Lectures on Nuclear and Hadron Physics

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Outline Lecture III

Introduction

1. The mesonic scalar sector has the vacuum quantum numbers $J^{PC} = 0^{++}$. Essential for the study of Spontaneous Breaking of Chiral Symmetry
2. The interactions with these quantum numbers are also essential for the masses of the lightest hadrons, the pseudo-Goldstone bosons (π, K, η), and ratio of light quark masses
   L. Roca, J.A.O. Eur. Phys. J. A **34** (2007) 371
3. In this sector meson interactions are really strong
   - Large unitarity loops (infrared enhancements and dynamical generation of resonances)
   - Channels that couple very strongly, e.g. $\pi\pi$-$K\bar{K}$, $\pi\eta$-$K\bar{K}$
   - Resonances with large widths and in coupled channels: NO Breit-Wigner form, NO Vector-Meson-Dominance, NO narrow-resonance approximation, etc
4. Okubo-Zweig-Izuka rule (large N_C QCD) has large corrections:

- NO ideal mixing multiplets
- Simple quark model fails
- Large unitarity loops

σ Resonance

- The "lightest" hadronic resonance in QCD

$\sqrt{s_\sigma}$ (MeV)	Reference
$469 - i\,203$	Oller, J.A.O. NPA620(1997)438; (E)NPA652(1999)307
$441^{+16}_{-8} - i\,271^{+9}_{-12}$	Caprini,Colangelo,Leutwyler PRL96(2006)132001
$484 \pm 14 - i\,255 \pm 10$	García-Martín, Peláez, Ynduráin PRD76(2007)074034
$456 \pm 6 - i\,241 \pm 7$	Albaladejo,J.A.O. PRL101(2008)252002
$440^{+3}_{-3} - i\,258^{+2}_{-3}$	Z.-H.Guo, J.A.O., PRD in press

Connection of the σ with chiral symmetry

The lowest order chiral perturbation theory Lagrangian

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger + U^\dagger \chi + \chi^\dagger U\right)$$
$$\chi = 2B_0(s + i\,p)$$

$$U = \exp\left\{i\sqrt{2}\Phi/F_\pi\right\}$$
$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

The lowest order $J^{PC} = 0^{++}$ $\pi\pi$ scattering amplitude
$\pi(p_1)\pi(p_2) \rightarrow \pi(p_3)\pi(p_4)$
Mandelstam variables: $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$

$$T_2 = \frac{s - m_\pi^2/2}{F_\pi^2}$$

It has a zero at $s = m_\pi^2/2$ (Adler zero)
For higher partial waves $\ell \geq 1$ the Adler zero is trivial. It is located at threshold.

Coming back to the N/D method without LHC:

$$T = \left[\sum_i \frac{\gamma_i}{s - s_i} + g(s) \right]^{-1}$$

To reproduce the Adler zero ONE CDD is needed

$$\gamma_1 = F_\pi^2$$
$$s_1 = m_\pi^2/2$$

$g(s)$ is the unitarity loop function

$$g(s) = g(s_0) - \int_{4m_\pi^2}^{\infty} ds' \frac{\rho(s')}{(s'-s_0)(s'-s-i\epsilon)}$$
$$= \frac{1}{(4\pi)^2}\left[a(\mu) + \log\frac{m_\pi^2}{\mu^2} + \sigma(s)\log\frac{\sigma(s)+1}{\sigma(s)-1}\right]$$

Fitting $\pi\pi$ phase shifts

$$a(M_\rho) = -0.79$$

$$T = \left[\frac{1}{T_2} + g(s)\right]^{-1} = \frac{T_2}{1+T_2 g(s)} = T_2 - T_2 g(s) T_2 + \dots$$

On the natural size for $a(\mu)$: First Unitarity Correction

$$\frac{T_2^2 g(s)}{T_2} \sim \frac{a}{16\pi^2}\frac{s-m_\pi^2/2}{F_\pi^2} \sim a\frac{s}{16\pi^2 F_\pi^2}$$

$|a|$ must be $\mathcal{O}(1)$. Dynamical Generated Resonance

Pole position of the σ

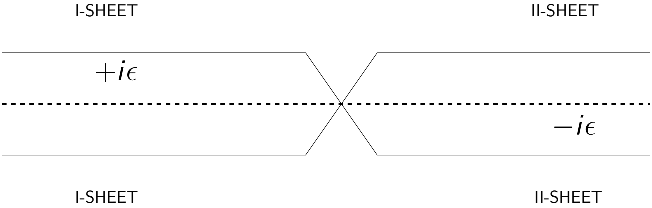
• Physical amplitudes have no pole positions in the complex s plane with imaginary part because hermiticity would be violated
The eigenvalues of a Hermitian operator like the Hamiltonian must be real.
The pole positions for physical amplitudes should be real (bound states)
K. Gottfried, Quantum Mechanics, W.A. Benjamin

• Scattering amplitudes can be extended to unphysical Riemann sheets.
There one can find poles corresponding to resonances or antibound states

Multi-valued function:

$$p = \sqrt{\frac{s}{4} - m_\pi^2}$$

Physical sheet:	$\Im\|\mathbf{p}(s)\| \geq 0$
Second Riemann sheet:	$\Im\|\mathbf{p}(s)\| \leq 0$



$$\Im g(s) = -\frac{|\mathbf{p}|(s)}{8\pi\sqrt{s}} \rightarrow \Im g_{II}(s) = \frac{|\mathbf{p}|(s)}{8\pi\sqrt{s}}$$

Schwartz's reflection principle:

$$g(s+i\epsilon) - g(s-i\epsilon) = 2i\Im g(s+i\epsilon) = -i\frac{|\mathbf{p}|}{4\pi\sqrt{s}}$$

$$g(s+i\epsilon) - g_{II}(s+i\epsilon) = -i\frac{|\mathbf{p}|}{4\pi\sqrt{s}}$$

$$g_{II}(s) = g(s) + i\frac{|\mathbf{p}|}{4\pi\sqrt{s}}$$

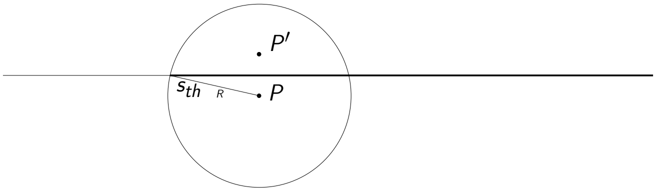
$$T_{II}(s) = \left[T_2^{-1} + g_{II}(s)\right]^{-1}$$

$$T_{II}(s)^{-1} = T(s)^{-1} + i\frac{|\mathbf{p}|}{4\pi\sqrt{s}}$$

The σ pole position is located in $T_{II}(s)$
Because $T_{II}(s^*) = T_{II}(s)^*$ there are indeed two poles at s_σ and s_σ^*

