Lectures on Nuclear and Hadron Physics

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July 28, 2011

Outline Lecture IV

Introduction

EFT with short- and long range few-nucleon interactions is quite advanced in vacuum

Pion-nucleon interactions in nuclear matter are already largely exploited, considering chiral Lagrangians

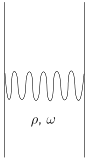
For a recent review: Epelbaum,Hammer,Meißner, Rev.Mod.Phys.**81**(2009)1773

Commonly, free parameters are fixed to nuclear matter properties

Many important results and studies of nuclear processes have been accomplished.

Nonetheless the development of a Chiral EFT in nuclear matter is a left problem of foremost importance:

- Need of a in-medium power counting to include both short- and long-range multi-N interactions



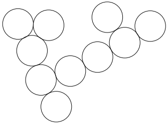
Boson exchange models: Short range interactions are due to exchanges of heavier mesons like ρ, ω, etc

- The power counting has to take into account
- In reducible diagrams for NN-interactions N-propagators are enhanced, $\frac{1}{k^0 - E_{\mathbf{k}} + i\epsilon} \sim \mathcal{O}(p^{-2})$

S. Weinberg, Nucl.Phys.B**363**(1991)3

- In-medium multi-N interactions must be taken into account consistently with the vacuum Machleidt, Entem, J. Phys. G 37 (2010) 064041: stress that 3NF is one of the most important outstanding issues in chiral EFT approach to nuclear forces

- Any number of closed nucleon loops can be arranged in any way



- Pion-nucleon interactions have to be included with the same requirements

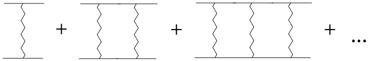
Chiral power counting

$$E_k = |\mathbf{k}|^2/2m$$

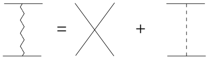
$$G_0(k)_{i_3} = \frac{\theta(|\mathbf{k}| - \xi_{i_3})}{k_0 - E_k + i\epsilon} + \frac{\theta(\xi_{i_3} - |\mathbf{k}|)}{k_0 - E_k - i\epsilon} = \frac{1}{k_0 - E_k + i\epsilon} + 2\pi i\, \delta(k_0 - E_k)\, \theta(\xi_{i_3} - k)$$

If $k_0 \sim \mathcal{O}(p)$: Standard counting $\longrightarrow$ $G_0(k) \sim \mathcal{O}(p^{-1})$ $1/|\mathbf{p}|$

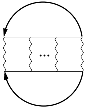
If $k_0 \sim \mathcal{O}(p^2)$: Non-standard counting $\longrightarrow$ $G_0(k) \sim \mathcal{O}(p^{-2})$ $1/|\mathbf{p}| \times 2m/|\mathbf{p}|$



where



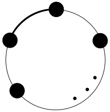
The NN reducible diagrams are very abundant in the nuclear medium



Every nucleon propagator $G_0(k)_{i_3} \sim \mathcal{O}(p^{-2})$

Despite this the chiral power counting is still bounded from below

Concept of in-medium generalized vertex (IGV):
thick line: Fermi sea insertion, thin lines: full in-medium nucleon propagator, filled circles: bilinear nucleon vertices



JAO,Phys.Rev.C65(2002)025204; Meißner,JAO,Wirzba,Ann.Phys.297(2002)27

Let $\mathbf{H}$ be an auxiliary field for heavy mesons responsible for short range $2N$, $3N, \ldots$ interactions, the $\mathbf{H}$-"propagator" counts as $\mathcal{O}(p^0)$[1]

Short-range interactions enter the counting via bilinear vertices of $\mathcal{O}(\geq p^0)$

$$\mathcal{L}_{eff} = \sum_{n=1}^{\infty} \mathcal{L}_{\pi\pi}^{(2n)} + \sum_{n=1}^{\infty} \mathcal{L}_{\pi N}^{(n)} + \sum_{n=0}^{\infty} \mathcal{L}_{NN}^{(2n)} + \ldots$$

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$$

- $\xi_F \sim M_\pi \sim q_\pi \sim \mathcal{O}(p) \quad , \quad \Delta_\pi(k^2) \sim \mathcal{O}(k^{-2})$
- $G_0(k) \sim \mathcal{O}(p^{-2})$
- $m \sim 4\pi f_\pi \sim C \sim \pi \sim f_\pi \sim \mathcal{O}(p^0)$

$$\nu = 4L_H + 4L_\pi - 2I_\pi + \sum_{i=1}^{V} d_i - \sum_{i=1}^{V_\rho} 2m_i + \sum_{i=1}^{V_\pi} \ell_i + \sum_{i=1}^{V_\rho} 3$$

V_ρ number of IGV.
m_i number of nucleon propagators (minus one) in the i_{th} IGV.
L_H, L_π number of heavy meson and pion loops.
I_π number of internal pion lines.
V number of bilinear vertices.
d_i chiral dimension of a bilinear vertex.
V_π number of purely mesonic vertices.
ℓ_i chiral dimension of a purely mesonic vertex.

A Cluster is a set of IGV joined by Hs. Its number is V_Φ.

$$L_H = I_H - \sum_{i=1}^{V_\Phi}(V_{\rho,i} - 1) = I_H - V_\rho + V_\Phi \ ,$$
$$L_\pi = I_\pi - V_\pi - V_\Phi + 1 \ ,$$
$$L_H + L_\pi = I_H + I_\pi - V_\pi - V_\rho + 1 \ .$$

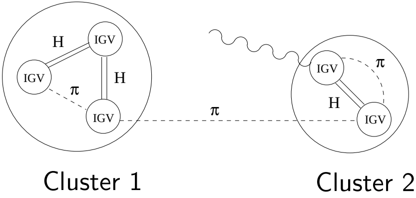


Figure: IGVs separated in two clusters. Here $V_\rho = 5$, $V_\Phi = 2$, $I_\pi = 3$, $I_H = 3$ and $E = 1$. $L_\pi = 2$ and $L_H = 0$.

