S-wave meson scattering up to 2 GeV and its spectroscopy

[M. Albaladejo, J.A. Oller arXiv:hep-ph/0801.4929]

M. Albaladejo J.A. Oller C. Piqueras

Universidad de Murcia, Spain

SCADRON70 (Lisbon): February 11-16, 2008

Introduction

Objective and tools:

- Problem: Scalar mesons identification. How many? Where? Nature?
- Very broad resonances, strongly coupled channels open up in the nearby of resonances, which have very different natures: dynamically generated, $q\overline{q}$, glueballs...
- Our objective: study of strongly interacting channels with quantum numbers $I = 0, I = 1/2\ J^{PC} = 0^{++}$ for $\sqrt{s} \leq 2$ GeV.
- We use Chiral Lagrangians, implementing Unitarity (UCHPT) in a standard way (N/D-type equations).

Lagrangians

Our lagrangian is:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{S_8} + \mathcal{L}_{S_1}$$

- In $SU(3)$ UCHPT, we have eight Goldstone bosons: π, K, η.
- Large N_c limit implies η' becomes the ninth Goldstone boson: $SU(3) \rightarrow U(3)$.
  [Herrera-Siklody et al, NP, B**497**, 345 (1997)], [Herrera-Siklody et al, PL, B**419**, 326 (1998)]

$$\begin{aligned}
\mathcal{L}_2 &= \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle + \frac{f^2}{4}\langle \chi^\dagger U + \chi U^\dagger\rangle - \underbrace{\frac{1}{2}M_1^2\eta_1^2}_{U_A(1)} \\
U(\phi) &= \exp\left(\mathrm{i}\sqrt{2}\Phi/f\right) \quad \Phi = \sum_{i=0}^{8}\frac{\lambda_i}{\sqrt{2}}\phi_i \quad \lambda_0 = \sqrt{\frac{2}{3}}\mathbf{I}_3 \\
D_\mu U &= \partial_\mu U - \mathrm{i} r_\mu U + \mathrm{i} U l_\mu = \partial_\mu U - \mathrm{i} g\left[v_\mu, U\right]
\end{aligned}$$

- In our case, $r_\mu = l_\mu = g v_\mu$: vectorial resonances nonet (Massive Yang-Mills fields)
- Mixing: $\eta_1, \eta_8 \rightarrow \eta, \eta'$. Mixing angle is $\theta \approx -20^\circ$.
- $\chi = 2B_0\mathcal{M}$, with $\mathcal{M}$ quark mass matrix

Lagrangians (II)

- Vector field v_μ is given by:

$$v_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}$$

- Assuming ideal mixing between ω_8, ω_1:

$$\frac{1}{\sqrt{2}}\omega = \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \qquad \phi = -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1$$

Derivative piece of $\mathcal{L}_2$ has interactions of: $\Phi\Phi$, $V\Phi\Phi$ y $VV\Phi\Phi$:

- $\mathcal{L}_2^{\Phi\Phi} = \frac{f^2}{4}\langle \partial_\mu U \partial_\mu U^\dagger \rangle$
- $\mathcal{L}_2^{VV\Phi\Phi} = g^2 \left\langle \Phi^2 v^\mu v_\mu - v_\mu \Phi v^\mu \Phi \right\rangle$
- $\mathcal{L}_2^{V\Phi\Phi} = -\frac{igf^2}{4}\langle \partial_\mu U [v^\mu, U^\dagger] + [v^\mu, U] \partial_\mu U^\dagger \rangle$
- g is determined through decay width $\rho \to \pi\pi$, from $\mathcal{L}_2^{V\Phi\Phi}$, being $g = 4.23$

Lagrangians (III)

- We introduce explicit resonances from RChPT [Ecker et al., NP, B**321**, 311 (1999)].
- Scalar resonances $J^{PC} = 0^{++}$:

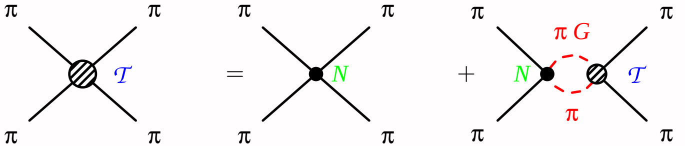
$$\begin{aligned}
\mathcal{L}_{S_8} &= c_d \left\langle S_8 u_\mu u^\mu \right\rangle + c_m \left\langle S_8 \chi_+ \right\rangle \\
\mathcal{L}_{S_1} &= \tilde{c}_d S_1 \left\langle u_\mu u^\mu \right\rangle + \tilde{c}_m S_1 \left\langle \chi_+ \right\rangle \\
\chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u, \\
U(x) = u(x)^2 & \qquad u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger
\end{aligned}$$

- $S_1^{(i)}$ singlet and $S_8^{(i)}$ octet scalar resonance, with M, c_d and c_m fitted to data.

$$S_8 = \begin{pmatrix} \frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & a_0^+ & K_0^{*+} \\ a_0^- & -\frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & K_0^{*0} \\ K_0^{*-} & \overline{K}_0^{*0} & -\frac{2}{\sqrt{6}} f_8 \end{pmatrix}.$$

- Channels to be considered:
  - $I = 0$: $\pi\pi$, $K\overline{K}$, $\eta\eta$, $\sigma\sigma$, $\eta\eta'$, $\rho\rho$, $\omega\omega$, $\eta\eta'$, $\omega\phi$, $\phi\phi$, $K^*\overline{K}^*$, $a_1(1260)\pi$, $\pi^\star\pi$
  - $I = 1/2$, $I = 3/2$: $K\pi$, $K\eta$ and $K\eta'$ [Jamin, Oller, Pich, NP, B**622**, 279 (2002)], [Jamin, Oller, Pich, NP, B**587**, 331 (2000)]

$\sigma\sigma$ channel amplitudes. Pion rescattering

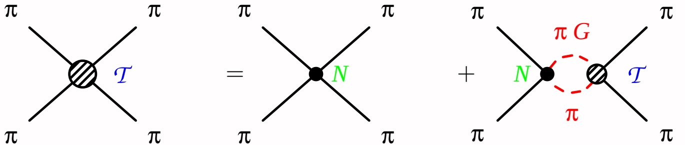


We want to obtain $\sigma\sigma$ amplitudes starting from our lagrangian. σ *is* S-wave $\pi\pi$ interaction, $|\sigma\rangle = |\pi\pi\rangle_0$, [Oller, Oset, NP, B**620**, 438 (1997)]