Laurent series

Physical sheet, s variable

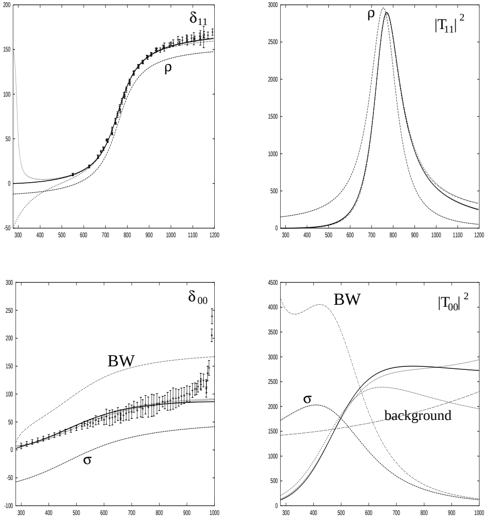


Because $T_{II}(s - i\epsilon) = T(s + i\epsilon)$ the pole P (with negative imaginary part) is effectively much closer to the physical region than the pole P' (with positive imaginary part)

$$T_{II}(s) = \underbrace{\frac{\gamma_0^2}{s - s_P}}_{pole} + \underbrace{\gamma_1 + \gamma_2(s - s_P)}_{non-resonant\ \ terms} + \ldots$$

The reason why the σ has been traditionally hard to establish experimentally is due to the large contribution of non-resonant terms. Due to the need of cancellation for having the Adler zero.

$$\begin{aligned}
s_\sigma &= (0.466 - i\,0.224)^2 \text{ GeV}^2 \\
\gamma_0^2 &= 5.3 + i\,7.7 \text{ GeV}^2 \\
\gamma_1 &= -8.1 + i\,36.9 \\
\gamma_2 &= 1.1 + i\,0.1 \text{ GeV}^2
\end{aligned}$$


δ11
ρ
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background

Breit Wigner formula

$$T \propto e^{i\delta} \sin\delta = \frac{\sin\delta}{\cos\delta - i\,\sin\delta} = \frac{1}{\cot\delta - i}$$

The partial wave T has a maximum at the resonance position, so that $\sin\delta = \pm 1 \rightarrow \cot\delta = 0$

$$\cot\delta = \cot\delta(E_R) + (E - E_R)(d\cot\delta/dE)|_{E=E_R}$$
$$(d\cot\delta/dE)|_{E=E_R} = -2/\Gamma$$

$$\frac{1}{\cot\delta - i} = \frac{1}{-2(E-E_R)/\Gamma - i} = \frac{-\Gamma/2}{E - E_R + i\Gamma/2}$$

If $\Gamma \ll E_R$ then it is a good approximation

$$E + E_R \simeq 2E_R, \quad s - s_R \simeq (E - E_R)2E_R$$

$$\frac{-\Gamma/2}{E - E_R + i\Gamma/2} \simeq \frac{-\Gamma E_R}{s - s_R + i\Gamma E_R} \qquad s_R = E_R^2$$

This is a Breit-Wigner (BW) formula for the amplitude

It is applied in many phenomenological studies to reproduce data

BW formula does not work well when typically the resonance is very wide (like the σ) or it is located just on top of a threshold, even more when several channels couple

BW works very bad for the σ indeed.

The lightest preexisting resonance $\rho(770)$

$M_\rho = 770$ MeV, $\Gamma_\rho = 150$ MeV

N/D method without LHC:

$$T = \left[\sum_i \frac{\gamma_i}{s - s_i} + g(s) \right]^{-1}$$

To reproduce the (Adler) zero at threshold ONE CDD is needed

$$\gamma_1 = 6F_\pi^2$$
$$s_1 = 4m_\pi^2$$
$$a = -\frac{6F_\pi^2}{M_\rho^2} 16\pi^2 \simeq -14.2$$

$|a| \gg 1$ Preexisting Resonance ($q\bar{q}$ resonance)

Another argument to appreciate the difference:

$$a + \log \frac{m_\pi^2}{\mu^2} = \frac{m_\pi^2}{\bar{\mu}^2} \rightarrow \bar{\mu} = \mu \exp(-a/2)$$

with $\mu \sim 800$ MeV

$$\bar{\mu}_\sigma = 1.76 \text{ GeV}$$
$$\bar{\mu}_\rho = 970 \text{ GeV} \sim 1 \text{ TeV}$$

To be compared with the natural scale $\Lambda_{\chi PT} = 4\pi F_\pi \sim 1.2$ GeV

- The ρ pole dominates over the background in the Laurent expansion
- The BW formula for the ρ compares very well with the full result

Pole:

$$\frac{\gamma_R^2}{s - s_R}$$

Its square root gives the coupling of the resonance to the $\pi\pi$ state
$|g_{\sigma\to\pi\pi}| = 3.1$ GeV
$|g_{\rho\to\pi\pi}| = 2.5$ GeV

Width in terms of the coupling:

$$T = -\frac{g^2}{s - M_R^2 + iM_R\Gamma_R}$$

Unitarity implies $\Im T^{-1} = -|\mathbf{p}|/8\pi\sqrt{s}$ at $s = M_R^2$

$$\Gamma_R = \frac{g^2|\mathbf{p}|}{8\pi M_R^2}$$

Including other cuts in the *algebraic* N/D method

$$T = \left[\sum_{i=1} \frac{\gamma_i}{s - s_i} + g(s) \right]^{-1} \equiv \left[\mathcal{R}^{-1} + g(s) \right]^{-1}$$

A CDD at infinity corresponds to a constant

$$\lim_{s_i \to \infty} \frac{\gamma_i}{s_i} = \text{ constant}$$

The crossed cuts (LHC for equal mass scattering) are included perturbatively in $\mathcal{R}$

This is accomplished by matching order by order with T calculated from the EFT, T_χ
U.-G. Meißner, J.A.O, Phys. Lett. B **500** (2001) 263

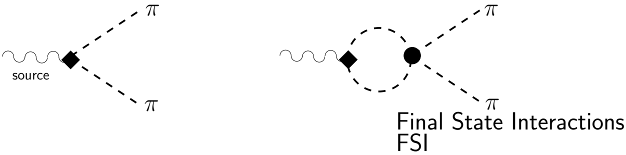
$$T_\chi = T_2 + T_4 + \ldots = \left[1 + \mathcal{R}g(s)\right]^{-1} \mathcal{R} = \mathcal{R}_2 + \mathcal{R}_4 - \mathcal{R}_2 g \mathcal{R}_2 + \ldots$$

$$\begin{aligned} \mathcal{R}_2 &= T_2 \\ \mathcal{R}_4 &= T_4 + \mathcal{R}_2 g \mathcal{R}_2 \end{aligned}$$

and son on for higher orders

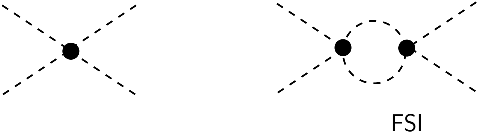
The resulting $\mathcal{R}$ has no RHC (that arises form $g(s)$)