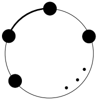
$\xi_F \sim M_\pi \sim q_\pi \sim \mathcal{O}(p)$

Order p^ν of a diagram:[1]

$$\nu = 4 - E_\pi + \sum_{i=1}^{V_\pi}(n_i + l_i - 4) + \sum_{i=1}^{V_B}(d_i + v_i + \omega_i - 2) + V_\rho$$

ν is bounded from below (modulo external sources):

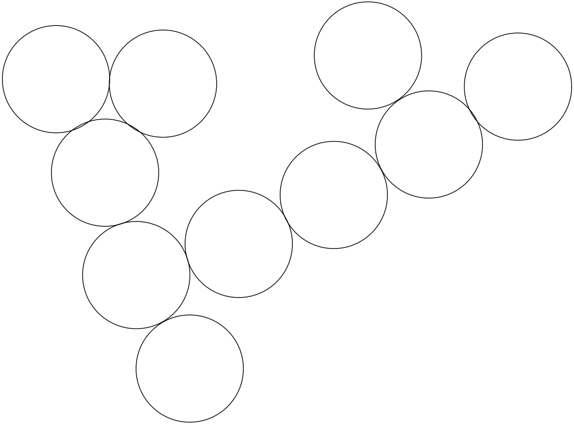
1. Adding pions to pionic vertices: $n_i \geq 2,\ l_i \geq 2$
2. Nucleon mass renormalization terms: $d_i \geq 2,\ \omega_i = 0,\ v_i \geq 0$
3. Adding pions to pion-nucleon vertices: $d_i \geq 1,\ \omega_i = 0,\ v_i \geq 1$
4. Adding heavy mesons to bilinear vertices: $d_i \geq 0,\ \omega_i \geq 1,\ v_i \geq 1$
5. $V_\rho \Rightarrow$ adding **1** IGV rises the counting at least by **1**

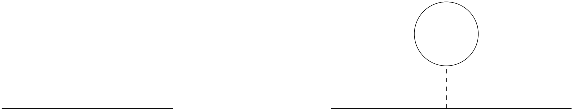


IGV

The actual order of a diagram may be higher, due to $\mathcal{O}(p^{-1})$ nucleon propagators from standard-χPT.

3 & 4 give rise to the resummation of certain types of diagrams!





Adding one extra IGV

$$\frac{2m}{k^2} \quad \& \quad \frac{2m}{k^2}\frac{k^3}{6\pi^2}\frac{2m}{k^2}\frac{g_A^2}{4f_\pi^2}$$

Relative factor (Extra factor of π^{-1}):

$$\frac{g_A^2 mk}{12\pi^2 f_\pi^2} \sim \frac{k}{2.3\pi f_\pi} = \frac{k}{\Lambda}$$

$\Lambda = \nu\pi f_\pi, \;\; \nu \sim \mathcal{O}(1)$

Twice iterated pion exchange:

$$\frac{m}{k^2} \quad \& \quad \frac{m}{k^2}\frac{k^3}{6\pi^2}\frac{2m}{k^2}\left(\frac{g_A^2}{4f_\pi^2}\right)^2 L_{10}$$

The same factor as before

$$\times \; - i\frac{g_A^2 m}{4\pi f_\pi}\frac{k}{4f_\pi} \lesssim 1$$

In the medium: imaginary part is suppressed (hole-hole part) and a real part $m\xi_F/4\pi^2$ stems

$$\times \; \frac{g_A^2 m}{4\pi f_\pi}\frac{\xi_F}{4\pi f_\pi} \sim \frac{\xi_F}{3\pi f_\pi}$$

$k \sim M_\pi \sim \xi_F$ is the most important region for this physics

$$T_{NN} \sim \frac{4\pi/m}{-\frac{1}{a} + \frac{1}{2}rk^2 + \ldots - ip}$$

$rk^2/2$ rapidly overcomes $-1/a$ ($|a| >> 1$)

Sum over states below the Fermi seas: The measure $\int k^2\, dk$ kills low three-momenta.

The power counting equation is applied increasing step by step V_ρ.

Augmenting the number of lines in a diagram **without increasing** the chiral power by adding:

1. Pionic lines attached to lowest order mesonic vertices, $\ell_i = n_i = 2$
2. Pionic lines attached to lowest order meson-baryon vertices, $d_i = v_i = 1$
3. Heavy mesonic lines attached to lowest order bilinear vertices, $d_i = 0$, $\omega_i = 1$.

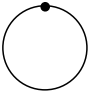
Source of **non-perturbative** physics. These rule give rise to infinite resummations.

Nuclear matter energy density - Contributions

$V_\rho = 1$

$\mathcal{O}(p^5)$

Leading Order

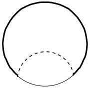


1

$V_\rho = 1$

$\mathcal{O}(p^6)$

Next-to-leading Order

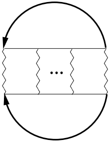


2

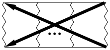
$V_\rho = 2$

$\mathcal{O}(p^6)$

Next-to-leading Order



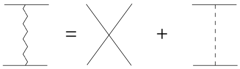
3.1

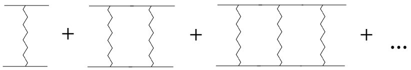


3.2

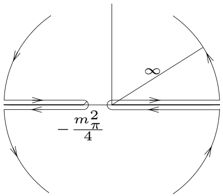
NN interactions:

LO in the chiral expansion: $d_i = 1,\ v_i = 1,\ \omega_i = 0$ OPE
$d_i = 0,\ v_i = 1,\ \omega_i = 1$ Local Terms





A NN Partial Wave has:
- Left-Hand Cut (LHC) $\mathbf{p}^2 < -m_\pi^2/4$
- Right-Hand Cut (RHC) $\mathbf{p}^2 > 0$
- Unitarity Cut



Elastic Case

$$\mathrm{Im} T_{JI}(\ell', \ell, S)^{-1} = -\frac{m|\mathbf{p}|}{4\pi} \quad \text{for} \quad \mathbf{p}^2 > 0$$

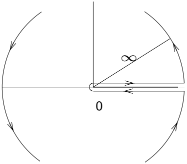
Fixed by kinematics

Once Subtracted Dispersion Relations

$$
\begin{aligned}
T_{JI}(\ell',\ell,S) &= \left[N_{JI}(\ell',\ell,S)^{-1} + \cdot g\right]^{-1} \\
&= \left[I + N_{JI}(\ell',\ell,S)\cdot g\right]^{-1}\cdot N_{JI}(\ell',\ell,S)
\end{aligned}
$$