- Pion rescattering, given by factor $D^{-1}(s) = (1+t_2 G(s))^{-1}$, with:
  - $t_2 = \frac{s-m_\pi^2/2}{f_\pi^2}$ basic $\pi\pi \to \pi\pi$ amplitude.
  - $(4\pi)^2 G(s) = \alpha + \log\frac{m_\pi^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}$, two pion loop.
- To isolate transition amplitude $N_{i\to\sigma\sigma}$:

$$\lim_{s_i\to s_\sigma}\frac{T_{i\to(\pi\pi)_0(\pi\pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i\to\sigma\sigma}g^2_{\sigma\pi\pi}}{(s_1-s_\sigma)(s_2-s_\sigma)}$$

σσ channel amplitudes. Pion rescattering



We want to obtain σσ amplitudes starting from our lagrangian. σ *is* S-wave ππ interaction, $|\sigma\rangle = |\pi\pi\rangle_0$, [Oller, Oset, NP, B**620**, 438 (1997)]

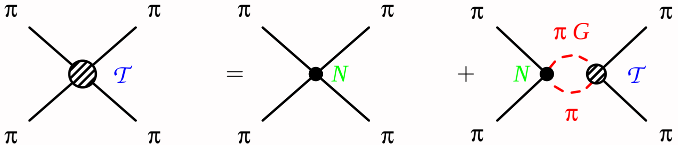
- To isolate transition amplitude $N_{i\to\sigma\sigma}$:

$$\lim_{s_i\to s_\sigma} \frac{T_{i\to(\pi\pi)_0(\pi\pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i\to\sigma\sigma}g^2_{\sigma\pi\pi}}{(s_1-s_\sigma)(s_2-s_\sigma)}$$

- Now as (σ pole) $D_{II}(s)^{-1} = (1+t_2G(s))^{-1} \approx \frac{\alpha_0}{s-s_\sigma} + \cdots$, then:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$

$\sigma\sigma$ channel amplitudes. Pion rescattering



We want to obtain $\sigma\sigma$ amplitudes starting from our lagrangian. σ *is* S-wave $\pi\pi$ interaction, $|\sigma\rangle = |\pi\pi\rangle_0$, [Oller, Oset, NP, B**620**, 438 (1997)]

- Now as (σ pole) $D_{II}(s)^{-1} = (1+t_2 G(s))^{-1} \approx \frac{\alpha_0}{s-s_\sigma} + \cdots$, then:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$

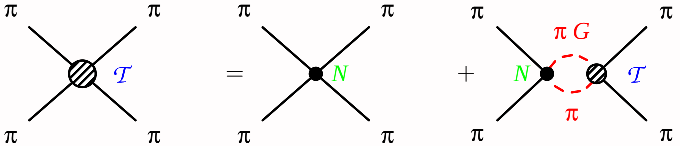
- To calculate $(\alpha_0/g_{\sigma\pi\pi})^2$, consider $\pi\pi$ elastic scattering,

$$V = \frac{t_2(s)}{1+t_2(s)G(s)} \approx -\frac{g^2_{\sigma\pi\pi}}{s-s_\sigma} + \cdots$$

So we can write, using that (σ pole) $g_{II}(s_\sigma) = -1/t_2(s_\sigma)$:

$$\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 = \frac{f^2}{1-G'_{II}(s_\sigma)f^2 t_2(s_\sigma)^2} \approx 1.1 f^2$$

σσ channel amplitudes. Pion rescattering



We want to obtain σσ amplitudes starting from our lagrangian. σ *is* S-wave ππ interaction, $|\sigma\rangle = |\pi\pi\rangle_0$, [Oller, Oset, NP, B**620**, 438 (1997)]

- To calculate $(\alpha_0/g_{\sigma\pi\pi})^2$, consider ππ elastic scattering,

$$V = \frac{t_2(s)}{1+t_2(s)G(s)} \approx -\frac{g^2_{\sigma\pi\pi}}{s-s_\sigma} + \cdots$$

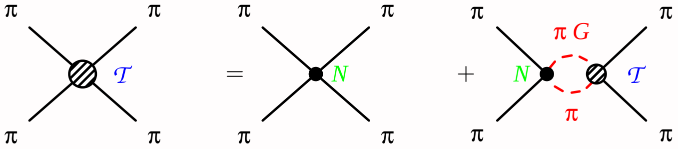
So we can write, using that (σ pole) $g_{II}(s_\sigma) = -1/t_2(s_\sigma)$:

$$\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 = \frac{f^2}{1-G'_{II}(s_\sigma)f^2 t_2(s_\sigma)^2} \approx 1.1f^2$$

- In conclusion, we follow a novel method to calculate amplitudes involving σσ, through:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$

Coupled channels Unitarization



Coupled channel partial wave

$$T = (I + N(s)g(s))^{-1} N(s)$$

- $N(s)$: N_{ij} amplitudes matrix, obtained from Lagrangian, $i,j = 1, \ldots 13$
- $g(s)$: Unitarization loops (diagonal) matrix (including all intermediate states).

$$g_i(s) = g_i(s_0) - \frac{s - s_0}{8\pi^2} \int_{s_{\text{th,i}}}^{\infty} \mathrm{d}s' \, \frac{p_i(s')/\sqrt{s'}}{(s' - s_0)(s' - s + i\varepsilon)}$$

We fold this $g(i)$ for the case of ρρ and σσ, following a mass distribution, due to large ρ and σ widths: $\Gamma_\rho \simeq 150$ MeV, $\Gamma_\sigma \simeq 500$ MeV

Results and data

	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1

- First octet ($M_8^{(1)}$, $c_d^{(1)}$, $c_m^{(1)}$) is fixed to the work in [Jamin, Oller, Pich, NP, B**622**, 279 (2002)], [Jamin, Oller, Pich, NP, B**587**, 331 (2000)]
- Second octet has only fixed its mass, $M_8^{(2)}$, but not its couplings.
- Singlet mass $M_1^{(2)} \lesssim 1\text{GeV}$, but for lower values, we can obtain the same physics with higher couplings.
- Since $SU(3)$ breaking is milder in the vector sector, we take just one substraction constant for the whole set of vector channels,

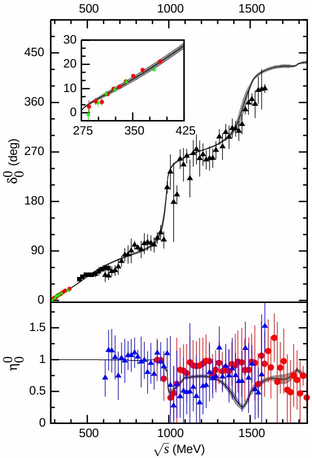
$$a_{\rho\rho} = a_{\omega\omega} = a_{K^\star \bar{K}^\star} = a_{\omega\phi} = a_{\phi\phi}$$

- Together, we have $a_{\pi\pi}$, $a_{K\bar{K}}$, $a_{\eta\eta}$, $a_{\eta\eta'}$, $a_{\eta'\eta'}$, $a_{\sigma\sigma}$ and $a_{\rho\rho}$

Total: just 12 free parameters, for 370 data points.