Production Process. E.g. Form factors



- ◆ Production vertex
- ● Full strong interaction vertex

Analogy with the scattering process



$$T = [1 + \mathcal{R}g]^{-1}\mathcal{R} \rightarrow F = [1 + \mathcal{R}g]^{-1}\mathcal{F}$$

Unitarity and analyticity can be used to obtain a more general derivation of this result

Oset, Palomar, J.A.O., Phys. Rev. D **63** (2001) 114009

1. $\mathcal{R}$ is known form the studies on the strong interactions
2. $\mathcal{F}$ is fixed by matching with F calculated up to a given order in the EFT

Pion and Kaon vector Form Factors

Strong amplitude: $J^{PC} = 1^{--}$

Matching with EFT:= CHPT+Resonances at Leading Order

Ecker, Gasser, Pich, de Rafael, Nucl. Phys. B **321** (1989) 311

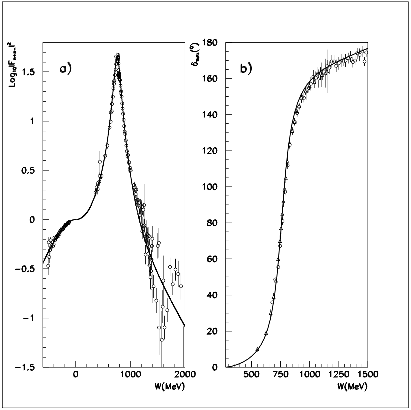
$$\mathcal{R}_2 = T^{LO}_{EFT}$$

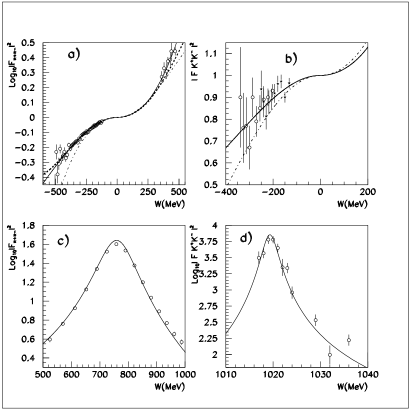
$$T^{LO}_{EFT}\Big|_{11} = \frac{2|\mathbf{p}|^2}{3F_\pi^2}\left[1 + \frac{2G_V^2}{F_\pi^2}\frac{s}{M_\rho^2 - s}\right]$$

Form Factors:
Matching with NLO EFT result on Form Factors (CHPT+Resonances)

$$\mathcal{F}_1 = 1 + \frac{F_V G_V}{F_\pi^2} \frac{s}{M_\rho^2 - s}$$

High energy QCD constraints were also imposed so that the subtraction constants are determined (they are introduced from the strong interactions)


a)
b)
W(MeV)
W(MeV)


a)
b)
c)
d)
W(MeV)
W(MeV)
W(MeV)
W(MeV)

Pion and kaon Scalar Form Factors

U.-G. Meißner, J.A.O., Nucl. Phys. A **679** (2001) 671
Strong amplitude: $J^{PC} = 0^{++}$
Matching with EFT:= CHPT at Leading Order
Gasser, Leutwyler, Nucl. Phys. B **250** (1985) 465

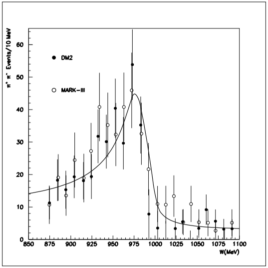
$$\mathcal{R}_2 = T^{LO}_{EFT}$$
$$\mathcal{R}_{11} = \frac{s - m_\pi^2/2}{F_\pi^2}$$

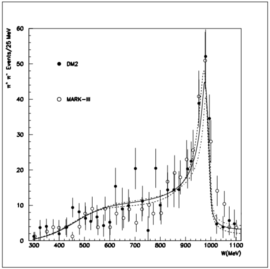
Form Factors

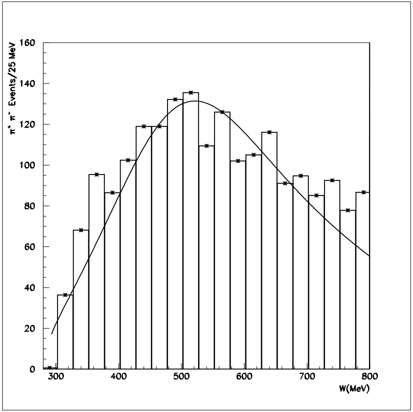
$$\langle 0|\bar{s}s|MM'\rangle \quad , \quad \langle 0|\bar{n}n|MM'\rangle$$
$$\bar{n}n = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$\mathcal{F}_1^{\bar{n}n} = \sqrt{\frac{3}{2}}\left\{1 + \frac{4(L_5+2L_4)}{F_\pi^2}s + \frac{16(2L_8-L_5)}{F_\pi^2}m_\pi^2 \right.$$
$$\left. + \frac{8(2L_6-L_4)}{F_\pi^2}(2m_K^2+3m_\pi^2) - \frac{m_\pi^2}{32\pi^2F_\pi^2} - \frac{1}{3}\mu_n \right\}$$

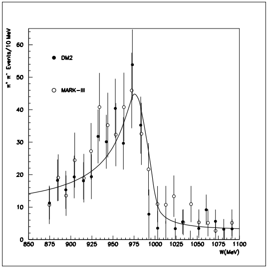
L_i: NLO CHPT Counterterms

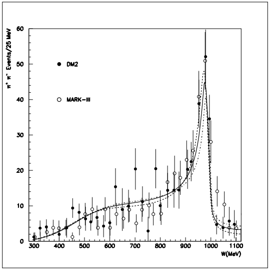


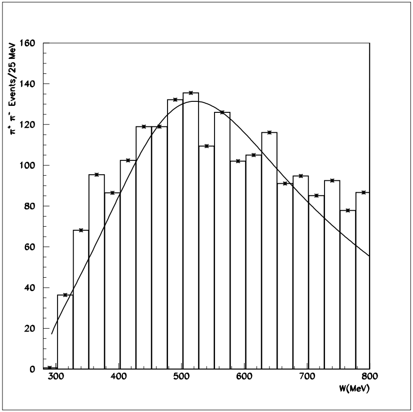




$J/\Psi \to \phi(w)\pi\pi$







$$J/\Psi \to \phi(w)\pi\pi$$

T.A. Lahde, U.-G. Meißner, Phys. Rev. D **74** (2006) 034021

L. Roca, J.E. Palomar, E. Oset, Nucl. Phys. A **744** (2004) 127

The Inverse Amplitude Method (IAM) with Coupled Channels

Oset, Peláez, J.A.O., Phys. Rev. Lett. 80 (1998) 3452

• The IAM was introduced originally for the elastic case (only one channel) in

Truong, Phys. Rev. Lett. 61 (1998) 2526
Dobado, Herrero, Truong Phys. Lett. B 235 (1990) 129
Dobado, Peláez, Phys. Rev. D 56 (1997) 3057

