Bonn, 11th September 2003

The Chiral Unitary Approach

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- **Introduction: The Chiral Unitary Approach**
- **Non perturbative effects in Chiral EFT**
- **Formalism**
- **Meson-Meson**
- **Meson-Baryon (E. Oset´s talk WG2)**
- **Nucleon-Nucleon**

The Chiral Unitary Approach

1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies).
   - S-wave meson-meson scattering: I=0 σ(500) $\pi\pi$

   Not at low energies, I=0 $f_0(980)$, I=1 $a_0(980)$, I=1/2 κ(700). Related by SU(3) symmetry.
   - S-wave Strangeness S=−1 meson-baryon interactions. I=0 Λ(1405) $\pi\Sigma, \bar{K}N$,...

   and other SU(3) related resonances.
   - $^1S_0$, $^3S_1$ S-wave Nucleon-Nucleon interactions.
2. Then one can study:
   - Strongly interacting coupled channels.
   - Large unitarity loops.
   - Resonances.
3. This allows as well to use the Chiral Lagrangians for higher energies. (BONUS)
4. The same scheme can be applied to productions mechanisms. Some examples:
   - Photoproduction: $\gamma\gamma \to \pi^0\pi^0, \pi^+\pi^-, \pi^0\eta, K^+K^-, K^0\bar{K}^0$
   - Decays:

   $\phi(1020) \to \gamma K^0\bar{K}^0, \gamma\pi^0\pi^0, \gamma\pi^0\eta \quad J/\Psi \to \phi(\omega)\pi\pi, K\bar{K}$

5. Connection with perturbative QCD, $\alpha_S\,(4\ \mathrm{GeV}^2)/\pi \approx 0.1$. (OPE). E.g. providing phenomenological spectral functions for QCD Sum Rules (going definitively beyond the sometimes insufficient hadronic scheme of narrow resonance+resonance dominance).
6. It is based in performing a chiral expansion, not of the amplitude itself as in Chiral Perturbation Theory (CHPT), or alike EFT´s (HBCHPT, KSW, CHPT+Resonances), but of a kernel with a softer expansion.

Chiral Perturbation Theory

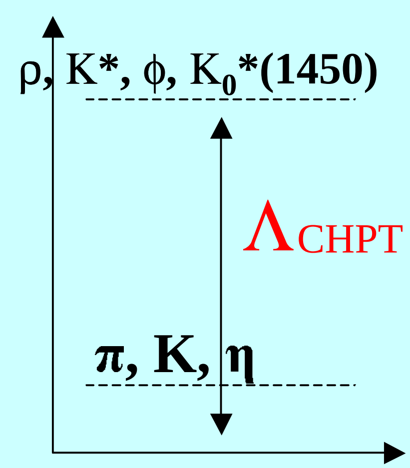
Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian → **Hilbert Space Physical States**

u, d, s massless quarks $SU(3)_L \otimes SU(3)_R$ —**Spontaneous Chiral Symmetry Breaking**→ $SU(3)_V$

Goldstone Theorem

Octet of massles pseudoscalars
π, K, η
Energy gap



$m_q \neq 0$. Explicit breaking of Chiral Symmetry

Non-zero masses
$m_P^2 \propto m_q$

Perturbative expansion in powers of the external four-momenta of the pseudo-Goldstone bosons over Λ^2_{CHPT}

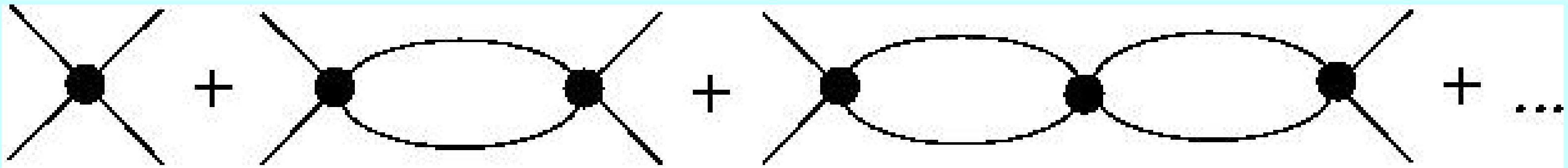
$$L = L_2 + L_4 + \dots \qquad \frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda^2_{CHPT}}\right) \qquad \Lambda_{CHPT} \approx 1\ \text{GeV} \approx M_\rho \approx 4\pi f_\pi \approx 1\ \text{GeV}$$

- When massive fields are present (Nucleons, Deltas, etc) the heavy masses (e.g. Nucleon mass) are removed and the expansion typically involves the quark masses and the small three-momenta involved at low kinetic energies.
- New scales or numerical enhancements can appear that makes definitively smaller the overall scale Λ_{CHPT}, e.g:
  - Scalar Sector (S-waves) of meson-meson interactions with I=0,1,1/2 the unitarity loops are enhanced by numerical factors.

P-WAVE S-WAVE

$$\frac{s-4m_\pi^2}{6f^2} \rightarrow \frac{s-m_\pi^2}{f^2}$$

Enhancement by a factor 6^L

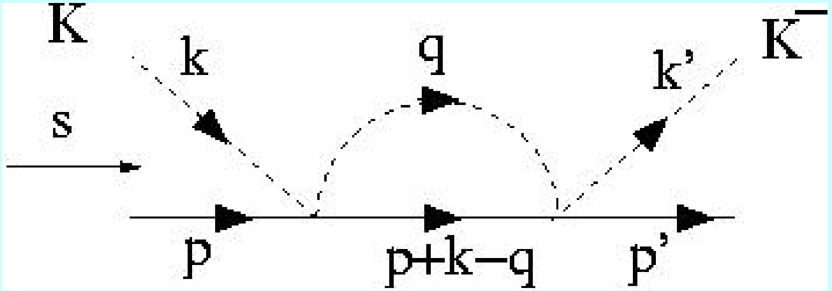


  - Presence of large masses compared with the typical momenta, e.g: Kaon masses in driving the appearance of the $\Lambda(1405)$ close to tresholed in $\bar{K}N$.

    This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass.

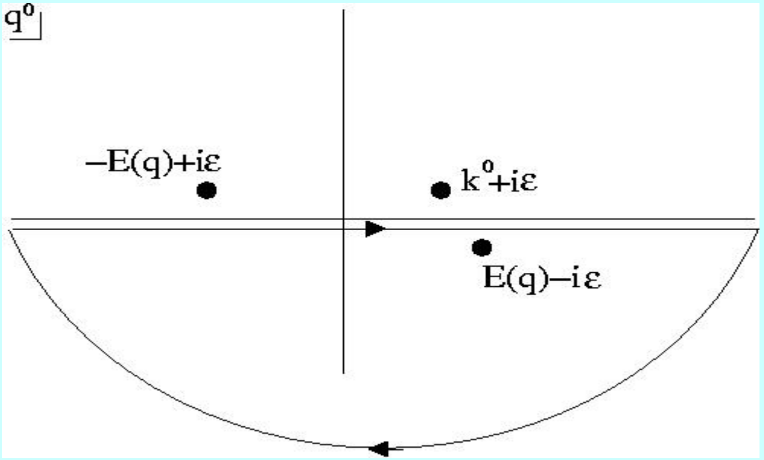
Let us keep track of the kaon mass, $M_K \approx 500$ MeV

We follow similar arguments to those of S. Weinberg in NPB363,3 (´91) respect to NN scattering (nucleon mass).



Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\varepsilon)(q^0 + E(q) - i\varepsilon)(q^0 - E(q) + i\varepsilon)}$$



$$\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \cong \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

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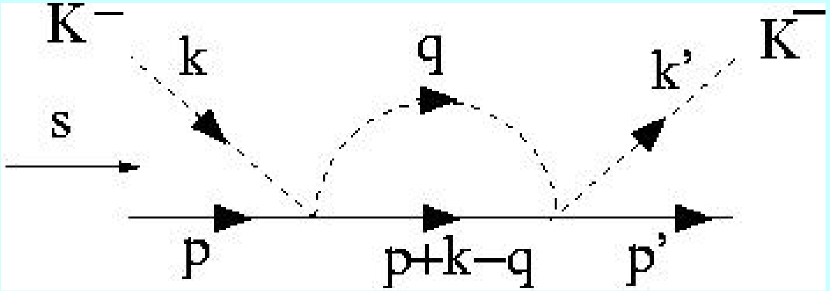
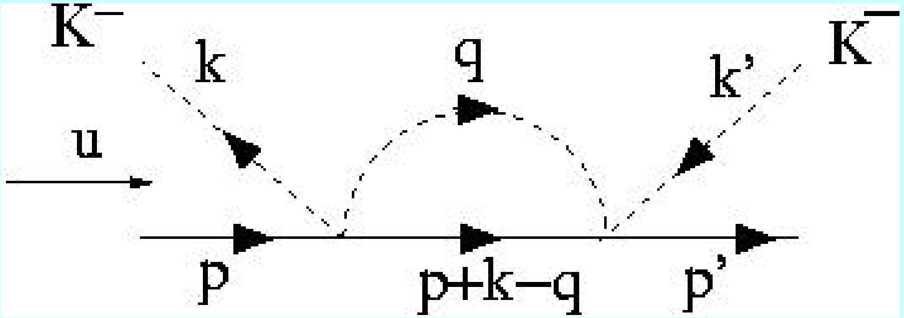


Unitarity Diagram

$$\frac{1}{k^0 - \mathrm{E}(q)}\frac{1}{2\mathrm{E}(q)} \cong \frac{2M_K}{k^2 - q^2}\frac{1}{2M_K}$$



Let us take now the crossed diagram

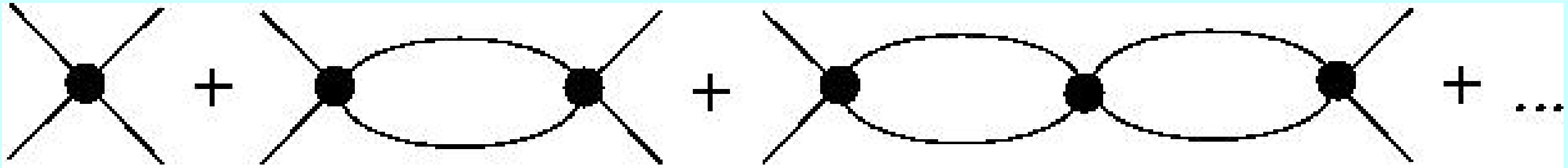
$$k \to -k$$

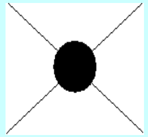
$$\frac{1}{k^0 + \mathrm{E}(q)}\frac{1}{2\mathrm{E}(q)} \cong \frac{1}{4M_K^2}$$

Unitarity&Crossed loop diagram: $\frac{4M_K^2}{k^2 - q^2}$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

In all these examples the unitarity cut (sum over the unitarity bubbles) is enhanced.