Results and data

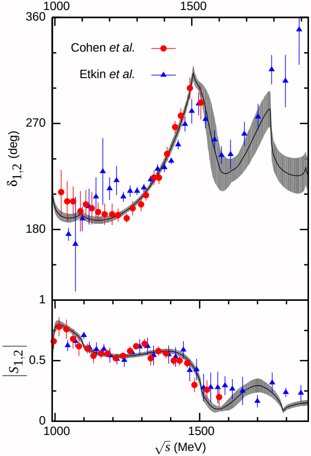
	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1



- $\pi\pi \to \pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still very good
- $\pi\pi \to K\bar{K}$: Near threshold, Cohen data are favored.
- $\pi\pi \to \eta\eta, \eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.
- $I = 1/2$ $K^-\pi^+ \to K^-\pi^+$ amplitude and phase from LASS.

Results and data

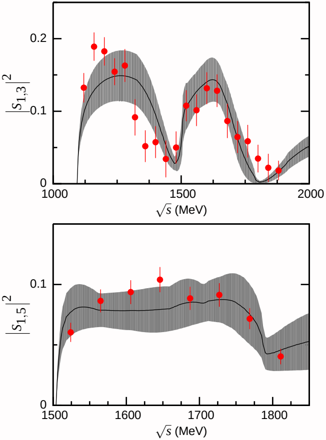
	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1



- $\pi\pi\to\pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still very good
- $\pi\pi\to K\bar{K}$: Near threshold, Cohen data are favored.
- $\pi\pi\to\eta\eta,\eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.
- $I=1/2$ $K^-\pi^+\to K^-\pi^+$ amplitude and phase from LASS.

Results and data

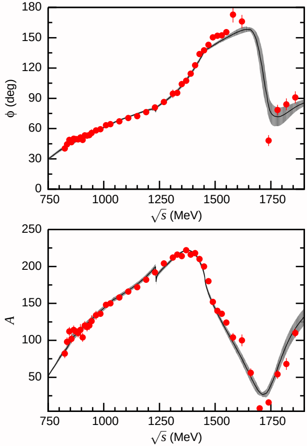
	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1



- $\pi\pi \to \pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still very good
- $\pi\pi \to K\bar{K}$: Near threshold, Cohen data are favored.
- $\pi\pi \to \eta\eta, \eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.
- $I = 1/2$ $K^-\pi^+ \to K^-\pi^+$ amplitude and phase from LASS.

Results and data

	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1



- $\pi\pi \to \pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still very good
- $\pi\pi \to K\bar{K}$: Near threshold, Cohen data are favored.
- $\pi\pi \to \eta\eta, \eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.
- $I = 1/2\ K^-\pi^+ \to K^-\pi^+$ amplitude and phase from LASS.

Results and data

	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1

No parametrization

We determine interaction kernels from Chiral Lagrangians, avoiding *ad hoc* parametrizations

Less free parameters

We have less free parameters, because of our chiral approach and our treatment of $\sigma\sigma$ amplitude

Higher energies

We have included enough channels to get at 2 GeV.

Compare with: [Lindenbaum, Longacre, PL, B**274**, 492 (1992)], [Kloet, Loiseau, ZP A**353**, 227 (1995)], [Bugg, NP B**471**, 59 (1996)]

Spectroscopy. Pole content: Summary

We move to the complex plane, exploring those Riemann sheets wich are continuosly connected to the physical s-real axis.

Resonance	$I = 0$ Poles (MeV)	M (PDG)	Γ (PDG)
$\sigma \equiv f_0(600)$	$456 \pm 6 - i\,241 \pm 7$		
$f_0(980)$	$983 \pm 4 - i\,25 \pm 4$	980 ± 10	$40 - 100$
$f_0(1370)$	$1466 \pm 15 - i\,158 \pm 12$	$1200 - 1500$	$200 - 500$
$f_0(1500)$	$1602 \pm 15 - i\,44 \pm 15$	1505 ± 6	109 ± 7
$f_0(1710)$	$1690 \pm 20 - i\,110 \pm 20$	1724 ± 7	137 ± 8
$f_0(1790)$ (BESII)	$1810 \pm 15 - i\,190 \pm 20$	1790^{+40}_{-30}	270^{+30}_{-60}

Resonance	$I = 1/2$ Poles (MeV)	M (PDG)	Γ (PDG)
$\kappa \equiv K_0^*(800)$	$708 \pm 6 - i\,313 \pm 10$	–	–
$K_0^*(1430)$	$1435 \pm 6 - i\,142 \pm 8$	1414 ± 6	290 ± 21
$K_0^*(1950)$	$1750 \pm 20 - i\,150 \pm 20$	–	–

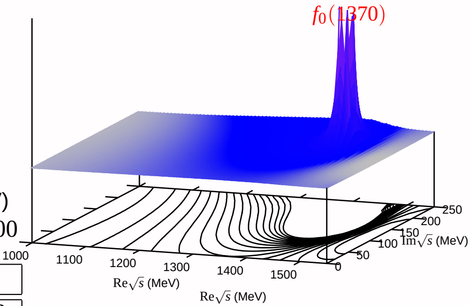
Spectroscopy. Pole content: $f_0(1370)$

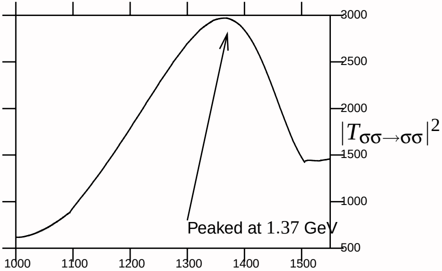
$f_0(1370)$

$(1466 \pm 15 - i\, 158 \pm 12)$ MeV

PDG	M (MeV)	Γ (MeV)
	$1200-1500$	$200-500$

Coupling	bare	final
$g_{\pi^+\pi^-}$	3.9	3.59 ± 0.18
$g_{K^0\bar{K}^0}$	2.3	2.23 ± 0.18
$g_{\eta\eta}$	1.4	1.70 ± 0.30
$g_{\eta\eta'}$	3.7	4.00 ± 0.30
$g_{\eta'\eta'}$	3.8	3.70 ± 0.40





Peaked at 1.37 GeV

- Shift in mass peak
- Strong couplings to $\sigma\sigma$, $\pi\pi$.