• Its generalization to coupled channels was given in

Oset, Peláez, J.A.O., Phys. Rev. Lett. 95 (2005) 172502
Oset, Peláez, J.A.O., Phys. Rev. D 59 (1999) 074001

- From the CHPT expansion up to NLO of T we work out the expansion of T^{-1}
- It reminds us of the *effective range approximation* of Quantum Mechanics

- In this way if the amplitude has a pole its inverse has only a zero
- The number of free parameter is the same as in CHPT at NLO: L_1, L_2, L_3, L_4, L_5, L_7, $2L_6 + L_8$ fitted to data for $\sqrt{s} \leq 1.2$ GeV

Explicit matrix language: Coupled Channels T_{ij} corresponds to $i \to j$

$$\Im T^{-1} = -\rho$$

$$\rho_i = \frac{|\mathbf{p}|_i}{8\pi\sqrt{s}}\theta(s - s_i)$$

$$T^{-1} = \Re T^{-1} - i\rho_i$$

$$T^{-1} = [T_2 + T_4]^{-1} = T_2^{-1} \cdot \left[I + T_4 \cdot T_2^{-1}\right]^{-1}$$

$$= T_2^{-1} \cdot \left[I - T_4 \cdot T_2^{-1}\right] = T_2^{-1} \cdot [T_2 - T_4] \cdot T_2^{-1}$$