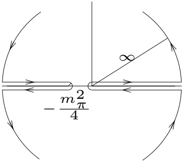
$$
g(A) = g(D) - \frac{m(A-D)}{4\pi^2}\int_0^\infty dk^2 \frac{k}{(k^2-A-i\epsilon)(k^2-D-i\epsilon)}
$$

$$
\equiv g_0 - i\frac{m\sqrt{A}}{4\pi}\ , \qquad D = 0
$$



$$
T_{JI}^{-1}(A) = T_{JI}^{-1}(D) - \frac{m(A-D)}{4\pi^2}\int_0^\infty dk^2 \frac{k}{(k^2-A-i\epsilon)(k^2-D)}
$$

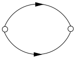
$$
-\frac{(A-D)}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\mathrm{Im}T_{JI}/|T_{JI}|^2}{(k^2-A-i\epsilon)(k^2-D)}
$$



Subtraction constants

One for every S-wave. Determined from the scattering lengths.
0 for higher partial waves. $T_{JI}(A=0)=0$ for ℓ or $\ell'>0$

Integral equation for $N_{JI}(\ell',\ell,S)$:

Interaction kernel: $N_{JI}(\ell',\ell,S)$ Unitarity loop g: 

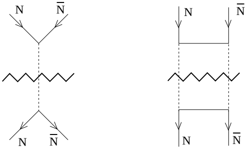
$T_{JI}=\left[N_{JI}^{-1}+g\right]^{-1}$:

$$\text{Im}N_{JI}=\frac{|N_{JI}|^2}{|T_{JI}|^2}\text{Im}T_{JI}=|1+gN_{JI}|^2\text{Im}T_{JI} \quad , \quad |\mathbf{p}|^2<-\frac{m_\pi^2}{4} \; .$$

$$N_{JI}(A)=N_{JI}(D)+\frac{A-D}{\pi}\int_{-\infty}^{-m_\pi^2/4}dk^2\,\frac{\text{Im}T_{JI}(k^2)\,|1+g(k^2)N_{JI}(k^2)|^2}{(k^2-A-i\epsilon)(k^2-D)}$$

The LHC Input: Im$T_{JI}(\mathbf{p}^2)$, $\mathbf{p}^2 < -m_\pi^2/4$

Intermediate states contain pionic lines.
It can be calculated perturbatively in CHPT.
Crossed dynamics $N\bar{N} \to N\bar{N}$
$t,\ u = m_\pi^2,\ > 4m_\pi^2, \ldots$



Two Subtraction Constants?: $g(D)$ and $N_{JI}(D)$

There is only one independent subtraction constant, $T_{JI}(D)$
Analogy with Renormalization Theory:
D is like the "Renormalization Scale"
$g(D)$ is like the "Renormalization Scheme"
$N_{JI}(D)$ depends on $g(D)$ but $T_{JI}(A)$ is $g(D)$ Independent

$D = 0$ is taken

S-waves: $N_{JI}(0) = \frac{-1}{g_0 + \frac{m}{4\pi a_s}}$

Higher partial waves: $N_{JI}(0) = 0$

The convergence in the iterative solution for N_{JI} improves for $g_0 \simeq -\frac{m m_\pi}{4\pi}$ or $g(-\frac{m_\pi^2}{4}) = 0$

$\frac{1}{N_{JI}(A)} + g_0 = F_{JI}(A)$ is independent of g_0

It is enough to know N_{JI}(A) for just one value of g_0

$$N_{JI}(A) = \frac{1}{F(A) - g_0}$$

Algebraic Approximate Solution for $N_{JI}(A)$

It yields Unitary χPT (UCHPT): Regulator Dependent Solutions

JAO,Oset,NPA620(1997)438; PRD60(1999)074023; JAO,Meißner,PLB500(2001)263.

$$\text{Im}N_{JI} = \frac{|N_{JI}|^2}{|T_{JI}|^2}\text{Im}T_{JI} = |1+gN_{JI}|^2\text{Im}T_{JI} \quad , \quad |\mathbf{p}|^2 < -\frac{m_\pi^2}{4} \; .$$

$$N_{JI}(A) = N_{JI}(D) + \frac{A-D}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\text{Im}T_{JI}(k^2)\,|1+g(k^2)N_{JI}(k^2)|^2}{(k^2-A-i\epsilon)(k^2-D)}$$

We take g_0 such that $g(\mathbf{p}^2)=0$ for $\mathbf{p}^2 \simeq -m_\pi^2 \rightarrow |\mathbf{p}| \simeq i m_\pi$

$$g(\mathbf{p}^2) = g_0 - i\frac{m|\mathbf{p}|}{4\pi} \approx 0 \quad \longrightarrow \quad \text{natural value:} \;\; g_0 \simeq -\frac{m m_\pi}{4\pi} = -0.54\; m_\pi^2$$

g is treated as small along low-energy LHC, $\sim \mathcal{O}(p)$

An approximate algebraic solution for N_{JI} results in powers of g

Two expansions: The Chiral one and that in powers of g

We join them simultaneously, $g(\mathbf{p}^2) \simeq \mathcal{O}(p)$

LO: $|1 + gN_{JI}|^2 \longrightarrow 1$

$$N_{JI}^{(0)}(A) = N_{JI}(D) + \frac{A-D}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\mathrm{Im}T_{JI}(k^2)}{(k^2 - A - i\epsilon)(k^2 - D)}$$

$$N_{JI} \doteq N_{JI}^{(0)} + N_{JI}^{(1)} + \mathcal{O}(p^2)$$
$$T_{JI} \doteq L_{JI}^{(0)} + L_{JI}^{(1)} + \mathcal{O}(p^2)$$

$$\begin{aligned} T_{JI} &= N_{JI} - N_{JI} \cdot g \cdot N_{JI} + \ldots \\ &= N_{JI}^{(0)} + N_{JI}^{(1)} - N_{JI}^{(0)} \cdot g \cdot N_{JI}^{(0)} + \mathcal{O}(p^2) + \ldots \end{aligned}$$

$$N_{JI}^{(0)} = L_{JI}^{(0)} \quad , \quad N_{JI}^{(1)} = L_{JI}^{(1)} + N_{JI}^{(0)} \cdot g \cdot N_{JI}^{(0)}$$

We are providing approximate solutions to

$$T_{JI} = \left[N_{JI}^{-1} + g\right]^{-1} \leftrightarrow N_{JI} = T_{JI}\left|1 + gN_{JI}\right|^2 - \left|N_{JI}\right|^2 g^*$$