UCHPT makes an expansion of an ``Interacting Kernel´´ 

from the appropriate EFT and then the unitarity cut is fulfilled to

all orders (non-perturbatively)

– Other important non-perturbative effects arise because of the presence of nearby resonances of non-dynamical origin with a well known influence close to threshold, e.g. the ρ(770) in P-wave ππ scattering, the Δ(1232) in πN P-waves,...

  Unitarity only dresses these resonances but it is not responsible of its generation (typical $q\bar{q}$,qqq, ... states)

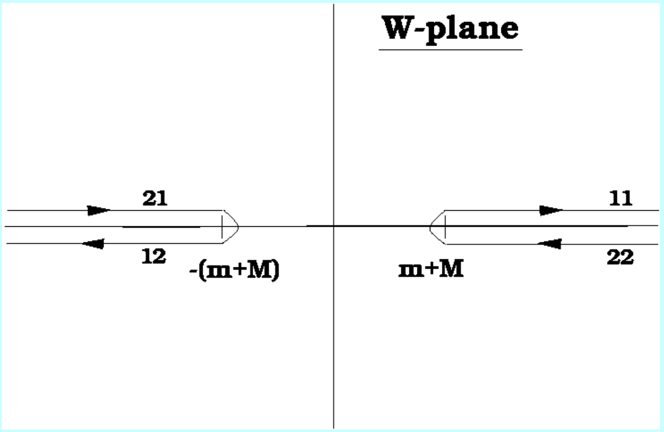
These resonances are included explicitly in the interacting kernel in a way consistent with chiral symmetry and then the right hand cut is fulfilled to all orders.

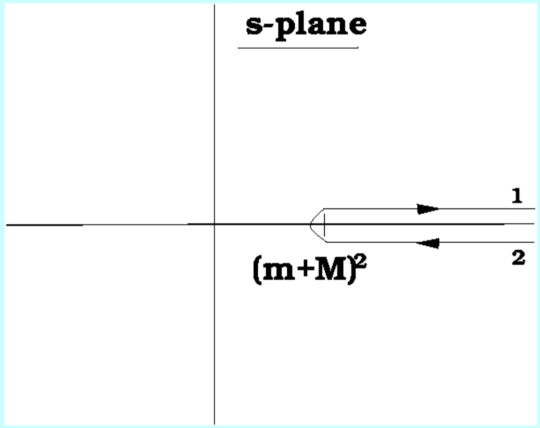
General Expression for a Partial Wave Amplitude

- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$Im T_{ij} = \sum_k T_{ik}\, \rho_k\, T^*_{kj} \quad \rightarrow \quad Im T^{-1}{}_{ij} = -\rho_i\, \delta_{ij}$$ Unitarity Cut

$W=\sqrt{s}$



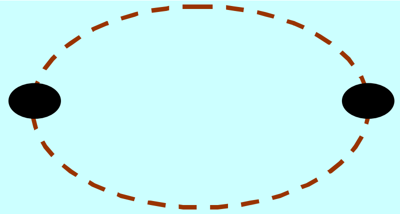


We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known)

$$T^{-1}_{ij} = R^{-1}_{ij} + \delta_{ij}\left(g(s_0)_i - \frac{s - s_0}{\pi} \int \frac{\rho(s')_i\, ds'}{(s' - s - i0^+)(s' - s_0)} \right)$$

The rest

g(s): Single unitarity bubble

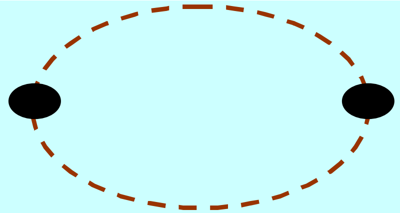
g(s)= 

$$g(s)=\frac{1}{4\pi^2}\left(a_{SL}+\sigma(s)\log\left(\frac{\sigma(s)+1}{\sigma(s)-1}\right)\right)$$

$$\sigma(s)=\frac{2q}{\sqrt{s}}$$

$$T=\left(R^{-1}+g(s)\right)^{-1}$$

1. T obeys a CHPT/alike expansion

g(s)= 

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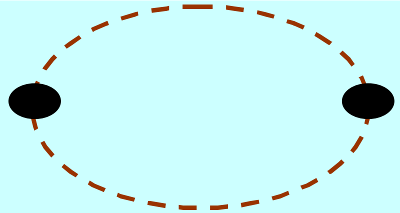
2. R is fixed by matching algebraically with the CHPT/alike
CHPT/alike+Resonances

expressions of T

In doing that, one makes use of the CHPT/alike counting for g(s)

The counting/expressions of R(s) are consequences of the known ones of g(s) and T(s) g(s)= 

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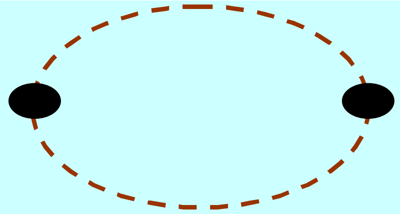
g(s)= 

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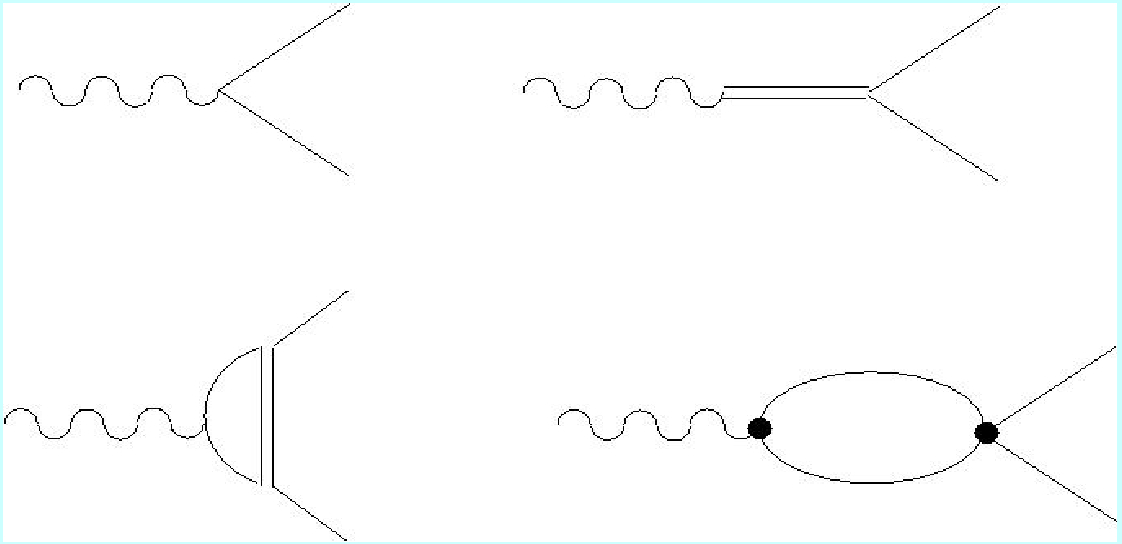
3. The CHPT/alike expansion is done to R(s). Crossed channel dynamics is included perturbatively.

4. The final expressions fulfill unitarity to all orders since R is real in the physical region (T from CHPT fulfills unitarity pertubatively as employed in the matching).

Production Processes

The re-scattering is due to the strong „final“ state interactions from some „weak“ production mechanism.



$$Im\, F_i = \sum_k F_k\, \rho_k\, T^*_{ki}$$

We first consider the case with only the right hand cut for the strong interacting amplitude, R^{-1} is then a sum of poles (CDD) and a constant. It can be easily shown then:

$$F = (I + R\, g(s))^{-1}\, \xi$$

Finally, ξ is also expanded pertubatively (in the same way as R) by the matching process with CHPT/alike expressions for F, order by order. The crossed dynamics, as well for the production mechanism, are then included pertubatively.

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Conection with the Inverse Amplitude Method (IAM)

UCHPT: $R=R_2+R_4+\ldots$, $T = (R^{-1}+g)^{-1}$

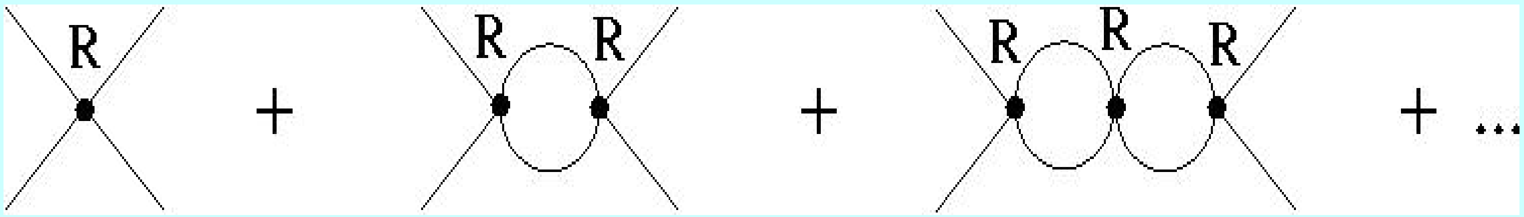
IAM: $R^{-1}=R_2^{-1} - R_4 R_2^{-2}+\ldots$, $T=(R_2^{-1} - R_4 R_2^{-2}+g)^{-1}$

In UCHPT one is just taking care of the unitarity cut (unitarity+analiticity)

In IAM one is doing extra assumptions.

Example: Consider a pure local theory, with just local terms (like NN when considering the pion field as heavy).

Within the CHPT-like counting one can calculate any given set of local vertices (with increasing number of derivatives) and from it is trivial to solve the Lippmann-Schwinger equation.



The answer for the resulting T-matrix is:

$$T = \left(R^{-1} + g(s) \right)^{-1}$$

like in UCHPT with $R=R_1+R_0+R_1+\ldots$ (EFT($\not\pi$) counting).

In IAM one performs an ad-hoc resummation at the ´tree level´ of the perturbative expansion of R (generating extra CDD poles).