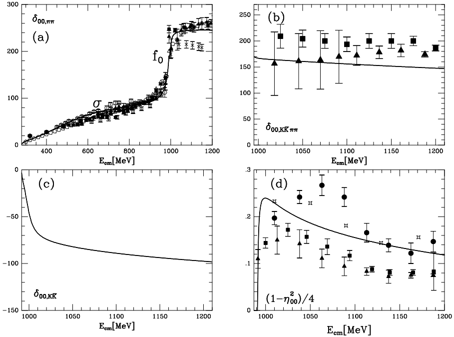


Figure: $L = 0$, $I = 0$

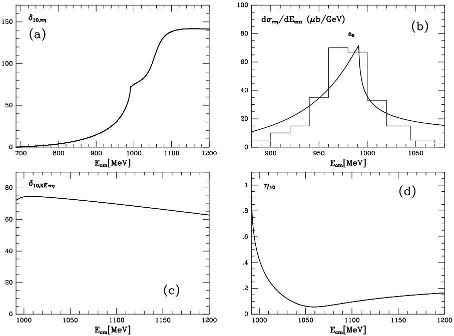


Figure: $L = 0$, $I = 1$

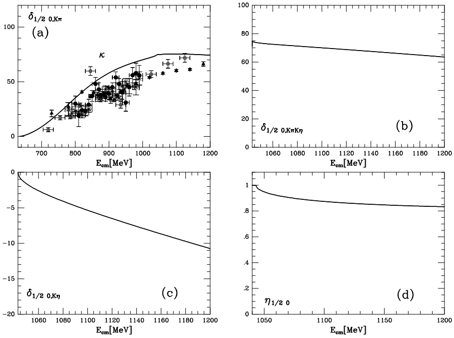


Figure: $L = 0$, $I = 1/2$

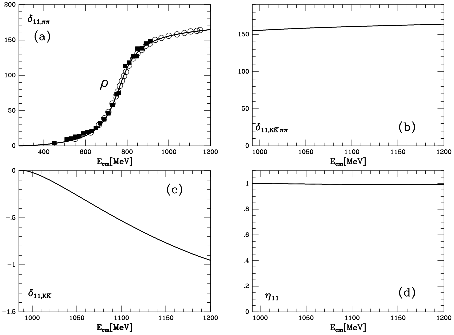


Figure: $L = 1$, $I = 1$

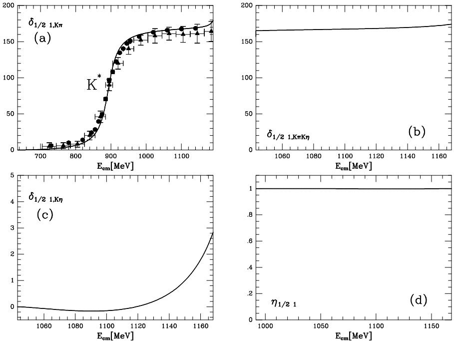


Figure: $L = 1$, $I = 1/2$

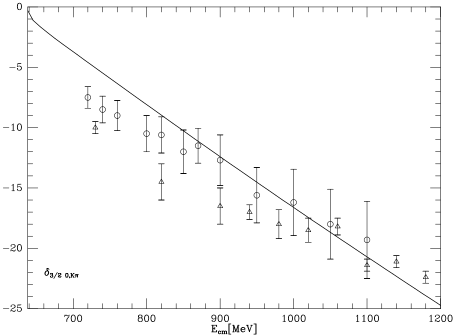


Figure: $L = 0$, $I = 3/2$

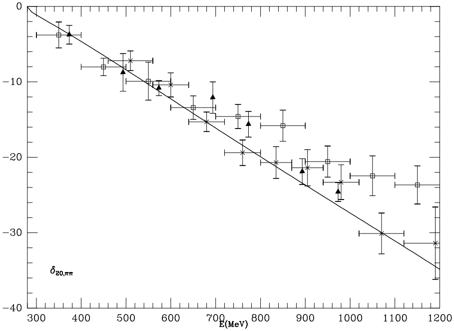


Figure: $L = 0$, $I = 2$

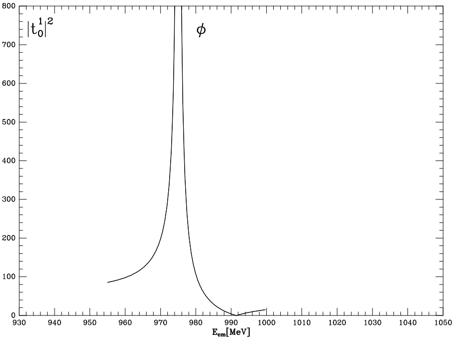


Figure: $L = 1$, $I = 0$

Resonances

Table III. Masses and partial widths in MeV

Channel (I, J)	Resonance	Mass from pole	Width from pole	Mass effective	Width effective	Partial Widths
$(0,0)$	σ	442	454	≈ 600	*very large*	$\pi\pi$ – 100%
$(0,0)$	$f_0(980)$	994	28	≈ 980	≈ 30	$\pi\pi$ – 65% $K\bar{K}$ – 35%
$(0,1)$	$\phi(1020)$	980	0	980	0	
$(1/2,0)$	κ	770	500	≈ 850	*very large*	$K\pi$ – 100%
$(1/2,1)$	$K^*(890)$	892	42	895	42	$K\pi$ – 100%
$(1,0)$	$a_0(980)$	1055	42	980	40	$\pi\eta$ – 50% $K\bar{K}$ – 50%
$(1,1)$	$\rho(770)$	759	141	771	147	$\pi\pi$ – 100%

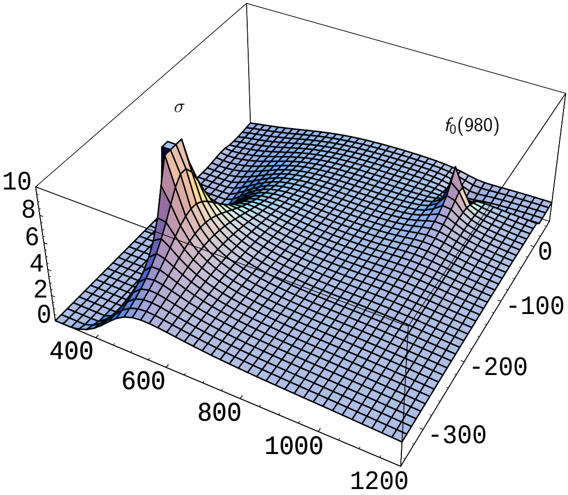


Figure: σ, $f_0(980)$

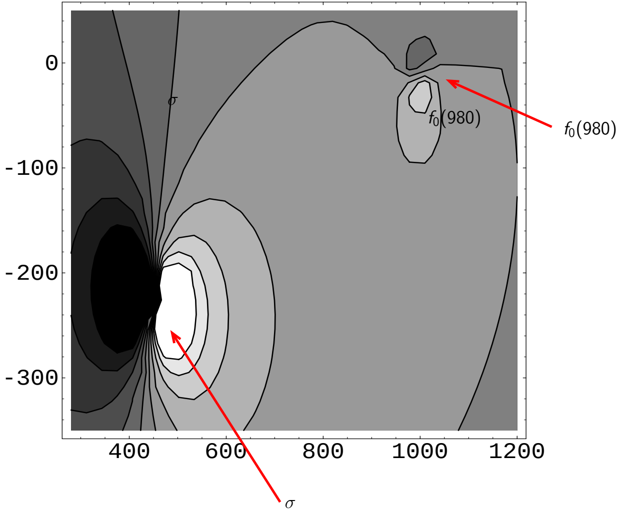


Figure: σ, $f_0(980)$

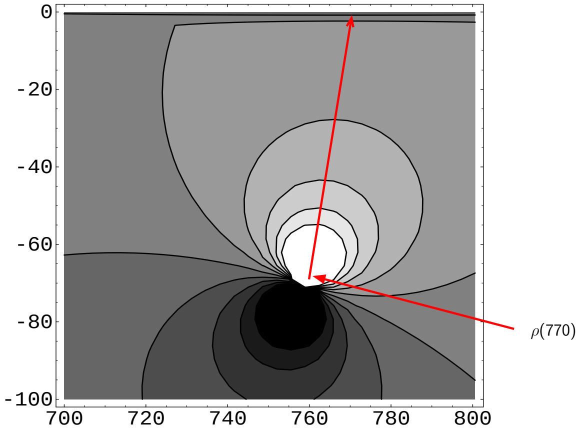


Figure: $\rho(770)$

The gradient is oriented towards higher values of the mass than the pole position

Mixing angle of the lightest scalar nonet

SU(3) analysis

1961 Gell-Mann and Ne'eman introduced a classification of hadrons based on SU(3) symmetry

SU(3) is the set of unitary transformations with determinant 1 in three dimensions acting on the quark flavors or species

$$q(x) = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

For standard rotations one has multiplets like scalars, vectors, axials, etc

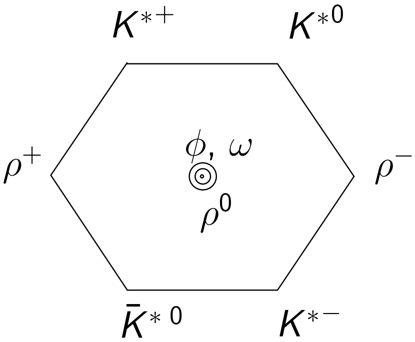
The QCD Lagrangian is invariant under SU(3) for equal quark masses

Different quark masses break SU(3) explicitly

$$\bar{q}(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q(x)$$

$m_s \gg m_{u,d}$
As a result SU(3) multiplets or irreducible representation mix

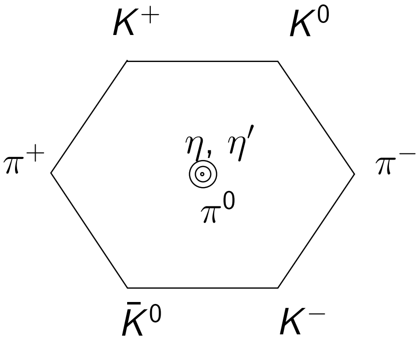
Well established octet+singlet of vector resonances. $I = 0$ resonances mix.



Ideal Mixing:

$$\phi = \bar{s}s \quad , \quad \omega = \frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)$$

Well established octet+singlet of lightest pseudoscalars. $I = 0$ states mix.



η and η' mix

$$\eta = \cos\theta_P\,\eta_8 - \sin\theta_P\,\eta_1$$
$$\eta' = \sin\theta_P\,\eta_8 + \cos\theta_P s\,\eta_1$$

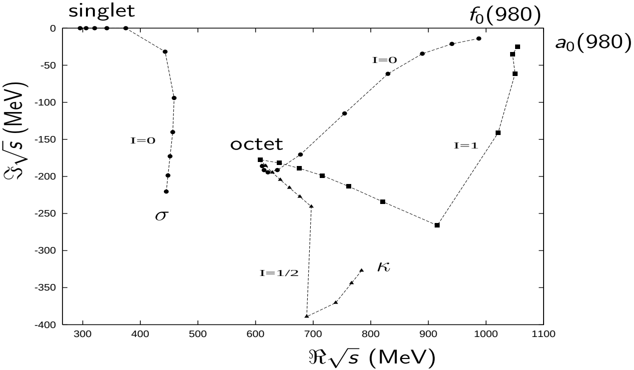
$\theta_p \sim -(15-20)^\circ$

What about the scalar resonances?