They coincide with those from the DR

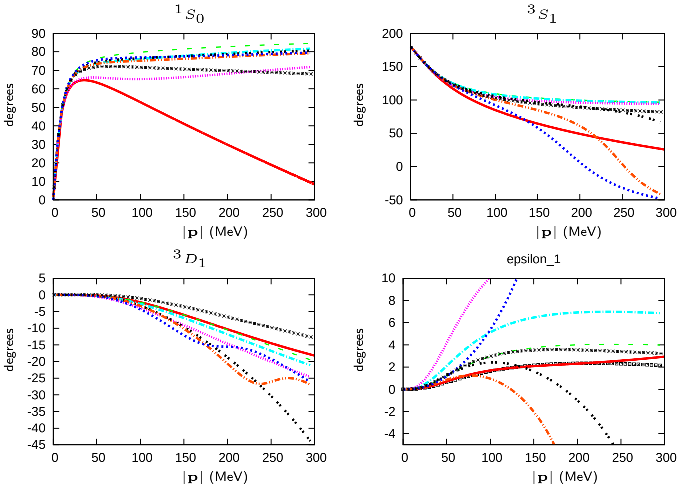
- $N_{JI}(A)$ only has LHC
- With the same discontinuity along the cut

Algebraic approximation. Chiral counterterms enter directly in N_{JI}

$$C_S = \frac{m}{16\pi}\frac{16\pi g_0/m + 3/a_s + 1/a_t}{(g_0 + m/(4\pi a_s))(g_0 + m/(4\pi a_t))}~,$$
$$C_T = \frac{m}{16\pi}\frac{1/a_s - 1/a_t}{(g_0 + m/(4\pi a_s))(g_0 + m/(4\pi a_t))}~.$$

$|g_0| \gg 1/a_t,~ |1/as| \longrightarrow |C_S| \sim 1/|g_0| \gg |C_T| = \mathcal{O}(m/16\pi a_t g_0^2)$
The $\mathcal{O}(p^0)$ counting for C_S, C_T is not spoiled by iterating them.

Green, Cyan, Magenta: LO
Blue, Orange, Black: NLO
Bursts: DR



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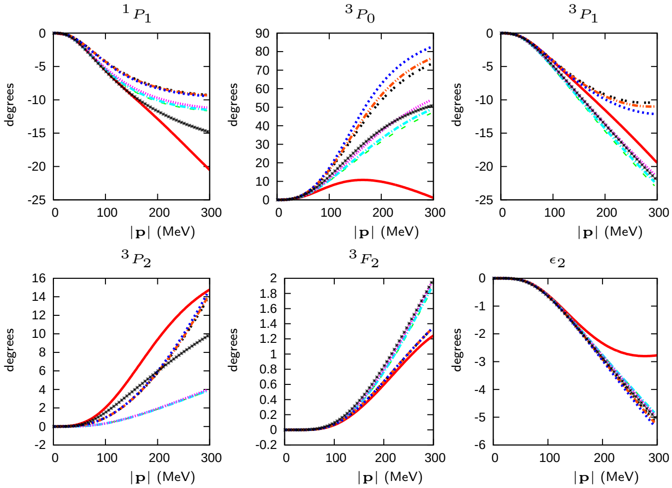

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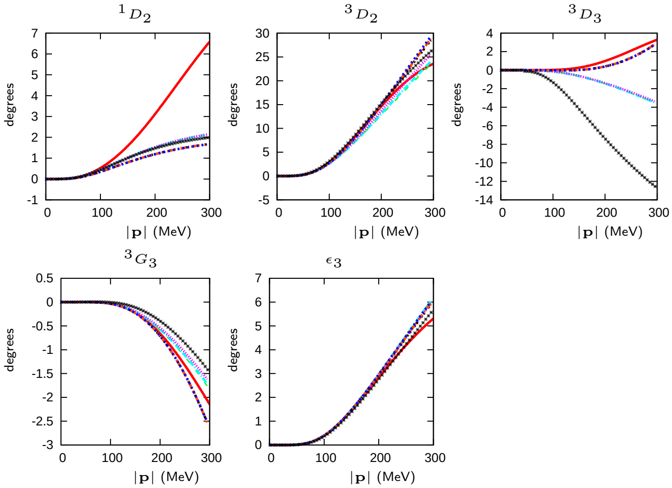
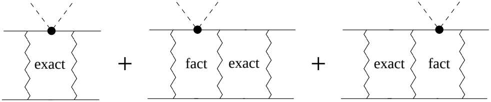


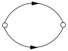
This formalism can also be applied to production diagrams
JAO, Oset NPA629(1998)739 , JAO PRD71(2005)054030

$$T_{JI} = D_{JI}^{-1} N_{JI} \rightarrow F_{JI} = D_{JI}^{-1} \cdot \xi_{JI} \quad , \quad D_{JI} = 1 + N_{JI} g \ ,$$

$$\xi_{JI} = \sum_{k=0}^{n} \xi_{JI}^{(k)}$$



$$\xi_{JI}^{(0)} + \xi_{JI}^{(1)} = DL_{JI}^{(1)} - \left\{ L_{JI}^{(1)} + {N_{JI}^{(0)}}^2 \cdot L_{10}, N_{JI}^{(0)} \right\} \cdot DL_{10}$$

In-medium unitarity loop  :

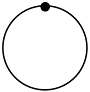
$$g \doteq L_{10,f} \quad \longrightarrow \quad L_{10} = L_{10,f} + L_{10,m}(\xi_1) + L_{10,m}(\xi_2) + L_{10,d}(\xi_1, \xi_2)$$
$$= L_{10,pp}(\xi_1, \xi_2) + L_{10,hh}(\xi_1, \xi_2)$$