LET US SEE SOME (ANALYTIC) APPLICATIONS

IMPORTANT CONSEQUENCES ON THE DYNAMICS AND SPECTROSCOPY OF THE SCALAR TWO MESON SECTOR

Meson-Meson Scalar Sector

1) **The mesonic scalar sector has the vacuum quantum numbers 0^{++}. Essencial for the study of Chiral Symmetry Breaking: Spontaneous and Explicit m_u, m_d, m_s .**

2) **In this sector the mesons really interact strongly.**
   1) **Large unitarity loops.**
   2) **Channels coupled very strongly, e.g. $\pi\pi$- $K\bar{K}$, $\pi\eta$- $K, \bar{K}$ ...**
   3) **Dynamically generated resonances, ~~Breit-Wigner formulae~~, ~~VMD~~, ...**

3) **OZI rule has large corrections.**
   1) **No ideal mixing multiplets.**
   2) **~~Simple quark model.~~**

Points 2) and 3) imply large deviations with respect to Large Nc QCD.

4) A **precise knowledge** of the scalar interactions of the lightest hadronic thresholds, $\pi\,\pi$ and so on, is often required.

- Final State Interactions (FSI) + Scalar Resonances in ε'/ε , de Rafael, Meißner, Gasser, Pich, Palante, Buras, Martinelli,... (Ecker´s talk WG1)
- Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)
- Fluctuations in order parameters of SχSB, Stern, Descartes talks in WG1.

5) **Recent and accurate** experimental data have establisehd the existence of the σ, κ (E791) and further constrains to the present models (CLOE).

6) **Lattice calculations** indicate that the lightest scalars are composed by four quarks (the size is not yet determined, $\bar{q}^2\,q^2$ bag or meson-meson resonances)

Alford, Jaffe, NPB578(00)367; hep-lat/0306037.

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- **Final State Interactions (FSI) + Scalar Resonances in ε'/ε**
- **Quark Masses (Scalar sum rules, Cabbibo suppressed Tau decays.)**
- **Fluctuations in order parameters of SχSB.**

Let us apply the chiral unitary approach

- LEADING ORDER:

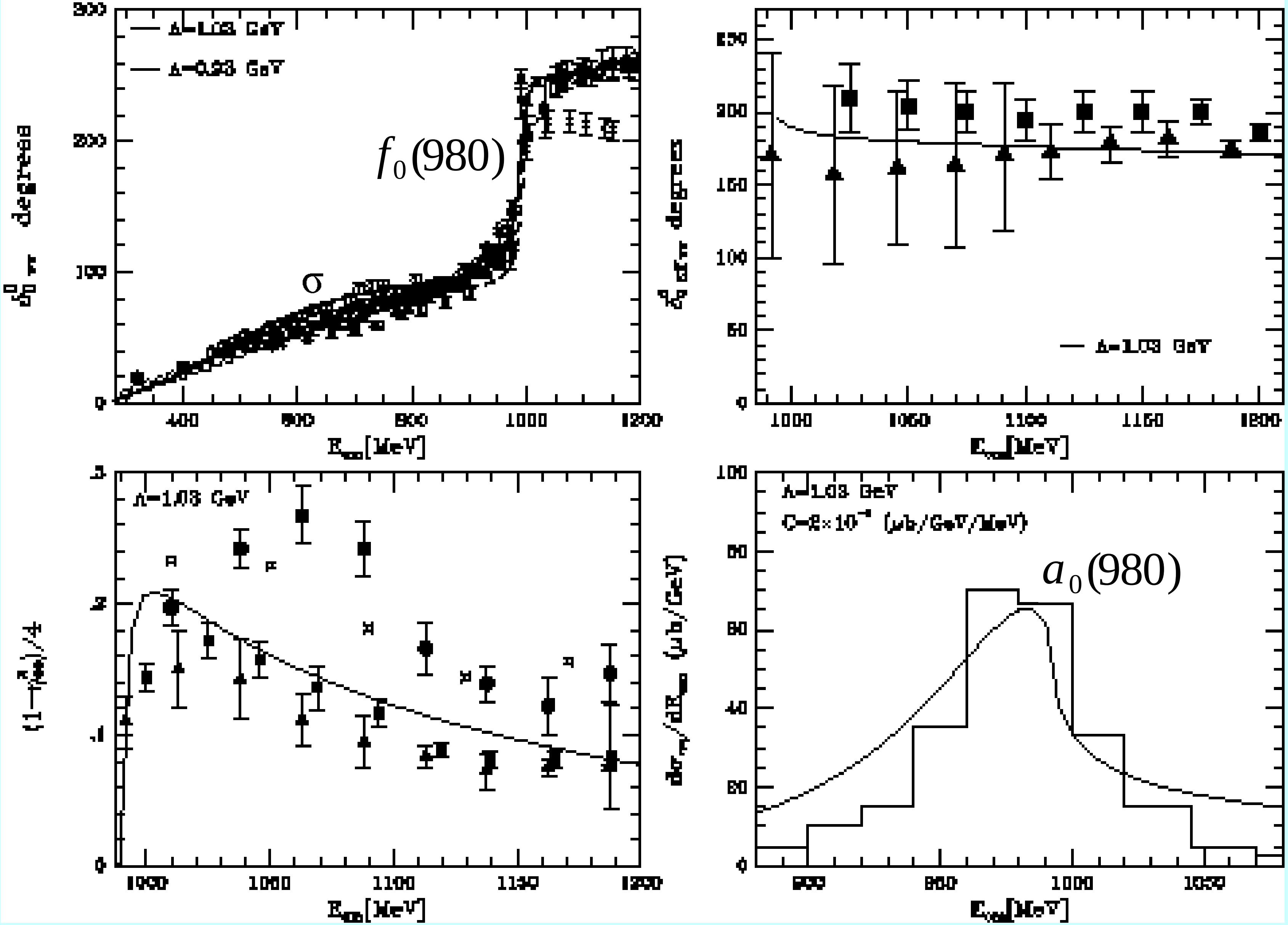
$$T = \left(R^{-1} + g(s) \right)^{-1}$$

g is order 1 in CHPT

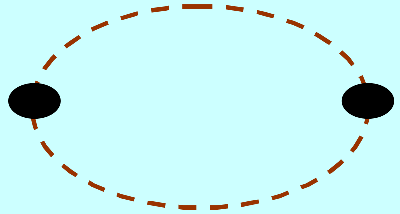
$$T = T_2 = R_2 - R_2 g R_2 + ..\qquad R = R_2 = T_2$$

Oset, J.A.O., NPA620,438(97)

$a_{SL} \cong -1$ only free parameter, equivalently a three-momentum cut-off $\Lambda \cong 1$ GeV



Λ=1.03 GeV
Λ=0.93 GeV
f0(980)
σ
E_ππ[MeV]
δ_0^0 ππ degrees
δ_0^0 KK̄ degrees
E_KK̄[MeV]
(1−η_00^2)/4
dσ_πη/dE_πη (μb/GeV)
C=2×10^-4 (μb/GeV/MeV)
a0(980)
E_πη[MeV]

g(s)= 

$$g(s)=\frac{1}{4\pi^2}\left(a_{SL} + log\frac{m^2}{\mu^2} + \sigma log\frac{\sigma+1}{\sigma-1}\right)$$

One can cut the three-momentum in the loop with a cut-off, and then:

$a_{SL}(\mu) = (-2\log 2Q/\mu)/16\pi^2 + O(m^2/Q^2)$; m=meson mass, Q=cut-off

- For $\mu \approx Q \approx M_\rho \approx \Lambda_{CHPT}$ one has $a_{SL} \approx -2\log(2) = -1.3$ → This is value the one that results from the fit !!

Notice the logarithmic dependence on Q

- Then a_{SL} is order 1 in large N_c and this makes that the „dynamically generated resonances“ disappear in large N_c, while the preexisting resonances do not.

- One can also include explicit resonances, but then the value of a_{SL} remains the same and the preexisting resonances are pushed to higher energies.

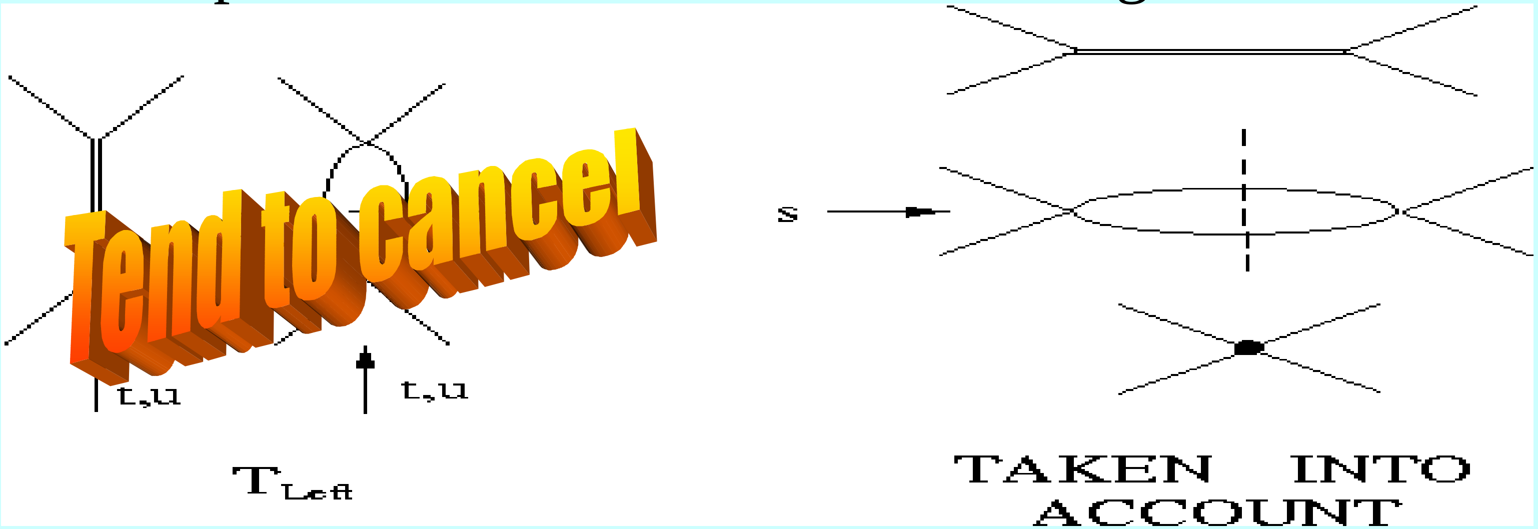
- Coming soon.

In Oset,J.A.O PRD60,074023(99) we studied the I=0,1,1/2 S-waves.