Mixing of the σ and $f_0(980)$

J.A.O. Nucl. Phys. A 727 (2003) 353

Continuous movement from an SU(3) symmetric point with equal masses m_0: $m_i(\lambda) = m_i + \lambda(m_0 - m_i)$ with $\lambda \in [0, 1]$



$a_0(980)$, κ: Pure octet states

Clebsch-Gordan decomposition:

$$g(a_0 \to [K\bar{K}]_{I=1}) = -\sqrt{\frac{3}{10}}g_8 \quad , \quad g(a_0 \to \pi\eta) = \frac{1}{\sqrt{5}}g_8$$

$$g(\kappa \to K\pi) = \frac{3}{\sqrt{20}}g_8 \quad , \quad g(\kappa \to K\eta) = \frac{1}{\sqrt{20}}g_8$$

Fraction	CG	Exp
$\frac{g(a_0 \to \pi\eta)}{g(a_0 \to [K\bar{K}]_1)}$	0.82	0.70 ± 0.04
$\frac{g(a_0 \to [K\bar{K}]_1]}{g(a_0 \to [K\pi]_{1/2})}$	0.82	1.10 ± 0.03

$$\frac{g(\kappa \rightarrow [K\pi]_{1/2})}{g(\kappa \rightarrow K\eta)} \quad 3 \quad 2.5 \pm 0.3$$

$$|g_8| = 8.7 \pm 1.3 \text{ GeV}$$

σ, $f_0(980)$ isosinglet states: Mixing

$$\sigma = \cos\theta S_1 + \sin\theta S_8$$
$$f_0 = -\sin\theta S_1 + \cos\theta S_8$$

Ideal Mixing: $\cos\theta^2 = 2/3 = 0.67$
No consensus in the literature

$$g(\sigma \to (\pi\pi)_0) = -\frac{\sqrt{3}}{4}\cos\theta g_1 - \sqrt{\frac{3}{10}}\sin\theta g_8 \ ,$$

$$g(\sigma \to (K\bar{K})_0) = -\frac{1}{2}\cos\theta g_1 + \frac{1}{\sqrt{10}}\sin\theta g_8 \ ,$$

$$g(\sigma \to (\eta_8\eta_8)_0) = \frac{1}{4}\cos\theta g_1 - \frac{1}{\sqrt{10}}\sin\theta g_8 \ ,$$

$$g(f_0 \to (\pi\pi)_0) = \frac{\sqrt{3}}{4}\sin\theta g_1 - \sqrt{\frac{3}{10}}\cos\theta g_8 \ ,$$

$$g(f_0 \to (K\bar{K})_0) = \frac{1}{2}\sin\theta g_1 + \frac{1}{\sqrt{10}}\cos\theta g_8 \ ,$$

$$g(f_0 \to (\eta_8\eta_8)_0) = -\frac{1}{4}\sin\theta g_1 - \frac{1}{\sqrt{10}}\cos\theta g_8 \ .$$

$$\frac{g_1}{g_8} = \sqrt{\frac{8}{5}} \tan\theta$$

$$\begin{aligned} \theta &= (+19 \pm 5)^\circ \\ g_8 &= (8.2 \pm 0.8)\ \text{GeV} \\ g_1 &= (3.9 \pm 0.8)\ \text{GeV} \end{aligned}$$

The sign of θ can be fixed from the peak value at the $f_0(980)$ resonance of the scalar form factors

$$\begin{aligned} f_n &= \langle 0 | \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) | K\bar{K} \rangle_0 \\ f_s &= \langle 0 | \bar{s}s | K\bar{K} \rangle_0 \\ |f_s/f_n| &\gg 1 \end{aligned}$$

Large N_C insight on the σ

QCD is a non-Abelian gauge QFT under SU(3)-color N_C

In real life $N_C = 3$, $q(x)_i \quad , \quad i = 1, \ldots, 3$

$$D_\mu = \partial_\mu + i \frac{g}{\sqrt{N_C}} A_\mu$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i \frac{g}{\sqrt{N_C}} [A_\mu, A_\nu]$$

$$\mathcal{L}_{QCD} = -\frac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \sum_{k=1}^{N_F} \bar{\psi}_k (i \not{D} - m_k) \psi_k$$

A non-trivial $N_C \to \infty$ limit exists:
G.'t Hooft, Nucl. Phys. B 72 (1974) 461; B 75 (1974) 461
E. Witten, Nucl. Phys. B 160 (1979) 57

The planar diagrams of QCD survive the large N_C limit. All the others are suppressed in large N_C

β-function of QCD

$$\mu \frac{dg}{d\mu} = -\left(\frac{11}{3} - \frac{2}{3}\frac{N_F}{N_C}\right)\frac{g^3}{16\pi^2} + \mathcal{O}(g^5)$$

It has a well defined limit for $N_C \to \infty$
Quark loops do not contribute $N_F/N_C \to 0$ because insertion of quark lines are suppressed in large N_C

$F_\pi \propto \sqrt{N_c}$

$$\langle 0| \sum_i^{N_c} \bar{q}_i \gamma^\mu \gamma_5 \lambda^a q_i |\pi^b(p)\rangle = i\,\delta^{ab} F_\pi\, p^\mu$$

The operator has N_c terms
A pion has the factor $1/\sqrt{N_c}$ because of normalization
$\sum_{i=1}^{N_c} \bar{q}_i q_i/\sqrt{N_c}$

σ from LO Unitarized CHPT Oset,J.A.O. Phys. Rev. D 60 (1999) 074023

$$\frac{F_\pi^2}{M_\sigma^2} + g_{II}(M_\sigma^2) = 0$$
$$M_\sigma^2 \propto F_\pi^2/g_{II}(M_\sigma^2) \sim N_c$$

According to the large N_C rules for a $\bar{q}q$ meson:

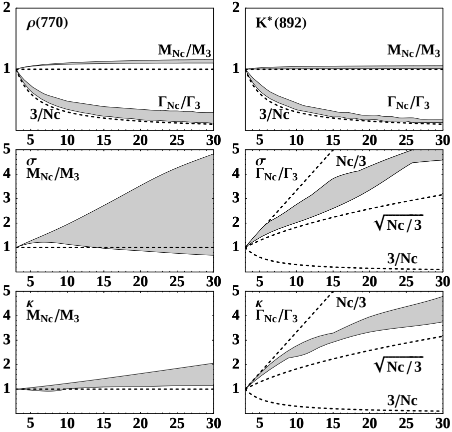
$$M \sim \mathcal{O}(N_C^0) \quad , \quad \Gamma \sim 1/N_C$$

which is not fulfilled by the σ ($a_0(980)$, κ) resonance.

