F_\pi^4} C_{12}^r\right]\frac{F_\pi^4}{8\Sigma_{K\pi}}$$

m=1: Fits 6.10K1, 6.11K1

Only one vanishing independent solution $G_1(s)=\{G_{11}(s), G_{12}(s)\}$

Normalized: $G_{11}(0)=1 \Rightarrow F_0(s) = F_0(0)\, G_{11}(s)$,
$F_0(s)' = F_0(0)\, G_{11}(s)'$, $F_0(s)'' = F_0(0)\, G_{11}(s)''$

$$C^r_{12} = [\, 2\bar{\Delta}_2 - F_0(0) G_{11}(0)'' \,] \frac{F_\pi^{\ 4}}{16}$$

$$C^r_{12} + C^r_{34} = \left[F_0(0) G_{11}(0)' - \bar{\Delta}_1 - \Sigma_{K\pi}\bar{\Delta}_2 - \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} + \Sigma_{K\pi} F_0(0) G_{11}(0)''\ 2 \right] \frac{F_\pi^4}{8\Sigma_{K\pi}}$$

But $F_0(0)$ appears as well as a necessary input... CONSISTENCY

$$F_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^{\ 4}} \left(C^r_{12} + C^r_{34} \right) \Delta^2_{K\pi}$$

We solve for $F_+(0)=F_0(0)$

$$F_+(0) = \left[1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \left(\bar{\Delta}_1 + \Sigma_{K\pi} \bar{\Delta}_2 + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} \right) \right] \times \left(1 + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \left(G_{11}(0)' + G_{11}(0)'' \frac{\Sigma_{K\pi}}{2} \right) \right)^{-1}$$

$$G_{11}(0)' = 0.803 \text{ GeV}^{-2}; \; G_{11}(0)'' = 1.661 \text{ GeV}^{-4}$$

$F_0(0) = 0.979 \pm 0.009$

$C^r_{12} = (2.8 \pm 2.9)\ 10^{-7}$

$C^r_{12} + C^r_{34} = (2.4 \pm 1.6)\ 10^{-6}$

$\mu = M_\rho$, in agreement with SMD estimates after running to $\mu \approx 1.2$-1.4 GeV

$$F_+(0)|_C = -\frac{8}{F_\pi^4} \left(C^r_{12} + C^r_{34} \right) \Delta_{K\pi}^2$$

$F_+(0)|_C = -\,0.013 \pm 0.009$ **The presently used estimated value is the one from Leutwyler&Ross ZPC25('84)91** **$F_+(0)|_C^{LR} = -\,0.016 \pm 0.008$**

Model depedent calculation based on the overlap between the Kaon and Pion quark wave functions in the light cone. Only analytic piece, no loops.

$F_0(0) = 0.979 \pm 0.009$

$F_0(0)=0.976 \pm 0.010$ BT03 employing $F_+(0)|_C^{LR}$

To be improved...

$$F_+(0) = \left[1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(\bar{\Delta}_1 + \Sigma_{K\pi}\bar{\Delta}_2 + \frac{\boxed{F_K/F_\pi} - 1}{\Delta_{K\pi}} \right) \right] \times \left(1 + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(G_{11}(0)' + G_{11}(0)'' \frac{\Sigma_{K\pi}}{2} \right) \right)^{-1}$$

We take $F_K/F_\pi = 1.22 \pm 0.01$ from Leutwyler, Roos ZPC25('84)91 (PDG source) $\Rightarrow F_0(0) = 0.979 \pm 0.009$.

LR84 already employed $F_+(0)$ ($O(p^4)$ CHPT + $F_+(0)|_C^{LR}$)

Our results for $F_+(0)|_C$ is biassed by the previous $F_+(0)|_C^{LR}$

$F_K/F_\pi = 1.20 \Rightarrow F_+(0)|_C = -0.028 \pm 0.009$ $F_0(0) = 0.965 \pm 0.009$

$F_K/F_\pi = 1.24 \Rightarrow F_+(0)|_C = +0.001 \pm 0.009$ $F_0(0) = 0.993 \pm 0.009$

One needs an independent determination of F_K/F_π

As pointed out in BT03, the loops corrections cancel partially the negative contribution from loops of $O(p^4)$. Thus:

$F_+(0)|_C \Rightarrow F_+(0)$ tends to be larger $\Rightarrow$ $|V_{us}|$ smaller

Unitarity of CKM then worses?

Depends on the taken F_K/F_π :

For $F_K/F_\pi \approx 1.19$ the unitarity problem disappears

Within GCHPT this points was also stressed in Fuchs,Knecht,Stern PRD62('00)033003 in a $O(p^4)$ GCHPT calculation

m=2: Fits 6.10K2, 6.10K3, 6.11K2, 6.11K3

Two vanishing independent solutions $G_1(s)=\{G_{11}(s), G_{12}(s)\}$,
$G_2(s)=\{G_{21}(s), G_{22}(s)\}$

Normalized: $G_{11}(0) = 1 \quad G_{21}(0) = 0$
$G_{21}(\Delta_{K\pi}) = 0 \quad G_{21}(\Delta_{K\pi}) = 0$

$$\Rightarrow F_0(s) = \boxed{F_0(0)}\, G_{11}(s) + \boxed{F_0(\Delta_{K\pi})}\, G_{21}(s$$