The input included leading order CHPT plus Resonances:

1. Cancellation between the crossed channel loops and crossed channel resonance exchanges. (Large Nc violation).

The loops were taken from next to leading CHPT for the estimation.



2. Dynamically generated renances ($M \sim N_c^{1/2}$). σ, $f_0(980)$, $a_0(980)$, κ(700)

The tree level or preexisting resonances move higher in energy (octet around 1.4 GeV). Pole positions were very stable under the improvement of the kernel R (convergence).

3. In the SU(3) limit we have a degenerate octet plus a singlet of dynamically generated resonances

In Jamin,Pich,J.A.O NPB587(02)331 we studied the I=1/2,3/2 S waves up to 2 GeV: $K\pi$, $K\eta$, $K\eta'$.

The input included nexto-to-leading order CHPT plus Resonances

The results were very stable regarding the previous study, e.g, existence of the $\kappa(700)$ pole very close to its previous position.

This analysis provides the basis to obtain the scalar form factors for $K\pi$ and $K\eta'$ (coupled channels) by solving the corresponding Muskhelishvilly-Omnès problem. Jamin,Pich,J.A.O NPB622(´02)279

They provided the phenomenological function to plug in scalar QCD sum rules to calculate a very reliable determination for the mass of the strange quark (going definitively beyond the hadronic approximation of narrow resonance approach+resonance dominance) In the MS scheme: $m_s(2\ \mathrm{GeV})=99\pm16$ MeV

CHPT ratios: $m_u(2\ \mathrm{GeV})=2.9\pm0.6$ MeV , $m_d(2\ \mathrm{GeV})=5.2\pm0.9$ MeV

Jamin,Pich,J.A.O EPJ C24(02)237.

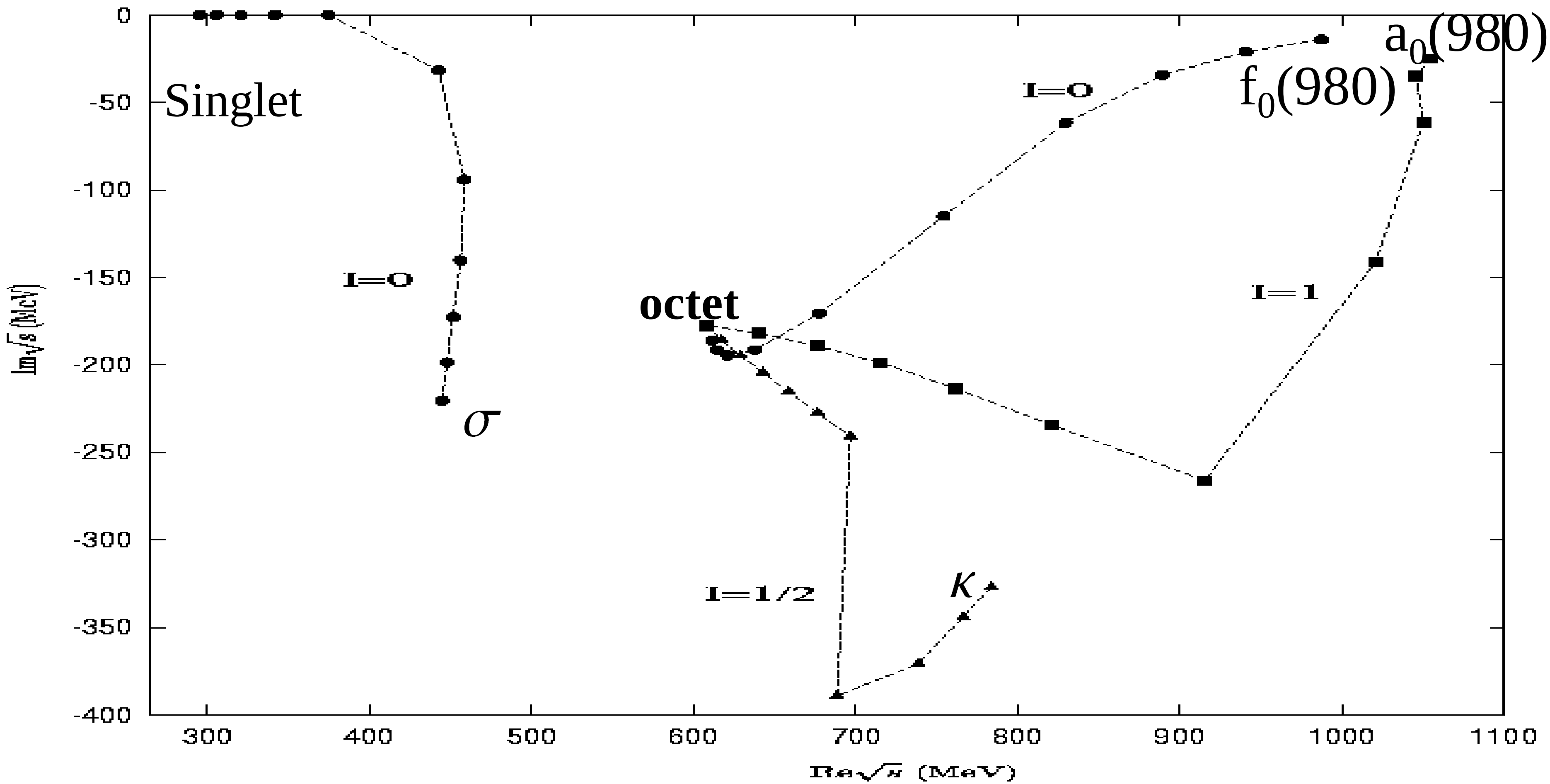
In J.A.O. hep-ph/0306031 (to be published in NPA) a SU(3) analysis of the couplings constants of the $f_0(980)$, $a_0(980)$, $\kappa(900)$, $f_0(600)$ and σ was done.

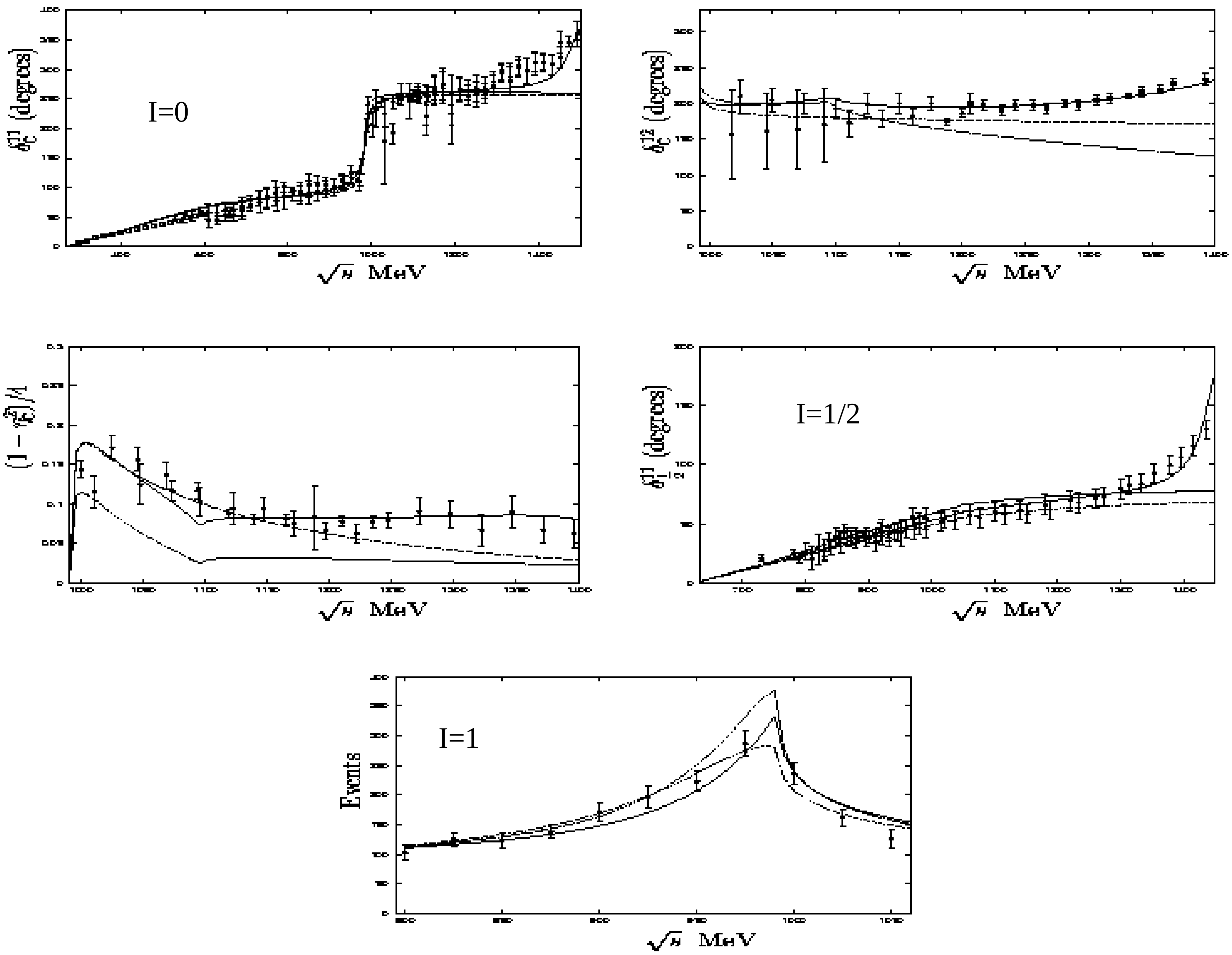
$0 \leq \lambda \leq 1$, $\lambda=0$ physical limit

$\lambda=1$ SU(3) Symmetric point SOFT EVOLUTION

$m_0=300\ MeV$

$m_\pi(\lambda)=m_\pi+\lambda(m_0-m_\pi)$; $m_K(\lambda)=m_K+\lambda(m_0-m_K)$; $m_\eta(\lambda)=m_\eta+\lambda(m_0-m_\eta)$





I=0
δ_C^{11} (degrees)
$\sqrt{s}$ MeV
δ_C^{12} (degrees)
$\sqrt{s}$ MeV
$(1-\eta_C^2)/4$
$\sqrt{s}$ MeV
I=1/2
$\delta_{\frac{1}{2}}^{11}$ (degrees)
$\sqrt{s}$ MeV
I=1
Events
$\sqrt{s}$ MeV

Solid lines: I=0 ($\pi\pi$, $K\bar{K}$, $\eta_8\eta_8$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

Singlet 1 GeV: $\tilde{c}_d$=20.9MeV ; $\tilde{c}_m$=10.6MeV; M_1=1021MeV

Octet 1.4 GeV: c_d=19.1MeV ; c_m=15±30MeV; M_8=1390MeV

Subtraction Constant: a=-0.75±0.20

Dashed lines: I=0 ($\pi\pi$, $K\bar{K}$, $\eta_8\eta_8$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

No bare Resonances

Subtraction Constant: a=-1.23

Short-Dashed lines: I=0 ($\pi\pi$, $K\bar{K}$) , I=1 ($\pi\eta_8$, $K\bar{K}$) , I=1/2 ($K\pi$, $K\eta_8$)

No bare Resonances

Several Subtraction Constants:

$$a_{\pi\pi}=-1.14;\ a_{K\bar{K}}=-1.64;\ a_{\pi\eta}=-0.5;\ a_{K\pi}=-0.75;\ a_{K\eta}=-0.75$$

Spectroscopy: Dynamically generated resonances.