Nuclear matter energy density - Contributions

$V_\rho = 1$

$\mathcal{O}(p^5)$

Leading Order

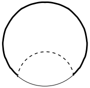


1

$V_\rho = 1$

$\mathcal{O}(p^6)$

Next-to-leading Order

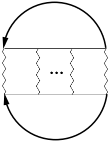


2

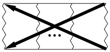
$V_\rho = 2$

$\mathcal{O}(p^6)$

Next-to-leading Order



3.1



3.2

Nuclear matter energy per particle

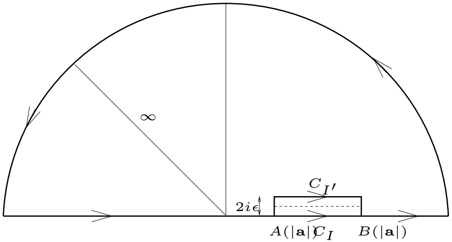
$$\mathcal{E}_3 = \frac{1}{2} \sum_{\sigma_1,\sigma_2} \sum_{\alpha_1,\alpha_2} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} e^{ik_1^0\eta} e^{ik_2^0\eta} G_0(k_1)_{\alpha_1} G_0(k_2)_{\alpha_2}$$
$$\times T_{NN}(k_1\sigma_1\alpha_1, k_2\sigma_2\alpha_2 | k_1\sigma_1\alpha_1, k_2\sigma_2\alpha_2) \ .$$

$a = \frac{1}{2}(k_1 + k_2) \quad , \quad p = \frac{1}{2}(k_1 - k_2)$

$$\int \frac{dp^0}{2\pi} G_0(a+p)_{\alpha_1} G_0(a-p)_{\alpha_2} = -i \left[\frac{\theta(|\mathbf{a}+\mathbf{p}| - \xi_{\alpha_1})\theta(|\mathbf{a}-\mathbf{p}| - \xi_{\alpha_2})}{2a^0 - E(\mathbf{a}+\mathbf{p}) - E(\mathbf{a}-\mathbf{p}) + i\epsilon} - \frac{\theta(\xi_{\alpha_1} - |\mathbf{a}+\mathbf{p}|)\theta(\xi_{\alpha_2} - |\mathbf{a}-\mathbf{p}|)}{2a^0 - E(\mathbf{a}+\mathbf{p}) - E(\mathbf{a}-\mathbf{p}) - i\epsilon} \right] \ .$$

$A = 2ma^0 - \mathbf{a}^2$

$$\mathcal{E}_3 = -4i \sum_{\sigma_1,\sigma_2} \sum_{\alpha_1,\alpha_2} \int \frac{d^3a}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{dA}{2\pi} e^{iA\eta}\, T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A) \left[\frac{1}{A-\mathbf{p}^2+i\epsilon} \right.$$
$$\left. - \frac{\theta(\xi_{\alpha_1} - |\mathbf{a}+\mathbf{p}|) + \theta(\xi_{\alpha_2} - |\mathbf{a}-\mathbf{p}|)}{A-\mathbf{p}^2+i\epsilon} - 2\pi i \delta(A-\mathbf{p}^2)\theta(\xi_{\alpha_1} - |\mathbf{a}+\mathbf{p}|)\theta(\xi_{\alpha_2} - |\mathbf{a}-\mathbf{p}|) \right] .$$



$$\int_{-\infty}^{+\infty} \frac{dA}{2\pi} \frac{e^{iA\eta}}{A-\mathbf{p}^2+i\epsilon} T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A) = \oint_{C_I} \frac{dA}{2\pi} \frac{e^{iA\eta}}{A-\mathbf{p}^2+i\epsilon} T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A)$$
$$- \oint_{C_{I'}} \frac{dA}{2\pi} \frac{e^{iA\eta}}{A-\mathbf{p}^2+i\epsilon} T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A)$$
$$= \int_{A(\alpha)}^{B(\alpha)} \frac{dA}{2\pi} \frac{T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A) - T^{\sigma_1\sigma_2}_{\alpha_1\alpha_2}(\mathbf{p},\mathbf{a};A+2i\epsilon)}{A-\mathbf{p}^2+i\epsilon} .$$

$$
\begin{aligned}
L_{10}^{i_3}(\mathbf{a}^2, A+2i\epsilon) - L_{10}^{i_3}(\mathbf{a}^2, A) &= -m\int \frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{a}+\mathbf{q}|)\theta(\xi_{\alpha_2} - |\mathbf{a}-\mathbf{q}|) \\
&\times \left(\frac{1}{A - \mathbf{q}^2 + i\epsilon} - \frac{1}{A - \mathbf{q}^2 - i\epsilon} \right) \\
&= i2\pi m \int \frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{a}+\mathbf{q}|)\theta(\xi_{\alpha_2} - |\mathbf{a}-\mathbf{q}|)\delta(A - \mathbf{q}^2) \ .
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}_3 = -4 \sum_{I,J,\ell,S} \sum_{i_3=\alpha_1+\alpha_2} (2J+1)\chi(S\ell I)^2 \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{q}|) \\
\times\, \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{q}|)\Big[T_{JI}^{i_3}\big|_{(\mathbf{q}^2,\mathbf{P}^2,\mathbf{q}^2)} \\
+\, m\int \frac{d^3p}{(2\pi)^3}\frac{1-\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{p}|) - \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{p}|)}{\mathbf{p}^2 - \mathbf{q}^2 - i\epsilon}\left|T_{JI}^{i_3}\right|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)}\Big]_{(\ell,\ell,S)}
\end{aligned}
$$

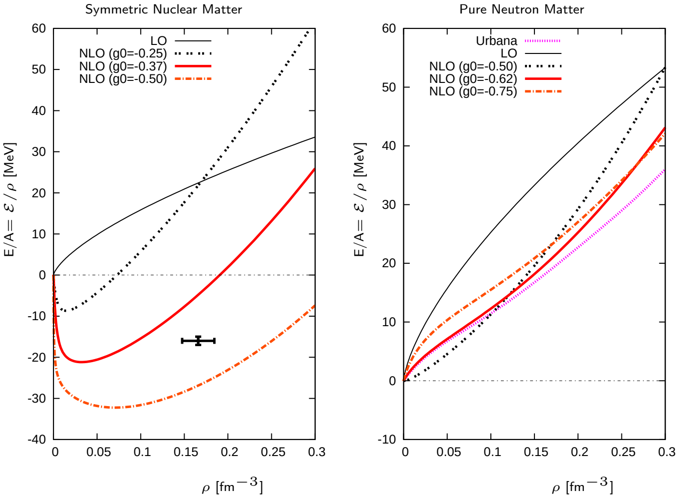
It is real because of unitarity and Pauli-exclusion principle, involving both terms between the square brackets.

$$m\int\frac{d^3p}{(2\pi)^3}\frac{1}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\left|T^{i_3}_{JI}\right|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)}$$

is divergent.
Expansion around $\mathbf{p}^2 \to \infty$.