Spectroscopy. Pole content: $f_0(1370)$

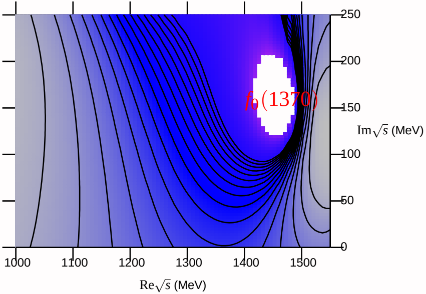
$f_0(1370)$

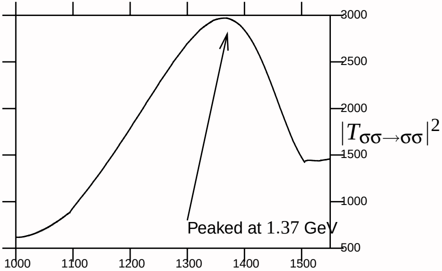
$(1466 \pm 15 - i\,158 \pm 12)$ MeV

PDG	M (MeV)	Γ (MeV)
	$1200 - 1500$	$200 - 500$

Coupling	bare	final
$g_{\pi^+\pi^-}$	3.9	3.59 ± 0.18
$g_{K^0\bar{K}^0}$	2.3	2.23 ± 0.18
$g_{\eta\eta}$	1.4	1.70 ± 0.30
$g_{\eta\eta'}$	3.7	4.00 ± 0.30
$g_{\eta'\eta'}$	3.8	3.70 ± 0.40

- Shift in mass peak
- Strong couplings to $\sigma\sigma$, $\pi\pi$.





Spectroscopy. Pole content: $f_0(1370)$

$f_0(1370)$ $(I=0)$			$K_0^*(1430)$ $(I=1/2)$		
Coupling	bare	final	Coupling	bare	final
$g_{\pi^+\pi^-}$	3.9	3.59 ± 0.18	$g_{K\pi}$	5.0	4.8
$g_{K^0\bar{K}^0}$	2.3	2.23 ± 0.18	$g_{K\eta}$	0.7	0.9
$g_{\eta\eta}$	1.4	1.70 ± 0.30	$g_{K\eta'}$	3.4	3.8
$g_{\eta\eta'}$	3.7	4.00 ± 0.30			
$g_{\eta'\eta'}$	3.8	3.70 ± 0.40			

- Bare coupling are very similar to the physical ones.
- Octet is $S_8^{(1)}$, with $M_8^{(1)} = 1290$, $c_d^{(1)} = c_m^{(1)} = 25.8$ MeV.
- The first scalar octet is a pure one, not mixed with the nearby $f_0(1500)$ and $f_0(1710)$

For instance, we have, for $\pi^+\pi^-$:

$$g_{\pi^+\pi^-} = \sqrt{\frac{2}{3}}\,\frac{c_d M_8^2 + 2(c_m - c_d)m_\pi^2}{f_\pi^2}$$

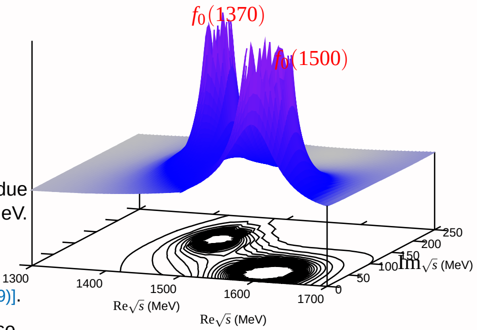
Spectroscopy. Pole content: $f_0(1500)$

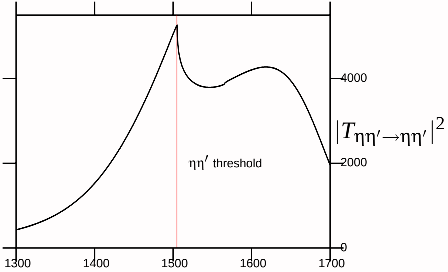
$f_0(1500)$

$(1602 \pm 15 - i\,44 \pm 15)$ MeV

- The peak position is at 1.5 GeV due to the $\eta\eta'$ threshold, $\sqrt{s} \approx 1.5$ GeV.
- This is similar to the $a_0(980)$ because the $K\bar{K}$ threshold [Oller, Oset, PR, D **60**, 0740023 (1999)].
- $\Gamma = 1.2 \times 88 \simeq 105$ MeV, because the BW at $(1.6 - i0.04)$ GeV is cut by the $\eta\eta'$ threshold.
- Complicated energy region. There are three interfering effects:
  - Pole at $(1.6 - i0.04)$ GeV
  - $f_0(1370)$ pole $(1.47 - i0.16)$ GeV
  - Nearby $\eta\eta'$, $\omega\omega$ thresolds

	M (MeV)	Γ (MeV)
PDG	1505 ± 6	109 ± 7





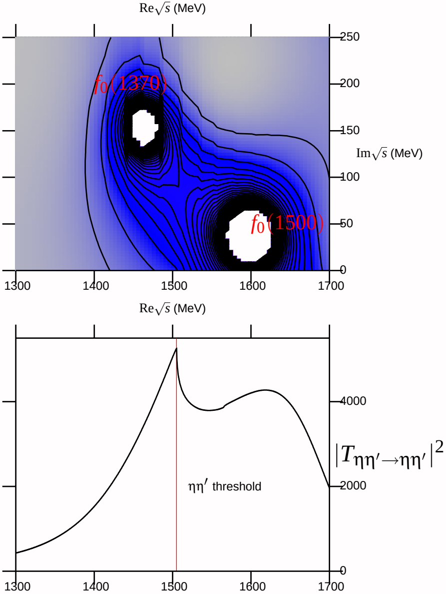
Spectroscopy. Pole content: $f_0(1500)$

$f_0(1500)$

$(1602 \pm 15 - i\,44 \pm 15)$ MeV

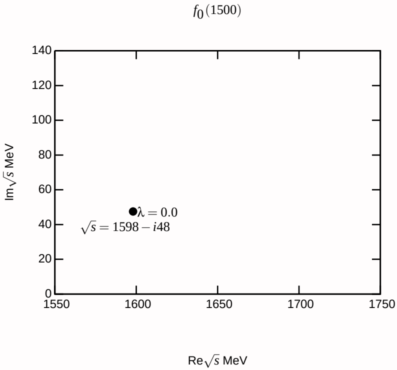
- The peak position is at 1.5 GeV due to the $\eta\eta'$ threshold, $\sqrt{s} \approx 1.5$ GeV.
- This is similar to the $a_0(980)$ because the $K\bar{K}$ threshold [Oller, Oset, PR, D **60**, 0740023 (1999)].
- $\Gamma = 1.2 \times 88 \simeq 105$ MeV, because the BW at $(1.6 - i0.04)$ GeV is cut by the $\eta\eta'$ threshold.
- Complicated energy region. There are three interfering effects:
  - Pole at $(1.6 - i0.04)$ GeV
  - $f_0(1370)$ pole $(1.47 - i0.16)$ GeV
  - Nearby $\eta\eta'$, $\omega\omega$ thresolds