σ	$0.445 - i\,0.220$	$0.443 - i\,0.213$	$0.442 - i\,214$
	$\lvert g_{\pi\pi}\rvert = 3.01$	$\lvert g_{\pi\pi}\rvert = 2.94$	$\lvert g_{\pi\pi}\rvert = 2.95$
	$\lvert g_{K\bar{K}}\rvert = 1.09$	$\lvert g_{K\bar{K}}\rvert = 1.30$	$\lvert g_{K\bar{K}}\rvert = 1.34$
	$\lvert g_{\eta_8\eta_8}\rvert = 0.09$	$\lvert g_{\eta_8\eta_8}\rvert = 0.04$	
$f_0(980)$	$0.988 - i\,0.014$	$0.983 - i\,0.007$	$0.987 - i\,0.011$
	$\lvert g_{\pi\pi}\rvert = 1.33$	$\lvert g_{\pi\pi}\rvert = 0.89$	$\lvert g_{\pi\pi}\rvert = 1.18$
	$\lvert g_{K\bar{K}}\rvert = 3.63$	$\lvert g_{K\bar{K}}\rvert = 3.59$	$\lvert g_{K\bar{K}}\rvert = 3.83$
	$\lvert g_{\eta_8\eta_8}\rvert = 2.85$	$\lvert g_{\eta_8\eta_8}\rvert = 2.61$	
$a_0(980)$	$1.055 - i\,0.025$	$1.032 - i\,0.042$	$1.030 - i\,0.086$
	$\lvert g_{\pi\eta_8}\rvert = 3.88$	$\lvert g_{\pi\eta_8}\rvert = 3.67$	$\lvert g_{\pi\eta_8}\rvert = 4.08$
	$\lvert g_{K\bar{K}}\rvert = 5.50$	$\lvert g_{K\bar{K}}\rvert = 5.39$	$\lvert g_{K\bar{K}}\rvert = 5.60$
κ	$0.784 - i\,0.327$	$0.804 - i\,0.285$	$0.774 - i\,0.338$
	$\lvert g_{K\pi}\rvert = 5.02$	$\lvert g_{K\pi}\rvert = 4.93$	$\lvert g_{K\pi}\rvert = 4.89$
	$\lvert g_{K\eta_8}\rvert = 3.10$	$\lvert g_{K\eta_8}\rvert = 2.96$	$\lvert g_{K\eta_8}\rvert = 3.00$

PLUS the values of the $\pi\pi$ AND $K\bar{K}$ scalar form factors in the $f_0(980)$ peak

Weighted Averages of the first and second SU(3) Analysis (Final results):

$$\sigma = \cos\theta\, S_1 + \sin\theta\, S_8$$

$$f_0 = -\sin\theta\, S_1 + \cos\theta\, S_8$$

$$\cos^2\theta = 0.925 \pm 0.013$$

$$\theta = 15.9^0 \pm 1.4^0$$

$$|g_8| = 8.6 \pm 0.5 \text{ GeV}$$

$$|g_1| = 3.7 \pm 0.5 \text{ GeV}$$

1. The σ is mainly the singlet state. The $f_0(980)$ is mainly the I=0 octet state. The $\kappa(700)$ the I=1/2 octet member and the $a_0(980)$ the isovector one.

2. Very similar to the mixing in the pseudoscalar nonet but inverted.

η Octet $\longrightarrow$ σ Singlet ; η' Singlet $\longrightarrow$ $f_0(980)$ Octet. (Anomaly)

Final State Interactions

- $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0, \pi\eta, K^+K^-, K^0\bar{K}^0$

  Oset, J.A.O NPA629(98)739.

- J/Ψ DECAYS

  Meissner, J.A.O NPA679(01)671.

  Chiang-bin Li, Oset, Vicente Vacas

- $\phi(1020)$ DECAYS

  J.A.O PLB426(98)7 ; NPA714(03)161

  Marco,Hirenzaki,Oset,Toki PLB470(99)20

  Palomar,Roca,Oset,Vicente Vacas, hep-ph/0306249

- η, η' DECAYS AND FSI INTERACTIONS

  STUDY OF EXOTIC RESONANCES AND D-Pseudoscalar RESONANCES

  Bass, Marco PRD65(02)057503 ; Borasoy´s talk

  Szczepaniak et al., hep-ph/0304095, hep-ph/0305060

  Oset,Peláez,Roca PRC67(03)073013

- Vector Form Factor of pions and kaons

  Palomar, Oset, J.A.O

- Scalar Form Factor of pions and kaons

  Meissner, J.A.O

  Pich, Jamin,J.A.O

- B decays

  Gardner,Meissner

  J.A.O

ETC....

S-Wave, S=-1 Meson-Baryon Scattering

Oset, Ramos, NPA635,99 (´98). **U.-G. Meißner, J.A.O**, PLB500, 263 (´01), PRD64, 014006 (´01); **A. Ramos, E. Oset, C. Bennhold**, PRL89(02)252001, PLB527(02)99; Jido,Hosaka et al**.,** PRC68(2003)018201, nucl-th/0305011, nucl-th/0305023, hep-ph/0309017,etc. **Jido, Oset, Ramos, Meißner, J.A.O, NPA725(03)181**

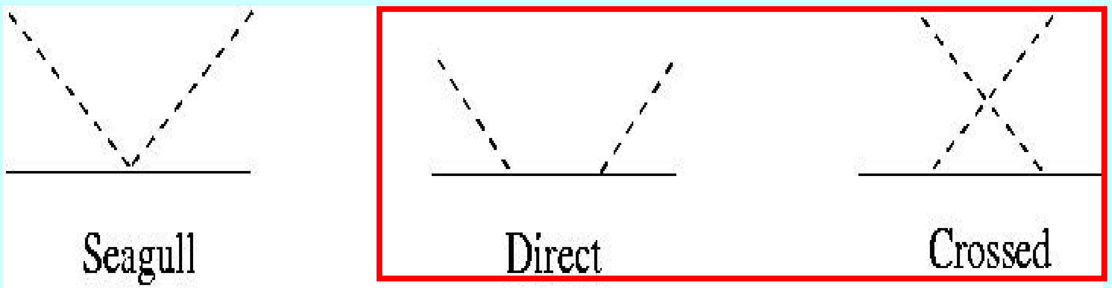
Kaiser,Weise,Siegel, NPA594(95)325

As in the scalar sector the unitarity cut is enhanced.

$$T = \left(R^{-1} + g(s) \right)^{-1} = \left(I + R . g(s) \right)^{-1} R(s)$$

$T=T_1=R_1$ LEADING ORDER: g is order p in (HB)CHPT (meson-baryon)

No bare resonances



Non-negligible for energies greater than 1.3 GeV

Many channels: $K^- p, \bar{K}^0 n, \pi^0\Sigma^0, p^+\Sigma^-, \pi^-\Sigma^+, \pi^0\Lambda, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$

Important isospin breaking effects due to cusp at thresholds, we work with the physical basis

In Meißner, J.A.O PLB500, 263 (´01), several poles were found.

1. All the poles were of dynamical origin, they disappear in Large Nc, because R.g(s) is order 1/Nc and is subleading with respect to the identity I.

$$T = (I + R.g(s))^{-1} R(s) \rightarrow R(s)$$

The subtraction constant corresponds to evalute the unitarity loop with a cut-off Λ of natural size (scale) around the mass of the ρ.

$$a_{SL} = -2 \,\mathrm{Log}\left(1 + \sqrt{1 + \frac{M_N}{\Lambda^2}}\right) \cong -2 \qquad M_N \rightarrow N_c$$

2. Two I=1 poles, one at 1.4 GeV and another one at around 1.5 GeV.

3. The presence of two resonances (poles) around the nominal mass of the Λ(1405).

These points were further studied in: Jido, Oset, Ramos, Meißner, J.A.O, nucl-th/0303062, taking into account as well another study of Oset,Ramos,Bennhold PLB527,99 (02).

SU(3) decomposition $\quad 8 \otimes 8 \rightarrow 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus 27$

Isolating the different SU(3) invariant amplitudes one observes de presence of poles for the Singlet (1), Symmetric Octet (8_S), Antisymmetric Octet (8_A).