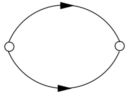
$$T^{i_3}_{JI\,(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} = T^{i_3}_{JI\,(+\infty,\mathbf{P}^2,\mathbf{q}^2)} + \mathcal{O}(|\mathbf{p}|^{-2})$$

$$m\int\frac{d^3p}{(2\pi)^3}\frac{1}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\left\{\left|T^{i_3}_{JI}\right|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)}-\left|T^{i_3}_{JI}\right|^2_{(+\infty,\mathbf{P}^2,\mathbf{q}^2)}\right\}$$
$$-\,g(\mathbf{q}^2)\left|T^{i_3}_{JI}\right|^2_{(+\infty,\mathbf{P}^2,\mathbf{q}^2)}$$

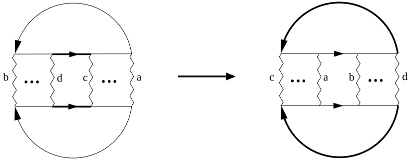


More stable for pure neutron matter (less dependent on g_0).

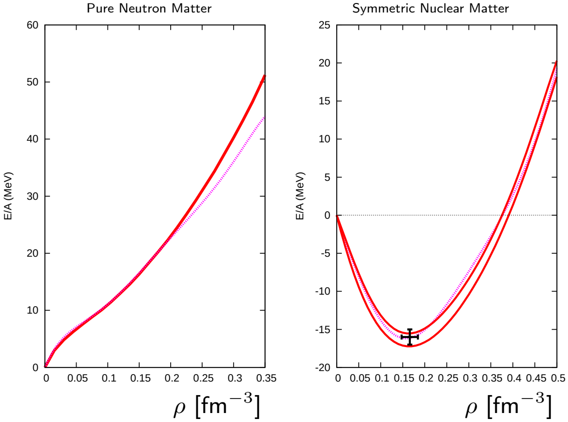
$$
\begin{aligned}
\mathcal{E}_3 = 4 \sum_{I,J,\ell,S} \sum_{i_3=\alpha_1+\alpha_2} & (2J+1)\chi(S\ell I)^2 \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{q}|) \\
& \times \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{q}|)\Bigg[-T_{JI}^{i_3}\Big|_{(\mathbf{q}^2,\mathbf{P}^2,\mathbf{q}^2)} \\
& + m\int \frac{d^3p}{(2\pi)^3}\frac{\theta(\xi_{\alpha_1} - |\mathbf{P}+\mathbf{p}|) + \theta(\xi_{\alpha_2} - |\mathbf{P}-\mathbf{p}|)}{\mathbf{p}^2 - \mathbf{q}^2 - i\epsilon}\left|T_{JI}^{i_3}\right|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} \\
& - m\int \frac{d^3p}{(2\pi)^3}\left\{ \frac{1}{\mathbf{p}^2-\mathbf{q}^2-i\epsilon}\left|T_{JI}^{i_3}\right|^2_{(\mathbf{p}^2,\mathbf{P}^2,\mathbf{q}^2)} - \frac{1}{\mathbf{p}^2}\left|T_{JI}^{i_3}\right|^2_{\mathbf{p}^2\to\infty}\right\} + \tilde{g}_0\left|T_{JI}^{i_3}\right|^2_{\mathbf{p}^2\to\infty}\Bigg]
\end{aligned}
$$



g_0 in NN scattering



$\tilde{g}_0$, particle-particle intermediate state



Akmal,Pandharipande,Ravenhall, PRC **58**(1998)1804

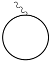
PNM: $g_0 = \tilde{g}_0 \simeq -0.6\ m_\pi^2$
SNM: $(g_0, \tilde{g}_0) \simeq (-1.0, -0.5)\ m_\pi^2$

$$K = \xi^2 \frac{\partial^2 \mathcal{E}/\rho}{\partial \xi^2}\bigg|_{\xi_0} = 240 - 260 \text{ MeV}$$

exp.250 ± 25 MeV

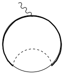
In-medium chiral quark condensate - Contributions

$V_\rho = 1$
$\mathcal{O}(p^5)$
Leading Order

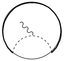


1

$V_\rho = 1$
$\mathcal{O}(p^6)$
Next-to-leading Order

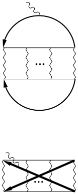


2

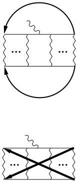


3

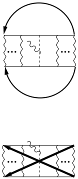
$V_\rho = 2$
$\mathcal{O}(p^6)$
Next-to-leading Order



4

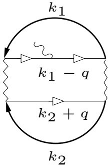


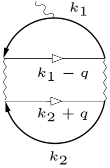
5



6

$$m_q\langle\Omega|\bar{q}_i q_j|\Omega\rangle = m_q\langle 0|\bar{q}_i q_j|0\rangle - m_q(\Xi_1 + \Xi_6)$$





$$\Xi_5^L = -\frac{1}{2}\sum_{\alpha_1,\alpha_2}\sum_{\sigma_1,\sigma_2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} e^{ik_1^0\eta} e^{ik_2^0\eta} G_0(k_1)_{\alpha_1} G_0(k_2)_{\alpha_2} \frac{\partial}{\partial k_1^0}\Big[i\sum_{\alpha_1',\alpha_2'}\int\frac{d^4k}{(2\pi)^4}$$

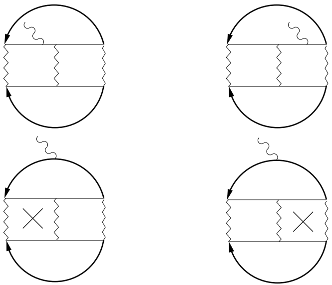
$$\times V_{\alpha_1\alpha_2;\alpha_1'\alpha_2'}(k) 2B\left[2c_1\delta_{ij} + c_5\tau^3_{ij}\tau^3_{\alpha_1'\alpha_1'}\right] G_0(k_1-q)_{\alpha_1'} G_0(k_2+q)_{\alpha_2'} V_{\alpha_1'\alpha_2';\alpha_1\alpha_2}(-k)\Big]$$

$$\Xi_4^L = \sum_{\alpha_1,\alpha_2}\sum_{\sigma_1,\sigma_2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} e^{ik_1^0\eta} e^{ik_2^0\eta} G_0(k_1)_{\alpha_1} 2B\left[2c_1\delta_{ij} + c_5\tau^3_{ij}\tau^3_{\alpha_1\alpha_1}\right] G_0(k_2)_{\alpha_2}$$

$$\times \frac{\partial}{\partial k_1^0}\Big[\frac{i}{2}\sum_{\alpha_1',\alpha_2'}\int\frac{d^4k}{(2\pi)^4} V_{\alpha_1\alpha_2;\alpha_1'\alpha_2'}(k) G_0(k_1-q)_{\alpha_1'} G_0(k_2+q)_{\alpha_2'} V_{\alpha_1'\alpha_2';\alpha_1\alpha_2}(-k)\Big]$$

For Ξ_5 the derivative acts directly in the scattering amplitude.
For Ξ_4 there is an integration by parts (extra sign).