Employing NLO CHPT

IAM results J.R.Peláez PRL 92(2004)102001

IAM results J.R.Peláez PRL 92(2004)102001



ρ, $K^*(892)$ standard $\bar{q}q$ resonances

σ does not correspond to such behavior

The QCD Lagrangian is invariant under $U(3)_L \otimes U(3)_R$ of quark flavors

If $U_A(1)$ were spontaneously broken (like $SU(3)_A$) then there should be an η_0 with a mass $< \sqrt{3} m_\pi$ Weinberg PRD'75 but the η is much heavier

The ninth axial singlet current has an anomalous divergence Adler PR'69 Fujikawa PRD'80

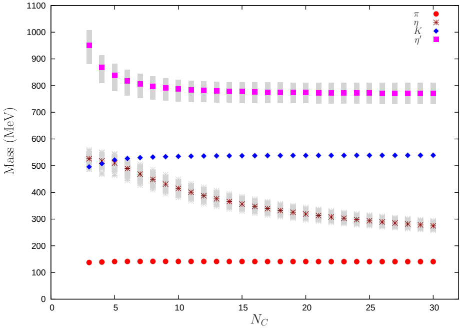
$$J_5^{\mu\,(0)} = \bar{q}\gamma_\mu\gamma_5 q$$
$$\partial_\mu J_5^{\mu\,(0)} = \frac{g^2}{16\pi^2}\frac{1}{N_c} Tr_c(G_{\mu\nu}\widetilde{G}^{\mu\nu})$$

Large N_c QCD $\rightarrow$ nonet of pseudoscalars (Goldstone theorem)

The previous studies are based on SU(3) CHPT which does not have the right number of degrees of freedom in large N_C

It has only 8 Goldstone bosons (π, K, η_8) but not 9 (including the η_1) as large N_C U(3) symmetry requires

Evolution of pseudoscalar masses with N_C Z.-H. Guo, J.A.O., PRD(2011) in press



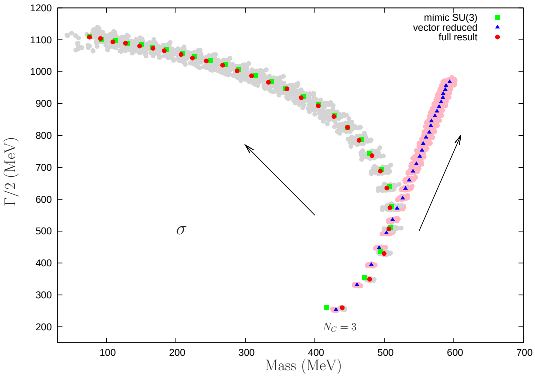
We make use $U(3)$ CHPT

Combined power expansion in light quark masses, soft external momenta and $1/N_C$

Di Vecchia, Veneziano, NPB'80
Rosenzweig, Schechter, Trahern, PRD'80
Witten, Ann.Phys.'80 Herrera-Siklody, Latorre, Pascual, Taron, NPB'97

δ-counting: $\delta \sim \mathcal{O}(p^2) \sim m_q \sim 1/N_C$

σ-Pole Trajectory



Oller, Oset PRD'99 $M_S^2 \propto f^2 \propto N_c$

Peláez *et al* '04,'06,...,'10 σ Mass always increases with N_c

At the two-loop order it moves to a pole with zero width at 1 GeV.

We also obtain such a pole but it comes from the bare scalar singlet $M_{S_1} \simeq 1$ GeV (At $N_c = 3$ it contributes to the $f_0(980)$.)

$$\frac{1}{M_V^2 - t} \to \frac{1}{M_V^2} ,$$

NLO local terms in χPT. *Vector Reduced* case (Blue Triangles)

The σ-pole trajectory is then more similar to that of the IAM one-loop study Peláez PRL'04
Sensitivity to higher order local terms.

When full vector propagators are kept their crossed exchange contributions cancel mutually with those from crossed loops along the RHC ($\sqrt{s} \lesssim 1$ GeV) Oller, Oset PRD'99

For increasing N_c loops are further suppressed and this cancellation is spoilt: More sensitivity to the LHC contributions.

The σ pole blows-up in the complex plane (not $q\bar{q}$). Dynamically generated resonance.

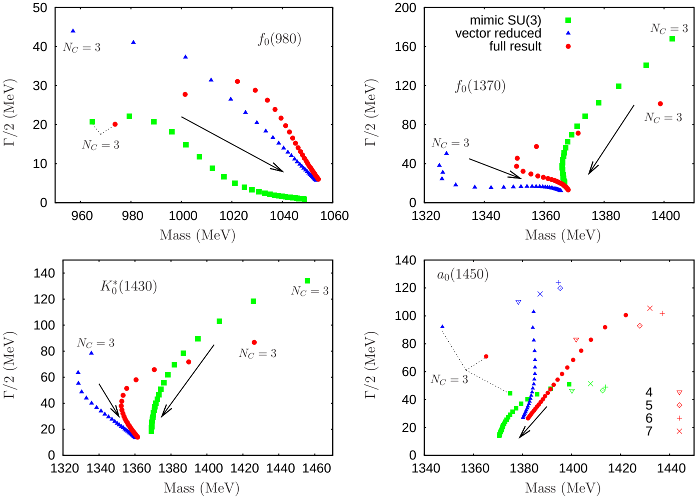
There is no influence on the σ N_C pole trajectory by the large reduction of the η mass because the σ does couple very little to states with η and η'

For the other scalar resonances this is not the case because they couple strongly with such states

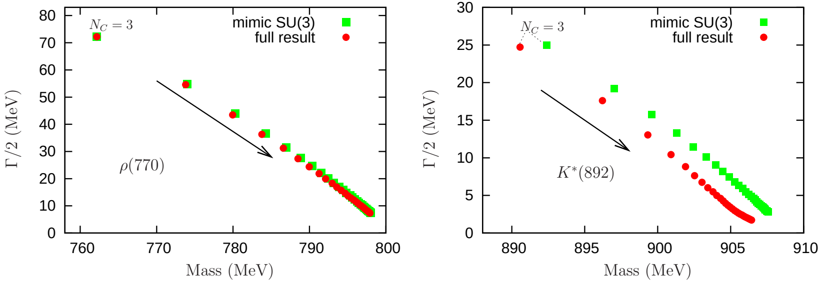
We also obtain the standard $q\bar{q}$ behavior for the lightest octet of vector resonances ρ, $K^*(892)$ and $\phi(1020)$

The $f_0(980)$, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ evolve at $N_C \to \infty$ to preexisting states.