	M (MeV)	Γ (MeV)
PDG	1505 ± 6	109 ± 7



Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

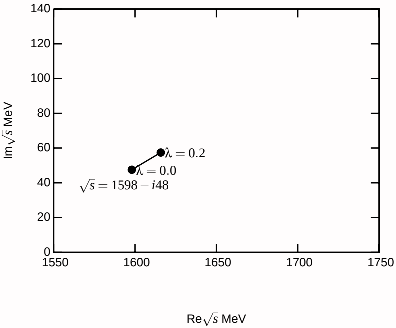
- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances



Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

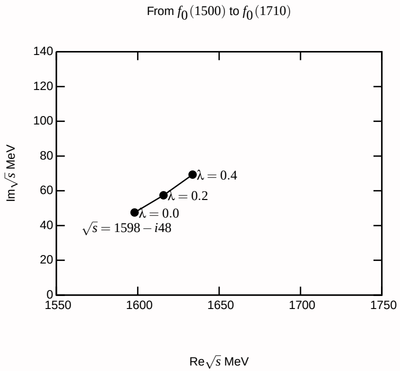
- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances

From $f_0(1500)$ to $f_0(1710)$



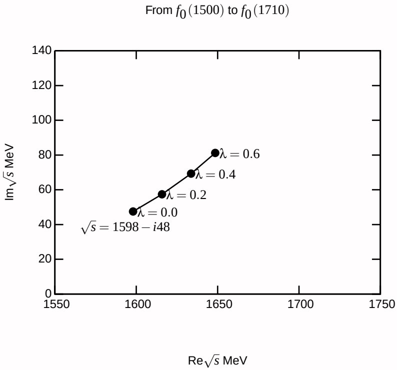
Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances



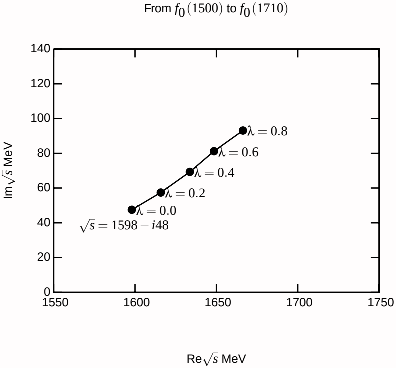
Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances



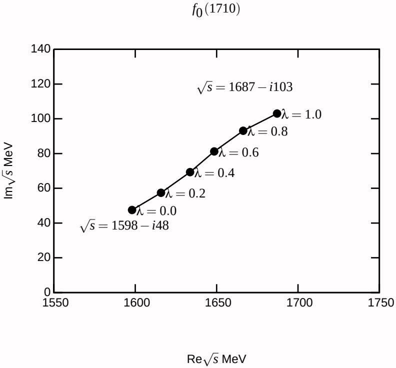
Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances

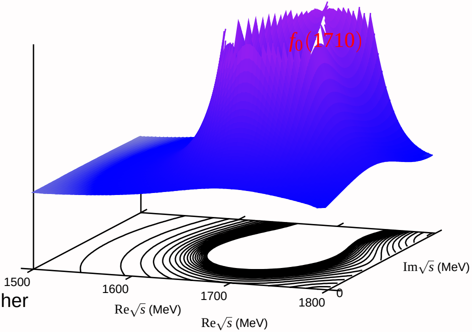


Spectroscopy. Pole content: $f_0(1500)$ moves to $f_0(1710)$

- The $f_0(1500)$ pole moves to $f_0(1710)$ when moving softly between involved sheets
- They are the same underlying pole, but reflected on different Riemann sheets
- They are not generated from preexisting resonances



Spectroscopy. Pole content: $f_0(1710)$

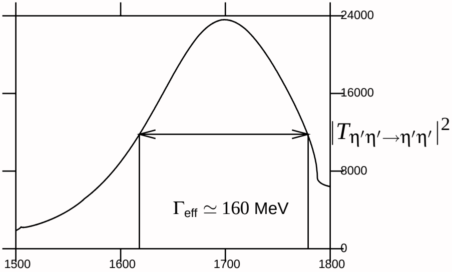


$f_0(1710)$

$(1690 \pm 20 - i\ 110 \pm 20)$ MeV

- The peak is shifted to slightly higher energy, at 1700 MeV.
- The effective width can depend on the processes.
- $\Gamma_{\text{eff}} \simeq 160$ MeV

	M (MeV)	Γ (MeV)
PDG	1724 ± 7	137 ± 8



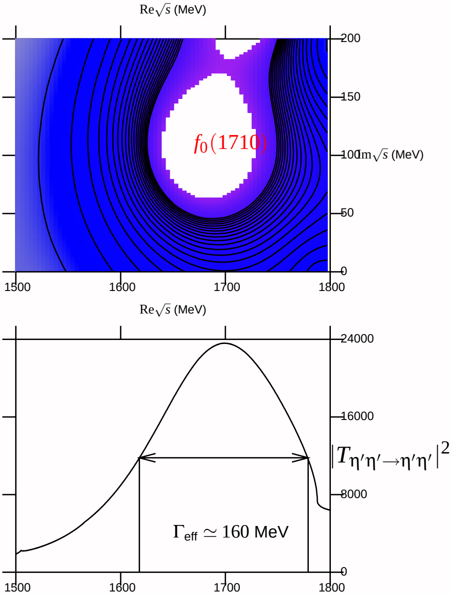
Spectroscopy. Pole content: $f_0(1710)$

$f_0(1710)$

$(1690 \pm 20 - i\, 110 \pm 20)$ MeV

- The peak is shifted to slightly higher energy, at 1700 MeV.
- The effective width can depend on the processes.
- $\Gamma_{\text{eff}} \simeq 160$ MeV

	M (MeV)	Γ (MeV)
PDG	1724 ± 7	137 ± 8



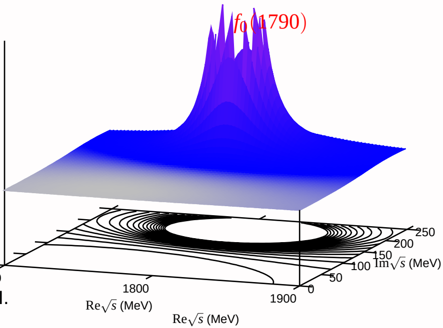
Spectroscopy. Pole content: $f_0(1790)$

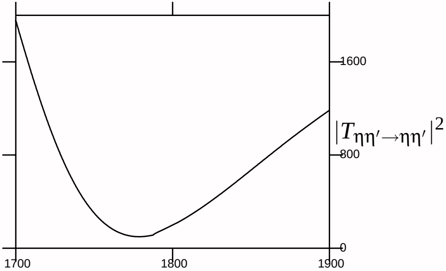
$f_0(1790)$

$(1810 \pm 15 - i\ 190 \pm 20)$ MeV

- Weak signal on the real axis
- It couples weakly to $K\bar{K}$, a major difference with respect to the $f_0(1710)$, as also observed by BESII.
- It is the partner of the pole at $1.75 - i0.15$ GeV in $I = 1/2$.
- These poles originate from the higher bare octet, $S_8^{(2)}$ with $M_8^{(2)} = 1905$, $c_d^{(2)} = 20.3$, $c_d^{(2)} = -13.9$ MeV.