This is a general argument following from the power counting



Cancellations happen explicitly for all orders in UχPT[3].

Feynman-Hellman theorem:

$$m_q \langle \Omega | \bar{q}_i q_j | \Omega \rangle - m_q \langle 0 | \bar{q}_i q_j | 0 \rangle = \frac{m_q}{2} \left(\delta_{ij} \frac{d}{d\hat{m}} + (\tau_3)_{ij} \frac{d}{d\bar{m}} \right) (\rho\, m + \mathcal{E}) \;,$$

$$\frac{\langle \Omega | \bar{q}_i q_j | \Omega \rangle}{\langle 0 | \bar{q}_i q_j | 0 \rangle} = 1 - \frac{\rho\, \sigma_N}{m_\pi^2 f_\pi^2} + \frac{2 c_5 (\tau_3)_{ij}}{f_\pi^2} (\rho_p - \rho_n) - \frac{1}{f_\pi^2} \frac{\partial \mathcal{E}(\rho, m_\pi)}{\partial m_\pi^2}$$

Long-range NN interactions dominate in the quark condensate calculation.

Kaiser, Homont, Weise PRC77(2008)025204; Plohl, Fuchs NPA798(2008)75

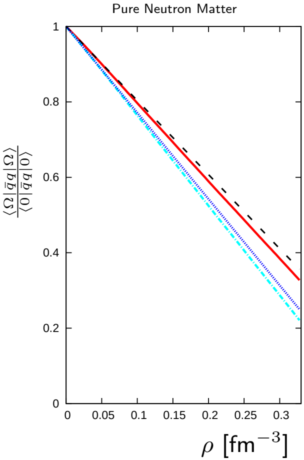
We offered an explanation for this observed fact:

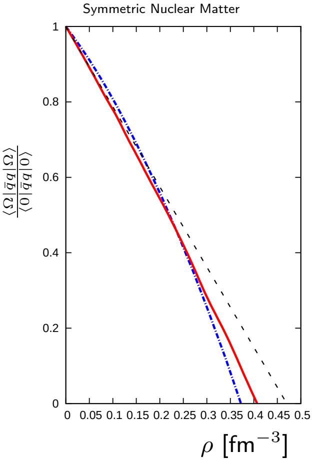
- The quark mass dependence of nucleon propagators cancels

$$\Xi_4 + \Xi_5 = 0$$

- The short distance part $|\mathbf{p}|^2 \to \infty$ cancels when taking the derivative

$$\frac{\partial \mathcal{E}(\rho, m_\pi)}{\partial m_\pi^2}$$





Left: $\langle\Omega|\bar{u}u|\Omega\rangle$ LO, NLO$(g_0 = -0.6m_\pi^2)$; $\langle\Omega|\bar{d}d|\Omega\rangle$ LO, NLO$(g_0 = -0.6m_\pi^2)$.

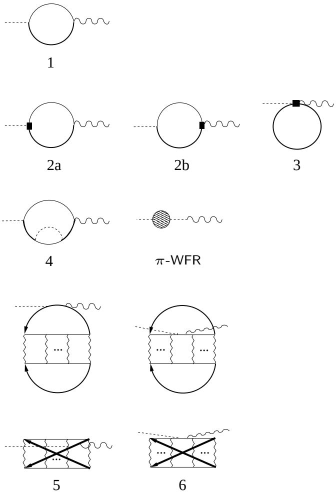
Right: $\langle\Omega|\bar{q}q|\Omega\rangle$ LO, NLO$(g_0 = -1.0m_\pi^2)$, NLO$(g_0 = -0.5m_\pi^2)$.
The quark condensate is independent of $\tilde{g}_0$

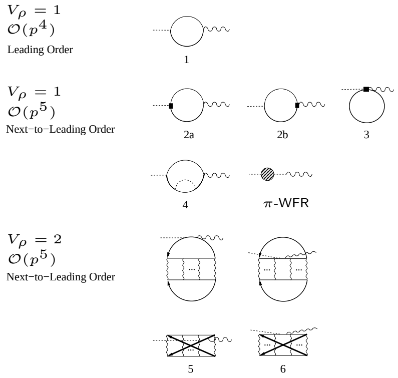
Contributions to the in-medium pion axial couplings

$V_\rho = 1$
$\mathcal{O}(p^4)$
Leading Order

$V_\rho = 1$
$\mathcal{O}(p^5)$
Next−to−Leading Order

$V_\rho = 2$
$\mathcal{O}(p^5)$
Next−to−Leading Order





Diagrams 1–3 discussed in Meißner, Oller, Wirzba ANP297(2002)27

Diagram with π-WFR also discussed there (NN interaction contributions cancel as shown in [1])

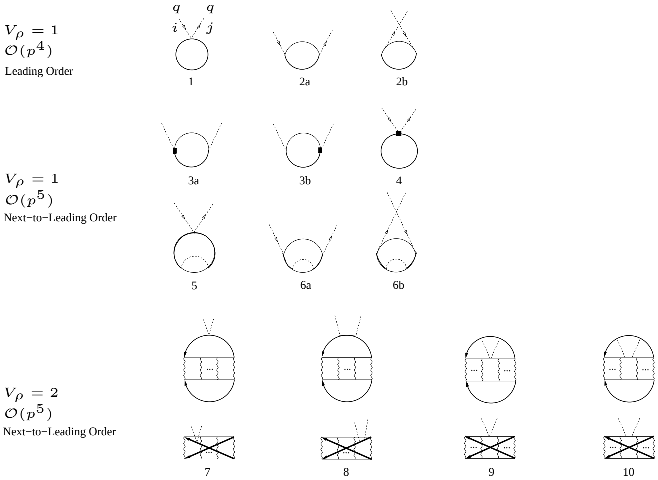
Diagram 4 is one order too high

Diagrams 5–6 mutually cancel

$$f_t = f_\pi \left\{ 1 - \frac{\rho}{\rho_0}(0.26 \pm 0.04) \right\}$$

$$f_s = f_\pi \left\{ 1 - \frac{\rho}{\rho_0}(1.23 \pm 0.07) \right\}$$

Contributions to the in-medium pion self-energy



NN-interactions cancel at $\mathcal{O}(p^5)$. Linear density approximation holds up to NLO