DEGENERATE

Table 3: Pole positions and couplings to $I = 0$ physical states from the model of Ref. [3]

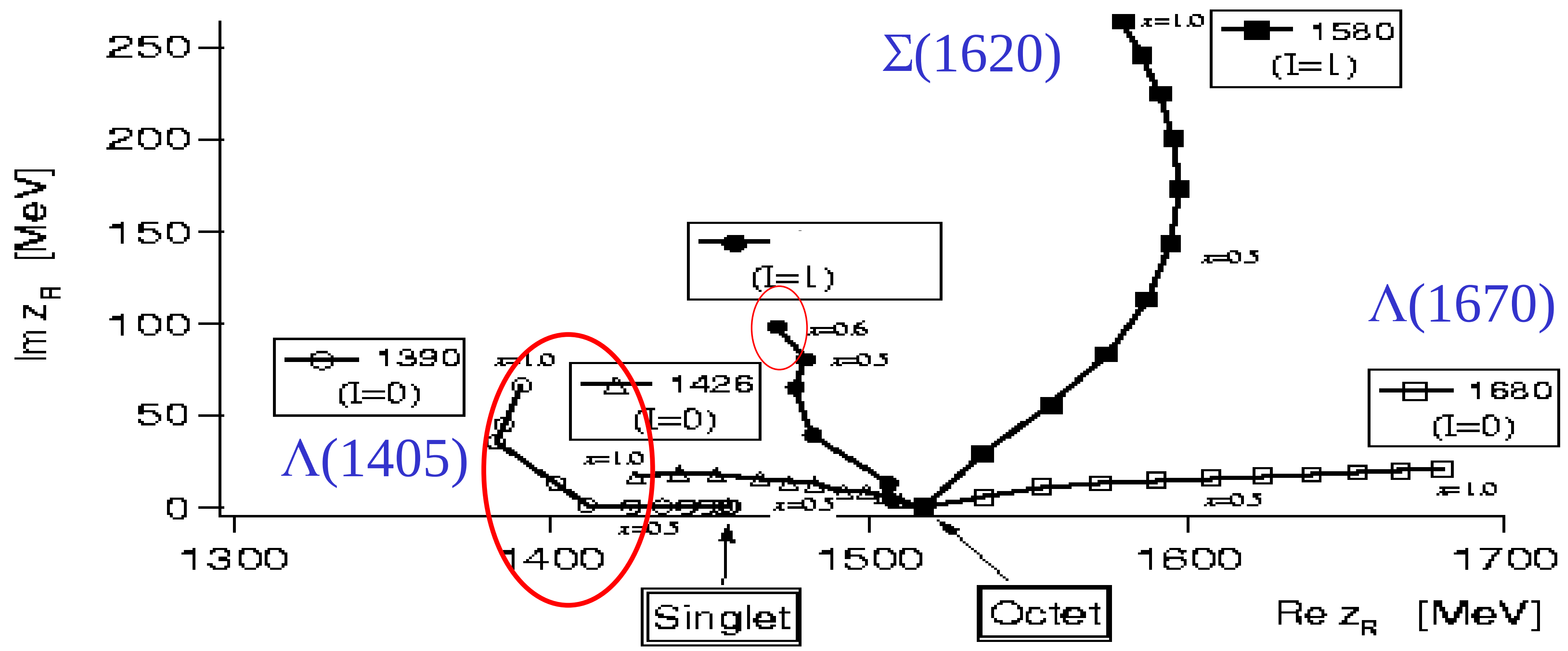
z_R $(I=0)$	$1379+27i$ g_i	$\lvert g_i \rvert$	$1434+11i$ g_i	$\lvert g_i \rvert$	$1692+14i$ g_i	$\lvert g_i \rvert$
$\pi\Sigma$	$-1.76-0.62i$	1.87	$-0.56-1.02i$	1.16	$-0.08-0.32i$	0.33
$\bar{K}N$	$0.86+0.70i$	1.11	$-1.74+0.63i$	1.85	$0.32+0.41i$	0.52
$\eta\Lambda$	$0.19+0.33i$	0.38	$-1.20+0.23i$	1.23	$-0.83-0.19i$	0.85
$K\Xi$	$-0.52-0.19i$	0.55	$-0.20-0.30i$	0.36	$3.87+0.05i$	3.87

a) b) Λ(1405) Λ(1670)

a) is more than twice wider than b)
(Quite Different Shape)

b) Couples stronger to $\bar{K}N$ than to $\pi\Sigma$ contrarily to a)

It depends to which resonance the production mechanism couples stronger that the shape will move from one to the other resonance

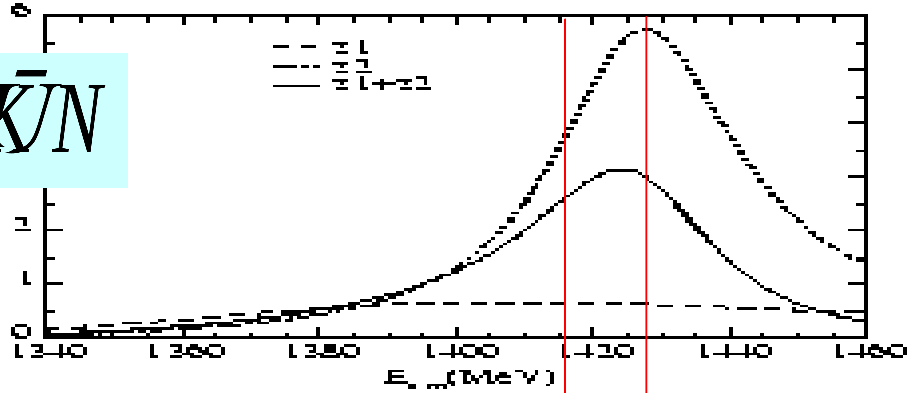


Simple parametrization of our own results with BW like expressions

$K^- p \rightarrow \gamma\, \Lambda(1405)$

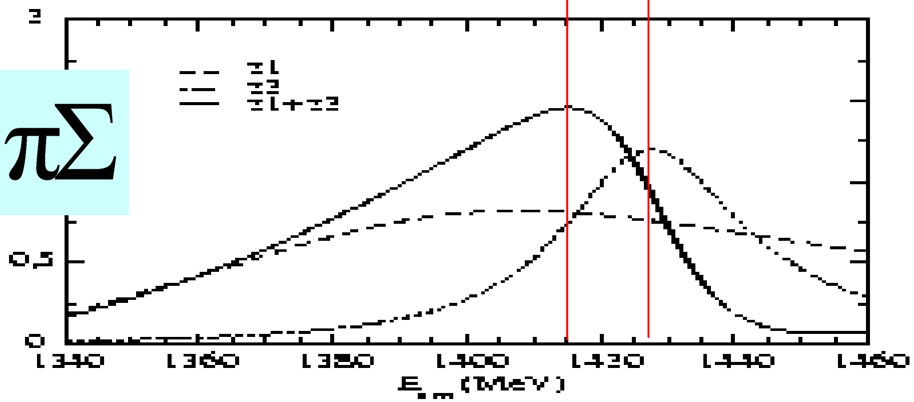
$$g^{R_1}_{\bar K N}\frac{1}{W-M_{R_1}+i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma}+g^{R_2}_{\bar K N}\frac{1}{W-M_{R_2}+i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma},$$

$\pi\Sigma \rightarrow \bar{K}N$



$$g^{R_1}_{\pi\Sigma}\frac{1}{W-M_{R_1}+i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma}+g^{R_2}_{\pi\Sigma}\frac{1}{W-M_{R_2}+i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma}.$$

$\pi\Sigma \rightarrow \pi\Sigma$



SU(3) Decomposition of the Physical Resonances

I=0

Pole (MeV)	C_1	C_{8_A}/C_1	C_{8_S}/C_1	$\|C_1\|^2$	$\|C_{8_A}\|^2$	$\|C_{8_s}\|^2$
1379+ i 27	0.96	0.15+ i 0.11	0.15- i 0.19	0.92	0.03	0.05
1434+ i 11	0.49	0.64+ i 0.77	0.71+ i 1.28	0.24	0.24	0.52
1692+ i 14	0.48	1.58+ i0.37	0.78+ i 0.16	0.23	0.63	0.14

I=1

Pole (MeV)	C_{8_A}	C_{8_S}/C_{8_A}	$\|C_{8_A}\|^2$	$\|C_{8_s}\|^2$
1401+ i 40	0.81	0.72+ i0.07	0.66	0.34
1488+ i 114	0.59	1.37- i 0.06	0.35	0.65

A more comprehensive and detailed view on meson-baryon scattering form UCHPT was given by E. Oset, in WG2

Nucleon-Nucleon Interaction

- Ideal system to apply the UCHPT
- At (very) low energies one finds already non-perturbative physics.
- Bound state (deuteron) and antibound state just below threshold (new and non-natural scale).
- Large nucleon masses that enhances the unitarity cut.

Nucleon-Nucleon Interaction

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- At (very) low energies one finds already non-perturbative physics.
- Bound state (deuteron) and antibound state just below threshold (new and non-natural scale).
- Large nucleon masses that enhances the unitarity cut.
- Weinberg scheme: The chiral counting is applied for calculating the NN potencial that then is iterated in a Lippmann-Schwinger equation.

S. Weinberg, PL B251(1990)288, NP B363(1991)3, PL B295 (1992)114 ;
C. Ordoñez, L. Ray, U. Van Kolck, PRC53(1996)2086
E. Epelbaum, W. Glöckle, U.-G. Meißner NP A671(2000)295, etc.

- Kaplan-Savage-Wise EFT: like in CHPT one works out directly the scattering amplitude folllowing a chiral like counting called the KSW counting. Problems with the convergence of the series.

D.B. Kaplan, M.J. Savage, M.B. Wise, NP A637(1998)107; NP B534(1998)329.
S. Fleming, T. Mehen, I. Stewart, NP A677(2000)313, PR C61(2000)044005, etc.

WE FIX ***R*** MATCHING WITH KSW:

Since $\boldsymbol{T} = \frac{4\pi}{M} \frac{1}{R^{-1} + g}$ is easier to fix ***R*** by matching with the inverses of the KSW amplitudes

${}^1S_0$,

$$g = -\nu - ip$$

O(p^o) O(p^1)

$$R = R_0 + R_1 + R_2 + R_3 + O(p^4)$$

O(p^o) O(p) O(p^2) O(p^3)

$$\frac{1}{A_{KSW}} = \frac{1}{A_{-1}} - \frac{A_0}{A_{-1}^2} + \frac{A_0^2 - A_1 A_{-1}}{A_{-1}^3} + O(p^4)$$