	M (MeV)	Γ (MeV)
BESII	1790^{+40}_{-30}	270^{+30}_{-60}





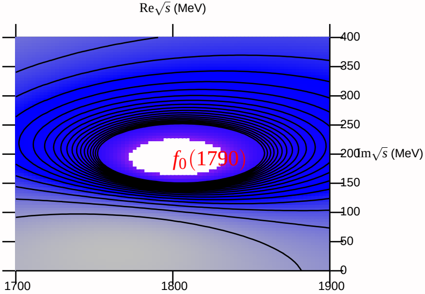
Spectroscopy. Pole content: $f_0(1790)$

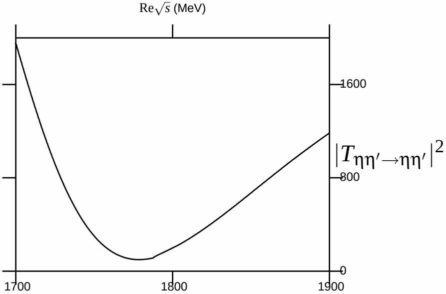
$f_0(1790)$

$(1810 \pm 15 - i\ 190 \pm 20)$ MeV

- Weak signal on the real axis
- It couples weakly to $K\bar{K}$, a major difference with respect to the $f_0(1710)$, as also observed by BESII.
- It is the partner of the pole at $1.75 - i0.15$ GeV in $I = 1/2$.
- These poles originate from the higher bare octet, $S_8^{(2)}$ with $M_8^{(2)} = 1905$, $c_d^{(2)} = 20.3$, $c_d^{(2)} = -13.9$ MeV.

	M (MeV)	Γ (MeV)
BESII	1790^{+40}_{-30}	270^{+30}_{-60}





Spectroscopy. Couplings. $f_0(1500)$ / $f_0(1710)$

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

This pattern suggests an enhancement in $s\bar{s}$ production.
With a pseudoscalar mixing angle $\sin\beta = -1/3$ for η and η':

$$\begin{aligned} g_{\eta'\eta'} &= \frac{2}{3}g_{ss} + \frac{1}{3}g_{nn} + \frac{2\sqrt{2}}{3}g_{ns} \\ g_{\eta\eta'} &= -\frac{\sqrt{2}}{3}g_{ss} + \frac{\sqrt{2}}{3}g_{nn} + \frac{1}{3}g_{ns} \\ g_{\eta\eta} &= \frac{1}{3}g_{ss} + \frac{2}{3}g_{nn} - \frac{2\sqrt{2}}{3}g_{ns} \end{aligned}$$

$$\begin{aligned} \eta &= -\frac{1}{\sqrt{3}}\eta_s + \sqrt{\frac{2}{3}}\eta_u \\ \eta' &= -\sqrt{\frac{2}{3}}\eta_s + \frac{1}{\sqrt{3}}\eta_u \end{aligned}$$

$$\eta_s = s\bar{s} \qquad \eta_u = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$$

The $G_0 \rightarrow q\bar{q} \propto m_q$ chiral suppresion [Chanowitz, PRL**95**, 172001 (2005)] implies $|g_{ss}| \gg |g_{nn}|$. This together with the OZI rule requires $|g_{ss}| \gg |g_{ns}|$

Spectroscopy. Couplings. $f_0(1500)$ / $f_0(1710)$

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
g_{ss}	11.5 ± 0.5	13.0 ± 1.0
g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

This pattern suggests an enhancement in $s\bar{s}$ production.
With a pseudoscalar mixing angle $\sin\beta = -1/3$ for η and η':

$$\begin{aligned} g_{\eta'\eta'} &= \frac{2}{3} g_{ss} + \frac{1}{3} g_{nn} + \frac{2\sqrt{2}}{3} g_{ns} \\ g_{\eta\eta'} &= -\frac{\sqrt{2}}{3} g_{ss} + \frac{\sqrt{2}}{3} g_{nn} + \frac{1}{3} g_{ns} \\ g_{\eta\eta} &= \frac{1}{3} g_{ss} + \frac{2}{3} g_{nn} - \frac{2\sqrt{2}}{3} g_{ns} \end{aligned}$$

$$\begin{aligned} \eta &= -\frac{1}{\sqrt{3}}\eta_s + \sqrt{\frac{2}{3}}\eta_u \\ \eta' &= -\sqrt{\frac{2}{3}}\eta_s + \frac{1}{\sqrt{3}}\eta_u \end{aligned}$$

$$\eta_s = s\bar{s} \qquad \eta_u = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

The $G_0 \to q\bar{q} \propto m_q$ chiral suppresion [Chanowitz, PRL**95**, 172001 (2005)] implies $|g_{ss}| \gg |g_{nn}|$. This together with the OZI rule requires $|g_{ss}| \gg |g_{ns}|$

This is precisely what we obtain from the previous couplings and equations

Spectroscopy. Couplings. $f_0(1500)$ / $f_0(1710)$

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
g_{ss}	11.5 ± 0.5	13.0 ± 1.0
g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

Now let us consider the $K\bar{K}$ coupling

- Valence quarks: K^0 corresponds to $\sum_{i=1}^{3} \bar{s}_i u^i/\sqrt{3}$, and analogously $\bar{K}^0$
- The production of a colour singlet $s\bar{s}$ requires the combination of the colour indices of K^0, $\bar{K}^0$
- Decompose $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^j$
- Only $s\bar{s}\,u\bar{u}$ contributes (factor $1/3$) and $s\bar{s}\,s\bar{s}$ has an extra factor 2 compared to $s\bar{s}\,u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$

Spectroscopy. Couplings. $f_0(1500)$ / $f_0(1710)$

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
g_{ss}	11.5 ± 0.5	13.0 ± 1.0
g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

Now let us consider the $K\bar{K}$ coupling

- Valence quarks: K^0 corresponds to $\sum_{i=1}^{3} \bar{s}_i u^i/\sqrt{3}$, and analogously $\bar{K}^0$
- The production of a colour singlet $s\bar{s}$ requires the combination of the colour indices of K^0, $\bar{K}^0$
- Decompose $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^j$
- Only $s\bar{s}u\bar{u}$ contributes (factor $1/3$) and $s\bar{s}s\bar{s}$ has an extra factor 2 compared to $s\bar{s}u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$

Reasons to identify the glueball

Wich is the origin of this $\bar{s}s$ enhanced production: chiral suppression mechanism for a glueball or the OZI rule for a $\bar{s}s$?