Application to neutron stars

Dobado, Llanes-Estrada, J.A.O., sent for publication to Phys. Rev. D

Spherical symmetry inside the star

Equation of Tolman-Oppenheimer-Vokoff for the hydrostatic equilibrium inside the star

Tolman, Phys. Rev. 55 (1939) 364

Oppenheimer, Volkoff, Phys. Rev. 55 (1939) 374

$$\frac{dP}{dr} = -\frac{G_N}{r^2} \frac{(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{1 - \frac{2G_N M(r)}{r}} \ .$$

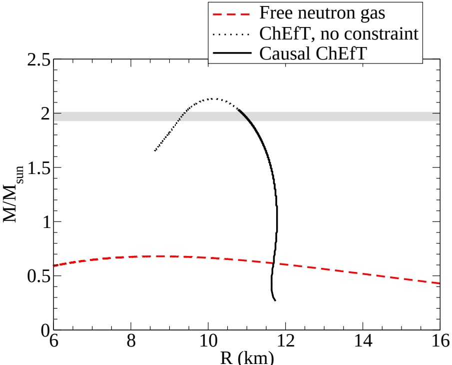
$M(r)$ is the mass accumulated at distance r
$\varepsilon(r)$ is the mass-energy density

Boundary conditions: $P(0)$, input ; $P(R) = 0$, with R the radius of the star

An equation of state is needed to relate $P(r)$ with $\varepsilon(r)$
This is provided by the chiral nuclear EFT.

Our study was triggered by the recent discovery of a neutron star with 1.97(4) solar masses.
Demorest *et al.* Nature 467 (2011) 1081
which is the new experimental upper bound for the mass of a neutron star

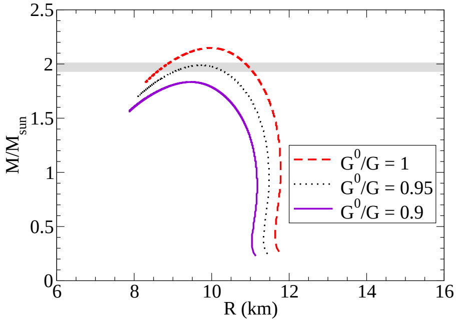
Does the chiral EFT support such a high mass for a neutron star?



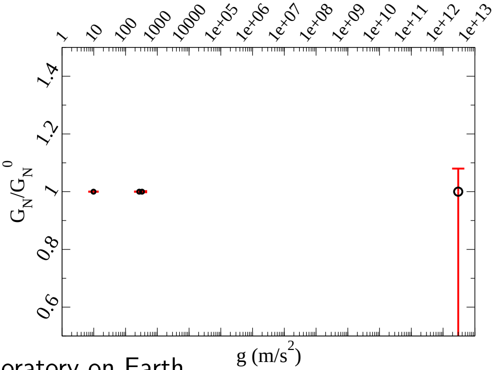
Upper limit on G in strong gravitational fields

We use the chiral EOS and the stiffest equation of state compatible with causality above $k_F \geq 600$ MeV

$$\partial P/\partial \rho = c$$
$$P = c^2(\rho - \rho_{max}) + P_{max}$$



The gravitational constant remains (so far) a constant.
Newton-Cavendish constant normalized by its accepted value $6.6738(8) N(m/kg)^2$.



Left point: laboratory on Earth.
Middle: orbital determinations of binary pulsars.
Right: existence of a neutron star with mass 1.97(4) solar masses.

At the intense gravitational field in such neutron star, the gravitational constant cannot exceed 8% of its value on Earth at 95% confidence level.

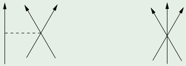
Summary & Outlook

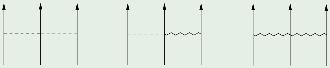
Summary

- It is developed a power counting scheme for nmEFT combining short- and long-range multi-N interactions
- LO Regulator independent NN partial-waves T_{JIS}.
- Nuclear matter energy density (up to NLO)
- In-medium chiral quark condensate (up to NLO)
- In-medium f_t, f_s (up to NLO)
- In-medium pion self-energy (up to NLO)
- EOS for neutron matter supports large mass for neutron star
- Quite good results at just NLO by applying non-perturbative methods of UχPT to NN-interactions

Outlook

- Exact solution of $N_{JI}(A)$
- $V_\rho = 3$ contributions, 3 nucleon force (N^{2}LO)
- Irreducible two-pion exchange (N^{3}LO)
- "Genuine" 3-nucleon force (N^{4}LO)





$d_2 + v_2 + w_2 - 2 = 2$ and $V_\rho = 3$
(instead of 0 and 1, respectively)

- Clarify the dependence on $\widetilde{g}_0$
- Neutron stars, supernovas, finite temperature, other N-point Green functions, adding strangeness...

Future perspectives?
Personal view:

- Follow a chiral power counting in nuclear medium systematically (this power counting should be valid also in vacuum).
- Do not regularize integrals with finite cut-offs (e.g. for the particle-particle parts in two-nucleon intermediate states.)
- This cut-off dependence should be replaced by subtraction constants (counterterms) of natural size for the low-energy regime at hand. $\Lambda \to m_\pi$.
- This is against standard “arguments” for particle-hole expansions.

Series of paper of the Munich group:
Kaiser, Mühlbauer, Weise EPJA 31(2007)53
Fritsch, Kaiser, Weise NPA 750(2005)259
Kaiser, Fritsch, Weise NPA 724(2003)47
Kaiser, Fritsch, Weise NPA 697(2002)255 ...

- Expansion in the number of loops (perturbative calculations).
- There is no chiral power counting.
- They always take the standard counting for the nucleon propagators $\sim \mathcal{O}(p^{-1})$.
  Infrared enhancements are not accounted for properly (They know and point out this in some of their works).
- No connection with vacuum NN scattering. Ad hoc cut-off parameter fitted to nucler matter properties.