O(p) O(p^2) O(p^3)

$$\frac{1}{R} + g = \left(\frac{1}{R_0} - \nu\right) + \left(-\frac{R_1}{R_0^2} + ip\right) - \left(\frac{R_0 R_2 - R_1^2}{R_0^3}\right) + \left(\frac{R_1^3 - 2R_0 R_1 R_2 + R_0^2 R_3}{R_0^4}\right) + O(p^4)$$

$$R_0 = \frac{1}{\nu} \qquad R_1 = \frac{1}{a_s \nu^2} \qquad R_2 = \frac{\frac{2\nu p^2}{\pi} + \gamma^2 M + \frac{\nu\, 4\pi\, A_0}{A_{-1}^2}}{M \nu^3}$$

$$\frac{R_1}{R_0} = \frac{1}{a_s \nu} \longrightarrow 0$$

etc

PHENOMENOLOGY

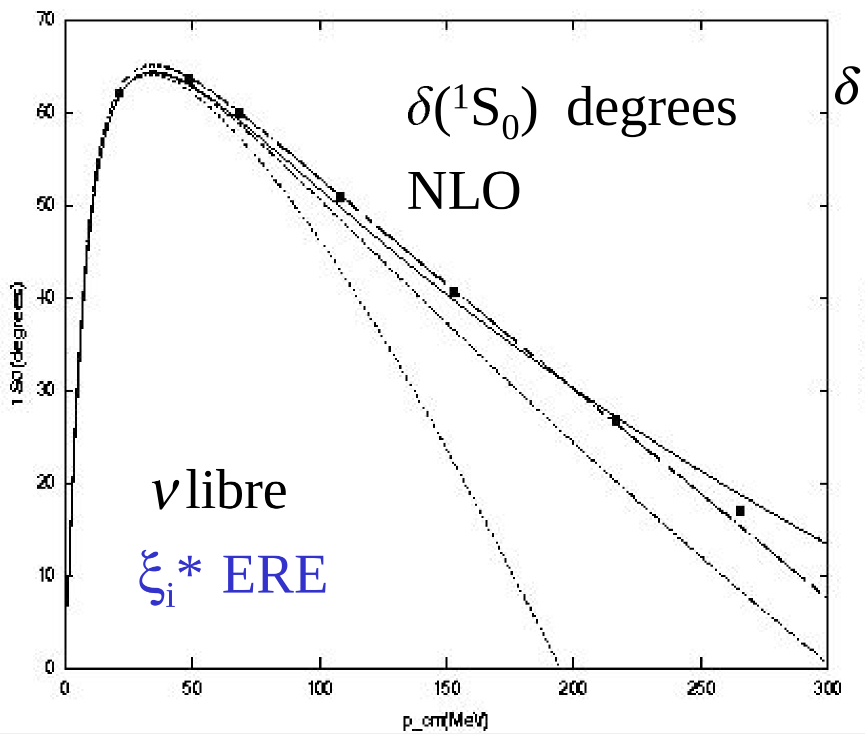
$^1S_0$

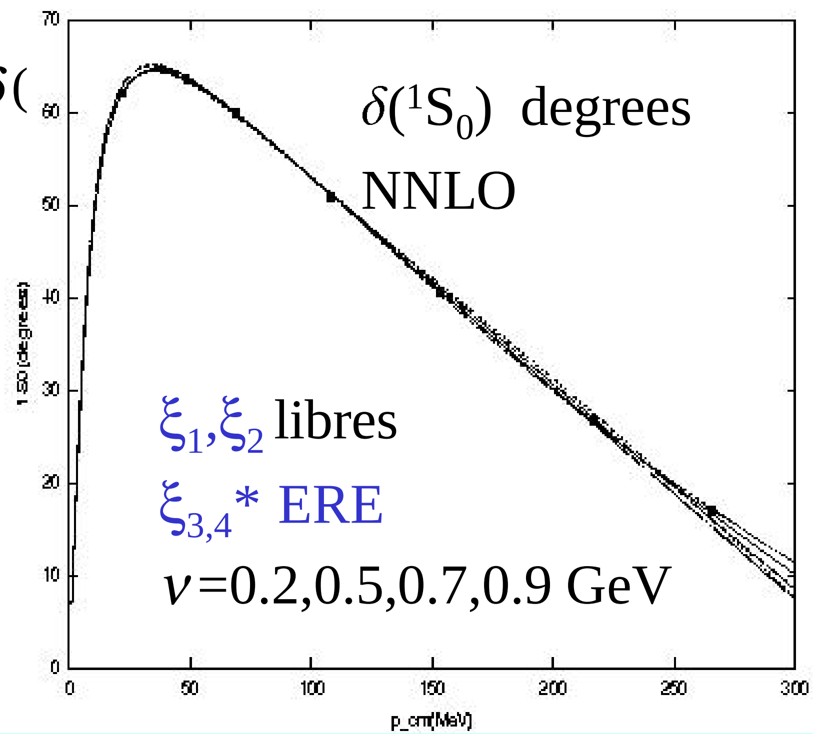
Counterterms: NLO: ξ_1, ξ_2 ; NNLO: ξ_3 , ξ_4

At every order in the expansion of ***R***, two counterterms are fixed in terms of a_s , r_0 and ν:

NLO: $\xi_1(a_s, r_0, \nu)$, $\xi_2(a_s, r_0, \nu)$

NNLO: $\xi_3(\xi_1, \xi_2, a_s, r_0, \nu)$, $\xi_4(\xi_1, \xi_2, a_s, r_0, \nu)$



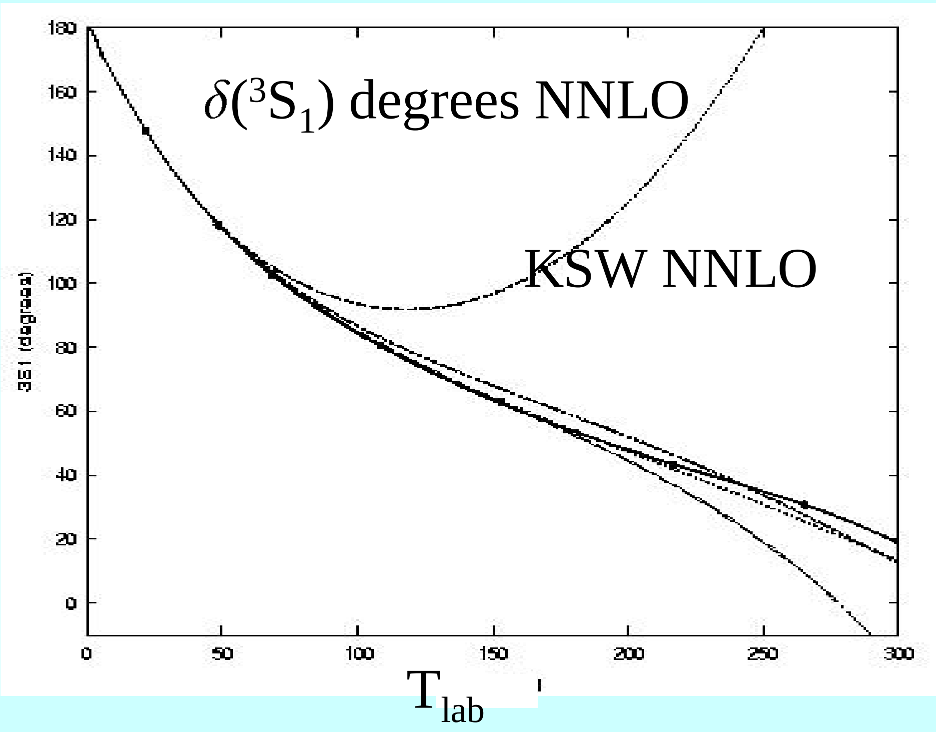


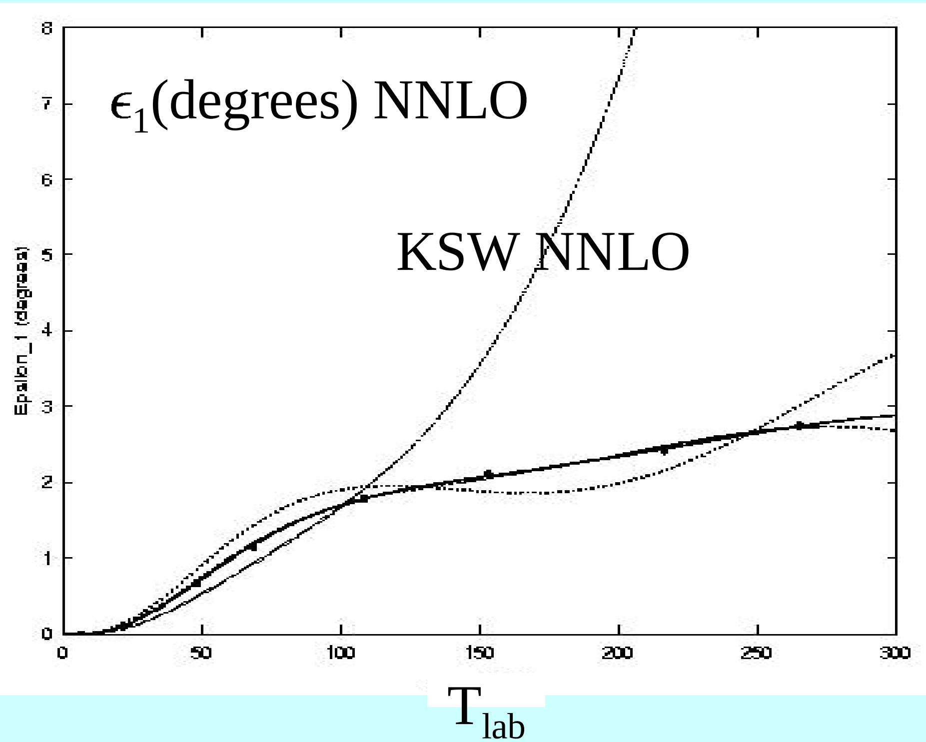
$^{3}S_{1}$ Counterterms: NLO: ξ_1, ξ_2 ; NNLO: ξ_3 , ξ_4 , ξ_5 , ξ_6

At every order in the expansion of ***R*,** two counterterms are fixed in terms of a_s , r_0 **and** ν:

NLO: $\xi_1(a_s, r_0, \nu)$, $\xi_2(a_s, r_0, \nu)$

NNLO: $\xi_3\ (\xi_1,\xi_2,\ a_s, r_0, \nu)$, $\xi_4\ (\xi_1,\xi_2,\ a_s, r_0, \nu)$





ν=500 MeV , γ=0.37 fm^{-1} , ξ_5 = 0.44, ξ_6 = 0.58

Summary

- Chiral Unitary Approach:
  - Systematic and general scheme to treat self-strongly interacting $\bar{K}N$ channels (Meson-Meson Scattering, Meson-Baryon Scattering and Nucleon-Nucleon scattering), through the chiral (or other appropriate EFT) expansion of an interaction kernel R.
  - Based on Analyticity and Unitarity.
  - The same scheme is amenable to correct from FSI Production Processes.
  - It treats both resonant (preexisting/dynamically generated) and background contributions.
  - It can also be extended to higher energies to fit data in terms of Chiral Lagrangias and, e.g., to provide phenomenological spectral function for QCD sum rules.