Arguments favouring the first interpretation for the $f_0(1500)$ - $f_0(1710)$:

1. $f_0(1370)$ does not mix, is pure $I = 0$ octet state:
    - No one can then mix with $f_0(1500)$, because $f_0(1710)$ is the same pole but on different Riemann sheets.

   Furthermore, if $f_0(1710)$–$f_0(1500)$ were a $s\bar{s}$ resonance, there should be an accompanying pole in $I = 1/2$, but that is not the case.
2. If $f_0(1370)$ were an $n\bar{n}$, then OZI rule would suggest $f_0(1500)$–$f_0(1710)$ to be an $s\bar{s}$. However, $f_0(1370)$ remains as pure octet.
3. Unquenched lattice calculations give the mass and the couplings [Sexton, Vaccarino, Weingarten, PRL **75**, 4563 (1995)] of the lightest 0^{++} glueball.
    - Mass [Chen et al, PRD, **73**, 014516 (2006)]:

      $$M^{\text{gb}}_{0^{++}} = 1.66 \pm 0.05 \text{ GeV} \qquad M_{f_0} = 1.69 \pm 0.02 \text{ GeV}$$
    - Couplings are calculated in the $SU(3)$ limit and are linear in the pseudoscalar mass squared in agreement with chiral suppresion mechanism [Chanowitz, PRL**95**, 172001 (2005)]

Reasons to identify the glueball

Wich is the origin of this $\bar{s}s$ enhanced production: chiral suppression mechanism for a glueball or the OZI rule for a $\bar{s}s$?

Arguments favouring the first interpretation for the $f_0(1500)$ - $f_0(1710)$:

1. $f_0(1370)$ does not mix, is pure $I=0$ octet state:
   - No one can then mix with $f_0(1500)$, because $f_0(1710)$ is the same pole but on different Riemann sheets.

   Furthermore, if $f_0(1710)$–$f_0(1500)$ were a $s\bar{s}$ resonance, there should be an accompanying pole in $I=1/2$, but that is not the case.
2. If $f_0(1370)$ were an $n\bar{n}$, then OZI rule would suggest $f_0(1500)$–$f_0(1710)$ to be an $s\bar{s}$. However, $f_0(1370)$ remains as pure octet.
3. Unquenched lattice calculations give the mass and the couplings [Sexton, Vaccarino, Weingarten, PRL **75**, 4563 (1995)] of the lightest 0^{++} glueball.
   - Mass [Chen et al, PRD, **73**, 014516 (2006)]:

$$M_{0^{++}}^{\text{gb}} = 1.66 \pm 0.05 \text{ GeV} \qquad M_{f_0} = 1.69 \pm 0.02 \text{ GeV}$$

   - Couplings are calculated in the $SU(3)$ limit and are linear in the pseudoscalar mass squared in agreement with chiral suppresion mechanism [Chanowitz, PRL**95**, 172001 (2005)]

Reasons to identify the glueball

Wich is the origin of this $s\bar{s}$ enhanced production: chiral suppression mechanism for a glueball or the OZI rule for a $s\bar{s}$?
Arguments favouring the first interpretation for the $f_0(1500)$ - $f_0(1710)$:

1. $f_0(1370)$ does not mix, is pure $I = 0$ octet state:
    - No one can then mix with $f_0(1500)$, because $f_0(1710)$ is the same pole but on different Riemann sheets.

   Furthermore, if $f_0(1710)$–$f_0(1500)$ were a $s\bar{s}$ resonance, there should be an accompanying pole in $I = 1/2$, but that is not the case.
2. If $f_0(1370)$ were an $n\bar{n}$, then OZI rule would suggest $f_0(1500)$–$f_0(1710)$ to be an $s\bar{s}$. However, $f_0(1370)$ remains as pure octet.
3. Unquenched lattice calculations give the mass and the couplings [Sexton, Vaccarino, Weingarten, PRL **75**, 4563 (1995)] of the lightest 0^{++} glueball.
    - Mass [Chen et al, PRD, **73**, 014516 (2006)]:

$$M^{\text{gb}}_{0^{++}} = 1.66 \pm 0.05 \text{ GeV} \qquad M_{f_0} = 1.69 \pm 0.02 \text{ GeV}$$

    - Couplings are calculated in the $SU(3)$ limit and are linear in the pseudoscalar mass squared in agreement with chiral suppresion mechanism [Chanowitz, PRL**95**, 172001 (2005)]

Reasons to identify the glueball

Wich is the origin of this $\bar{s}s$ enhanced production: chiral suppression mechanism for a glueball or the OZI rule for a $\bar{s}s$?

Arguments favouring the first interpretation for the $f_0(1500)$ - $f_0(1710)$:

1. $f_0(1370)$ does not mix, is pure $I=0$ octet state:
    - No one can then mix with $f_0(1500)$, because $f_0(1710)$ is the same pole but on different Riemann sheets.

   Furthermore, if $f_0(1710)$–$f_0(1500)$ were a $s\bar{s}$ resonance, there should be an accompanying pole in $I=1/2$, but that is not the case.

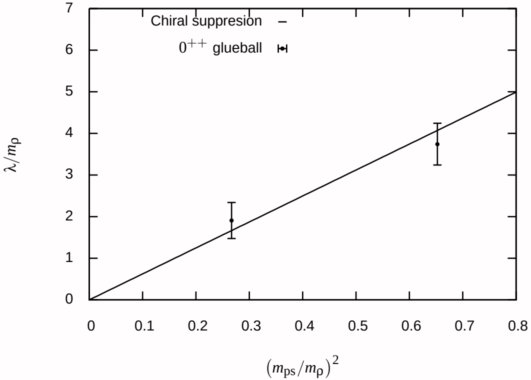
2. If $f_0(1370)$ were an $n\bar{n}$, then OZI rule would suggest $f_0(1500)$–$f_0(1710)$ to be an $s\bar{s}$. However, $f_0(1370)$ remains as pure octet.

3. Unquenched lattice calculations give the mass and the couplings [Sexton, Vaccarino, Weingarten, PRL **75**, 4563 (1995)] of the lightest 0^{++} glueball.
    - Mass [Chen et al, PRD, **73**, 014516 (2006)]:

$$M^{\text{gb}}_{0^{++}} = 1.66 \pm 0.05 \text{ GeV} \qquad M_{f_0} = 1.69 \pm 0.02 \text{ GeV}$$

    - Couplings are calculated in the $SU(3)$ limit and are linear in the pseudoscalar mass squared in agreement with chiral suppresion mechanism [Chanowitz, PRL**95**, 172001 (2005)]

Reasons to identify the glueball



Summary

- We have performed the first coupled channel study of the meson-meson S-waves for $I=0$ and $I=1/2$ up to 2 GeV with 13 coupled channels
- We determine our interaction kernels from Chiral Lagrangians, implemented in N/D-type coupled channel equations
- We have many less free parameters, compared with previous work in the literature, for a vast quantity of data
- We generate all scalar resonances below 2 GeV: σ, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ and $f_0(1790)$ for $I=0$ and κ, $K_0^*(1430)$ and $K_0^*(1950)$ for $I=1/2$
- The structure of the couplings to pairs of pseudoscalars implies that:
  - $f_0(1370)$, $K_0^*(1430)$ (and $a_0(1450)$) remain as pure octet
  - $f_0(1500)$ and $f_0(1710)$, which are the same pole reflected on different sheets
  - Their decays show an enhanced $s\bar{s}$ production, in agreement with the chiral supression mechanism of [Chanowitz, PRL**95**, 172001 (2005)], so
  - They should be considered as the lightest 0^{++} glueball

Summary

Thank you for your attention!