Dashen-Weinstein relation: $F_0(\Delta_{K\pi}) = F_K/F_\pi + \Delta_{CT}$

$\Delta_{CT} = O(m_s m_u, m_s m_d) = -3\ 10^{-3}$ in Gasser,Leutwyler NPB250('85)517

The extra dependence in this case on $F_0(\Delta_{K\pi})$ introduces more uncertainty (4.5×errorbar) in our results for $F_0(0)|_C$ following the scheme developed for **m=1**

Consistency Check: We take $F_0(0) = 0.979 \pm 0.009$ $(\mathbf{0.976 \pm 0.010})_{\mathbf{BT03}}$

$$F_K/F_\pi = 1.22 \pm 0.01$$

Input(m=1)/Output(m=2) compatible ?

Input: $F_0(0) = 0.979 \pm 0.009$ $F_K/F_\pi = 1.22 \pm 0.01$

Output: $F_0(0) = 0.977 \pm 0.010$

$C^r_{12} = (1 \pm 5)\ 10^{-7}$

$C^r_{12} + C^r_{34} = (2.7 \pm 1.4)\ 10^{-6}$

$F_+(0)|_C = -\ 0.015 \pm 0.008$ $F_+(0)|_C^{LR} = -\ 0.016 \pm 0.008$

$F_+(0)|_C = -\ 0.013 \pm 0.009$ $m=1$

Resulting Slope:

$F_0(0)' = 0.804 \pm 0.048\ \text{GeV}^{-2}$

$<r^2_{K\pi}> = (0.192 \pm 0.012)\ \text{fm}^2$; $\lambda_0 = 0.016 \pm 0.001$

CHPT $O(p^4)$: $< r^2_{K\pi} > = 0.20 \pm 0.05\ fm^2$; $\lambda_0 = 0.017 \pm 0.004$

PDG02: $K^+_{\mu 3}$ $\lambda_0 = 0.030 \pm 0.005$

$K^0_{\mu 3}$ $\lambda_0 = 0.013 \pm 0.005$

4. Conclusions

Present value for $F_K/F_\pi = 1.22 \pm 0.01$

$C^r_{12} = (2.8 \pm 2.9)\ 10^{-7}$

$C^r_{12} + C^r_{34} = (2.4 \pm 1.6)\ 10^{-6}$

$F_+(0)|_C = -\ 0.014 \pm 0.009$ $\quad (F_+(0)|_C^{LR} = -\ 0.016 \pm 0.008)$

$$F_+(0) = 1 - 0.0227[p^4] + 0.0113\ [p^6\ \text{loops}] + 0.0033[p^6\text{-}L_i] - 0.014\ [p^6 - C_i]$$
$$\pm\ 0.0057[\text{loops}] \pm 0.0028[L_i] \pm 0.0028[C_i]$$

$$= 0.978 \pm 0.009$$

It **seems** that Unitarity problem of CKM gets worse

$F_K/F_\pi \approx 1.19$ (without considering E865 K^+_{e3})

1. m=1

$$F_+(0) = \left[1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \left(\bar{\Delta}_1 + \Sigma_{K\pi}\bar{\Delta}_2 + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} \right) \right] \times \left(1 + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \left(G_{11}(0)' + G_{11}(0)'' \frac{\Sigma_{K\pi}}{2} \right) \right)^{-1}$$

m=2

$$F_+(0) = \left[1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}^2} \left(\bar{\Delta}_1 + \Sigma_{K\pi}\bar{\Delta}_2 + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} - \frac{F_K}{F_\pi} \left(G_{21}(0)' + G_{21}(0)'' \frac{\Sigma_{K\pi}}{2} \right) \right) \right] \times \left(1 + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \left(G_{11}(0)' + G_{11}(0)'' \frac{\Sigma_{K\pi}}{2} \right) \right)^{-1}$$

By equating both results we can solve for F_K/F_π in terms of $\Delta(0)$, $\bar{\Delta}_i(0)$, $G_{i1}(0)'$ and $G_{i1}(0)''$

$$F_K/F_\pi = 1.19 \pm 0.06$$

$$F_+(0) = 0.96 \pm 0.05$$

The large errors are mainly due to

$\Delta(0) = -0.0080 \pm 0.0064$ BT03

For a 5% of error for $\Delta(0)$, similar size as those of $\bar{\Delta}_i(0)$, $0.06 \rightarrow 0.02$
$0.05 \rightarrow 0.016$

Within this method one needs to improve definitely the knowledge of $\Delta(0)$ ..... We must face then the problem of diposing of more precise L_i^r

Had we followed the strategy of m=1 fits (solving for $F_0(0)$) then:

$F_0(0) = 0.96 \pm 0.04$

$F_+(0)|_C = -0.03 \pm 0.03$

Ynduráin PLB578 ('04)99: $< r^2_{K\pi} > = 0.31 \pm 0.06 \ fm^2$

2σ higher, incompatible

Y04 claims that this is due to κ(800) resonance...

We also have a κ(800) in our amplitudes. BWs, employed by Y04, are not appropriate in the scalar sector to parameterize phase shifts