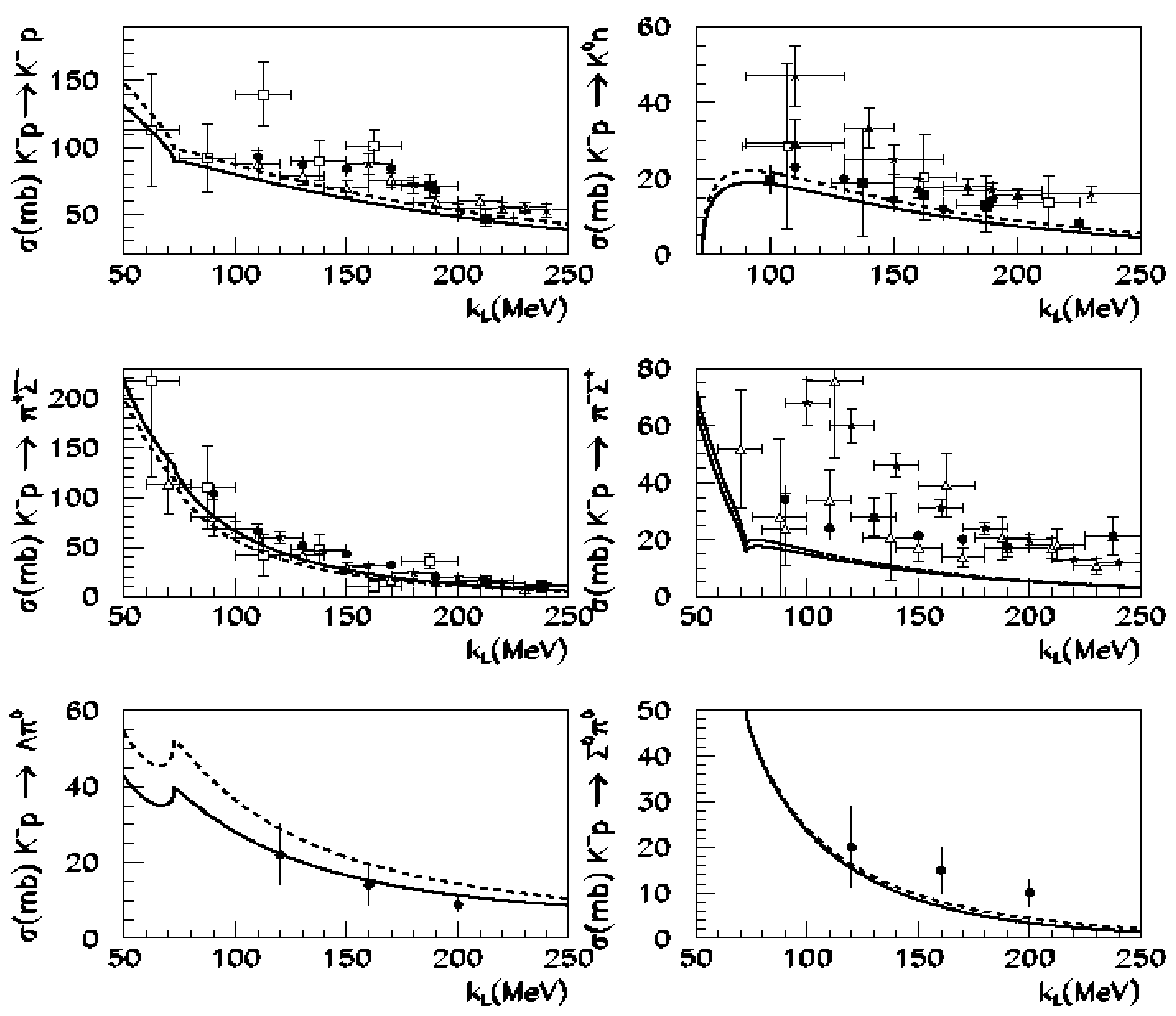
- Free Parameters:
  - a_{SL} subtraction constant.
  - M_0 Mass of the lightest baryon octet in the chiral limit.
  - f, weak pseudoscalar decay constant in the SU(3) chiral limit $(m_u = m_d = m_s = 0)$

  Natural Values (Set II):

  - M_0 =1.15 GeV, from the average of the masses in the baryon octet.
  - f=86.4 MeV, known value of f in the SU(2) chiral limit.
  - a=-2, the subtraction constant is fixed by comparing g(s) with that calculated with a cut-off around 700 MeV, Oset, Ramos, NPA635,99 (´98).

  Fitted Values (Set I):

  - M_0 =1.29 GeV
  - f=74 MeV
  - a=-2.23



σ(mb) K⁻p → K⁻p
σ(mb) K⁻p → K⁰n
σ(mb) K⁻p → π⁺Σ⁻
σ(mb) K⁻p → π⁻Σ⁺
σ(mb) K⁻p → Λπ⁰
σ(mb) K⁻p → Σ⁰π⁰
k_L(MeV)

$\pi\Sigma$ Mass Distribution

$$K^- p \rightarrow \Sigma^+ \pi^- \pi^+$$

Typically one takes: $$\frac{dN_{\pi^-\Sigma^+}}{dE} = C |T_{\pi\Sigma \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$$

As if the process were elastic

E.g: Dalitz, Deloff, JPG 17,289 (´91); Müller,Holinde,Speth NPA513,557(´90), Kaiser, Siegel, Weise NPB594,325 (´95); Oset, Ramos NPA635, 99 (´89)

But the $\bar{K}N$ threshold is only 100 MeV above the $\pi\Sigma$ one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. The presciption is ambiguos, why not?

$$\frac{dN_{\pi^-\Sigma^+}}{dE} = C |T_{\bar{K}N \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$$

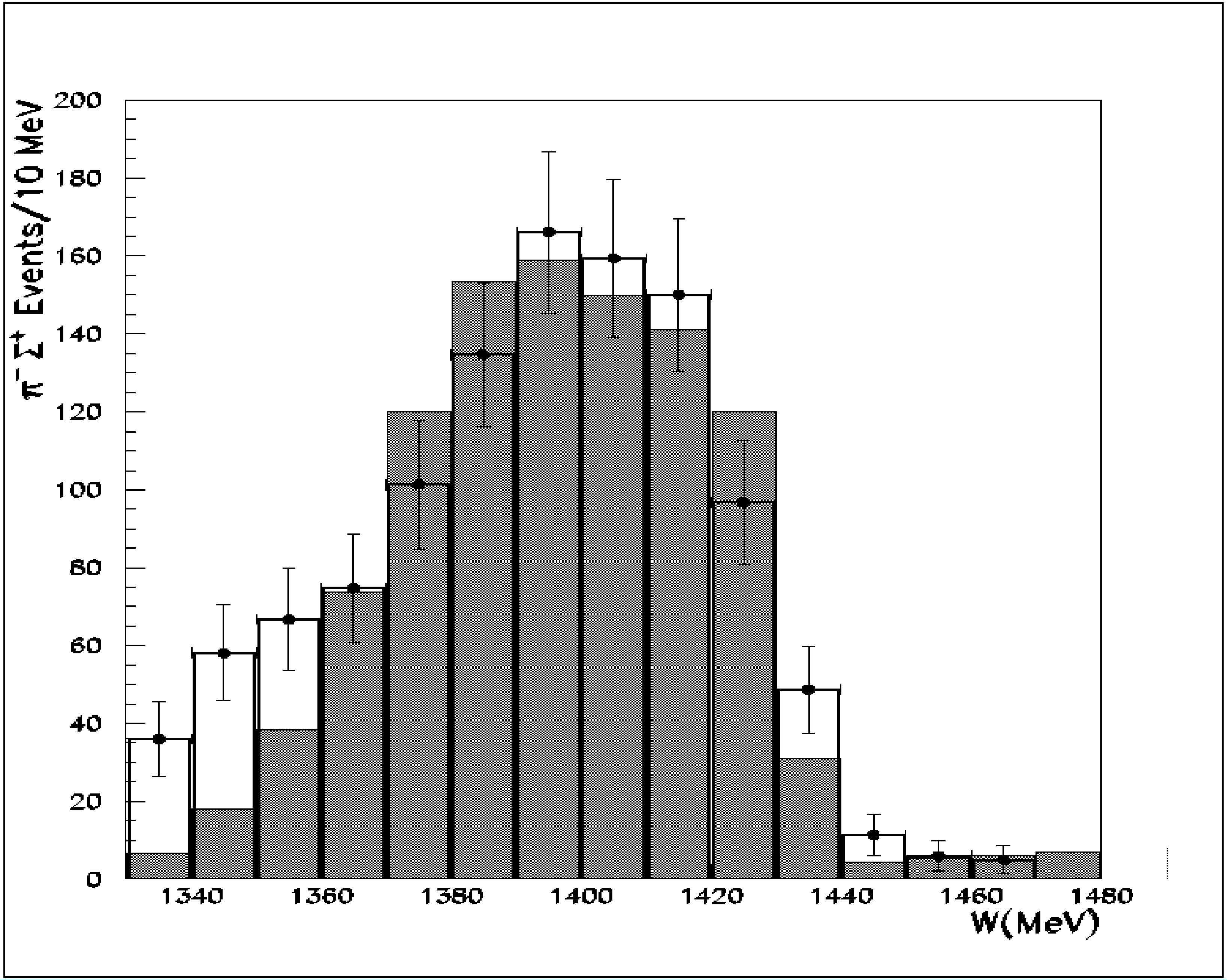
We follow the Production Process scheme previously shown:

$$F = (I + R\,g)^{-1} \xi$$

$$\xi^T = (r_1, r_1, r_2, r_2, r_2, 0,0,0,0,0)$$ I=0 Source

$$\frac{r_1}{r_2} = 1.42$$

r_1=0 (common approach)



$\pi\Sigma$ Mass Distribution

	Our Results
$\gamma=\dfrac{\Gamma(K^- p \to \pi^+\Sigma^-)}{\Gamma(K^- p \to \pi^-\Sigma^+)}=2.36\pm0.04$	2.33
$R_c=\dfrac{\Gamma(K^- p \to Charged)}{\Gamma(K^- p \to All)}=0.664\pm0.011$	0.645
$R_n=\dfrac{\Gamma(K^- p \to \pi^0\Lambda)}{\Gamma(K^- p \to Neutral)}=0.189\pm0.015$	0.227

- Scattering Lengths: $a_0=-0.58 + i1.19 \quad fm$

  $a_0=-0.53 + i0.95 \quad fm$ Isospin Limit

Data:

Kaonic Hydrogen: $a_0=(-0.78\pm0.15\pm0.03) + i(0.49\pm0.25\pm0.12) fm$

Isospin Scattering Lengths: $a_0=(-0.68\pm0.10) + i(0.64\pm0.10) fm$