σ width effects on $g(s)$

With s_σ complex, $N_{i\to\sigma\sigma}$ would be complex, thus violating unitarity. So we take s_i real, but varying according to a mass distribution.

- Lehman propagator representation (dispersion relation):

$$P(s) = -\frac{1}{\pi}\int_{\sqrt{s_{\text{th}}}}^{\infty} \mathrm{d}s' \, \frac{\mathrm{Im}P(s')}{s-s'+i\varepsilon},$$

- In a first approach:

$$\begin{aligned}
\mathrm{Im}P(s') &\propto \mathrm{Im}\left(\frac{1}{s'-m_\sigma^2+i\,m_\sigma\Gamma_\sigma(s')}\right) \\
\Gamma_\sigma(s') &= \Gamma_\sigma\sqrt{\frac{1-4m_\pi^2/s'}{1-4m_\pi^2/m_\sigma^2}} \\
\int_{\sqrt{s_{\text{th}}}}^{\infty} \mathrm{d}s'\,\mathrm{Im}P(s') &= 1
\end{aligned}$$

- $g(s)$ (unitarity loop) for σ channel can be written as:

$$\int_{\sqrt{s_{\text{th}}}}^{\infty} \mathrm{d}s_1 \int_{\sqrt{s_{\text{th}}}}^{\infty} \mathrm{d}s_2\,\mathrm{Im}P(s_1)\mathrm{Im}P(s_2)\; g_4(s;s_1,s_2)\ ,$$

Spectroscopy, poles and Riemann sheets

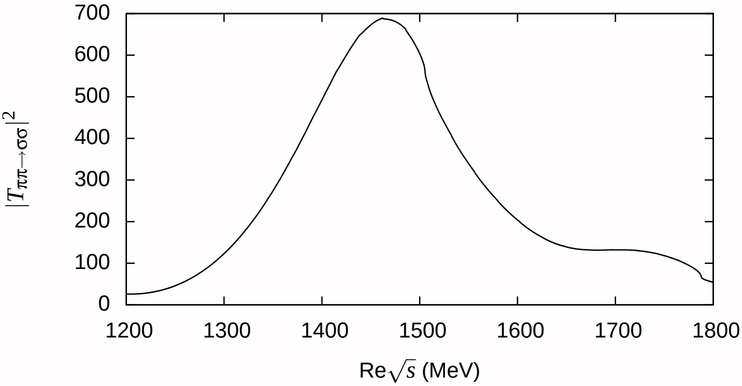
- When parameters are fitted to data, we extrapolate our amplitudes to the s-complex plane, finding poles, and we identify them with resonances through $\sqrt{s_0} \approx M - i\Gamma/2$.
- Analytical extrapolations are needed in the T_{ij} to the different Riemann sheets, which appear because of the cuts in $G(s)$ at opening thresholds:

$$p(s) = \frac{\sqrt{s-(m_a+m_b)^2}\sqrt{s-(m_a-m_b)^2}}{2\sqrt{s}}$$

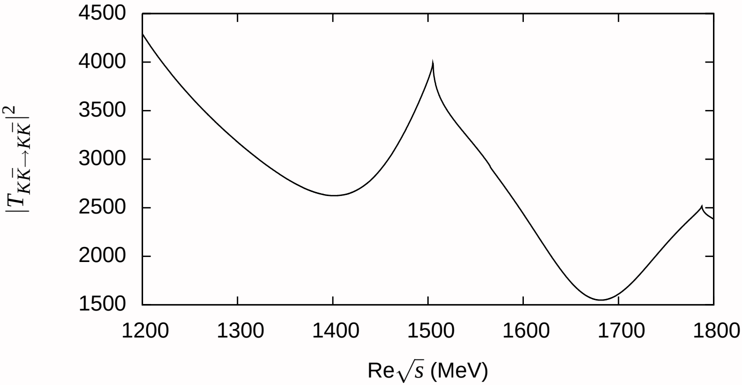
- Using continuity, for $\sqrt{s}$ real, $> m_a + m_b$, we have:

$$G^{II}(s+i\varepsilon) = G(s-i\varepsilon) = G(s+i\varepsilon) - 2i\mathrm{Im}G(s+i\varepsilon) = G(s+i\varepsilon) + \frac{i}{4\pi}\frac{p(s)}{\sqrt{s}}$$

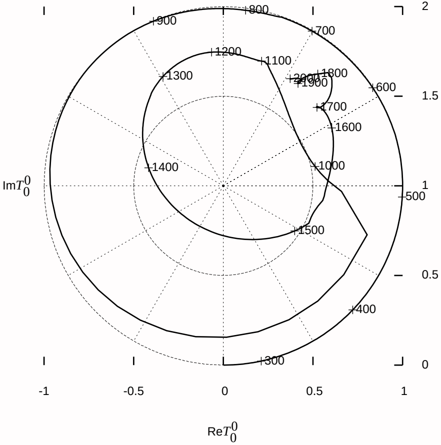
Other amplitudes



Other amplitudes



Argand diagram



References

- M. Albaladejo, J.A. Oller hep-ph/0801.4929
- M. Chanowitz, Phys. Rev. Lett. **95**, 172001 (2005) [arXiv:hep-ph/0506125].
- J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)] [arXiv:hep-ph/9702314].
- J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999) [arXiv:hep-ph/9809337].
- M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **587** (2000) 331 [arXiv:hep-ph/0006045].
- M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **622**, 279 (2002) [arXiv:hep-ph/0110193].
  P. Herrera-Siklody, J. I. Latorre, P. Pascual and J. Taron,
- P. Herrera-Siklody, J. I. Latorre, P. Pascual and J. Taron, Nucl. Phys. B **497**, 345 (1997) [arXiv:hep-ph/9610549].
- P. Herrera-Siklody, J. I. Latorre, P. Pascual and J. Taron, Phys. Lett. B **419**, 326 (1998) [arXiv:hep-ph/9710268].
- G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).
- W. M. Yao et al. [Particle Data Group], J. Phys. G **33**, 1 (2006).