Mimic SU(3) case: Mixing is set to zero and η_1 is kept in the loops. η_8, η_1 masses are frozen. Differences highlight the role of η'. Differences for the σ case are negligible.



$\rho(770)$, $K^*(892)$



$q\bar{q}$ trajectories: Mass $\mathcal{O}(N_c^0)$ and Width $\mathcal{O}(1/N_c)$ Peláez PRL'04

Above $N_c = 13$ the $K\eta$ threshold becomes lighter than the $K^*(892)$ mass (kink in the residue to $K\eta$ at $N_c = 14$).
For the $\phi(1020)$ the situation is the same. There is sensitivity to the slight movement of the nearby $K\bar{K}$ threshold with N_c.

On the lightest Glueball

M.Albaladejo, J.A.O., Phys. Rev. Lett. 101 (2008) 252002

A new state of matter made only of gluons
Due to the non-Abelian nature of QCD gluons interact between each other and can make hadrons without valence quarks

Predicted by quenched lattice QCD with $M = (1.66 \pm 0.05)$ GeV

Experimentally one finds the $f_0(1500)$ and $f_0(1710)$ near this predicted mass.

Multi-coupled-channel study:

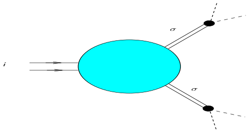
- $I = 0$: $\pi\pi$, $K\overline{K}$, $\eta\eta$, $\sigma\sigma$, $\eta\eta'$, $\rho\rho$, $\omega\omega$, $\eta\eta'$, $\omega\phi$, $\phi\phi$, $K^*\overline{K}^*$, $a_1(1260)\pi$, $\pi^\star\pi$
- $I = 1/2$, $I = 3/2$: $K\pi$, $K\eta$ and $K\eta'$ Jamin, Pich, J.A.O., NPB 587 (2000) 331

$\sigma\sigma$ channel amplitudes. Pion rescattering

We want to obtain $\sigma\sigma$ amplitudes starting from chiral Lagrangians.
σ *is* S-wave $\pi\pi$ interaction, $|\sigma\rangle = |\pi\pi\rangle_0$, Oset,J.A.O., NPA620'97

- Pion rescattering is given by the factor $D^{-1}(s) = (1 + t_2 G(s))^{-1}$, with:
  - $t_2 = \frac{s - m_\pi^2/2}{f_\pi^2}$ basic $\pi\pi \to \pi\pi$ amplitude.
  - $(4\pi)^2 G(s) = \alpha + \log\frac{m_\pi^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}$, two pion loop.
- Isolate the transition amplitude $N_{i\to\sigma\sigma}$:

$$\lim_{s_i \to s_\sigma} \frac{T_{i\to(\pi\pi)_0(\pi\pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i\to\sigma\sigma}\, g^2_{\sigma\pi\pi}}{(s_1 - s_\sigma)(s_2 - s_\sigma)}$$



- Around the σ pole $D_{II}(s)^{-1} = (1 + t_2 G(s))^{-1} \approx \frac{\alpha_0}{s - s_\sigma} + \cdots$, then:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$

- To calculate $(\alpha_0/g_{\sigma\pi\pi})^2$, consider $\pi\pi$ elastic scattering,

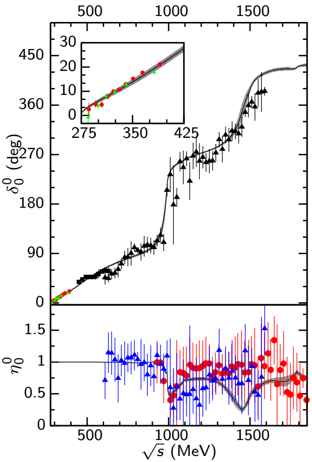
$$T = \frac{t_2(s)}{1 + t_2(s)G(s)} \approx -\frac{g_{\sigma\pi\pi}^2}{s - s_\sigma} + \cdots$$

So we can write, using that at the σ pole $g_{II}(s_\sigma) = -1/t_2(s_\sigma)$:

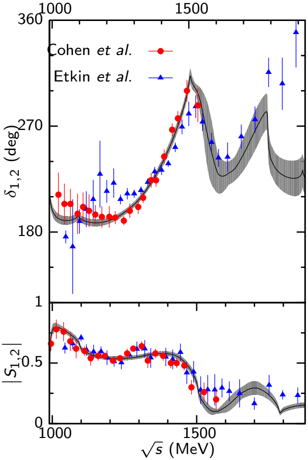
$$\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 = \frac{F_\pi^2}{1 - G'_{II}(s_\sigma)F_\pi^2 t_2(s_\sigma)^2} \approx 1.1 F_\pi^2$$

- In conclusion, we follow a novel method to calculate amplitudes involving $\sigma\sigma$, through:

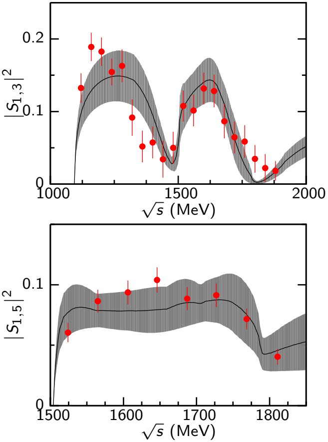
$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$



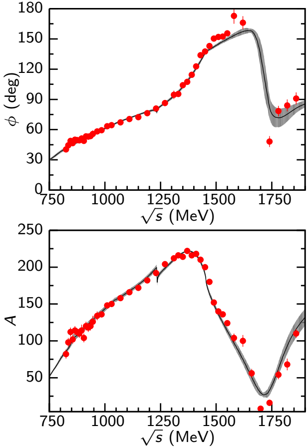
$\pi\pi \to \pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still quite good



$\pi\pi \to K\bar{K}$: Near threshold, Cohen data are favored.



$\pi\pi \to \eta\eta, \eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.



$I = 1/2\ K^- \pi^+ \to K^- \pi^+$
amplitude and phase from LASS.

No parameterization

We determine interaction kernels from Chiral Lagrangians, avoiding *ad hoc* parameterizations

Less free parameters

We have less free parameters, because of our chiral approach and our treatment of $\sigma\sigma$ amplitude

Higher energies

We have included enough channels to get at 2 GeV.

Compare with: Lindenbaum,Longacre PLB274'92; Kloet, Loiseau, ZPA353'95; Bugg, NPB471'96

Spectroscopy. Pole content: Summary

All the scalar resonances with mass up to 2 GeV are reproduced

$I = 0$ Poles (MeV)	Effect	M (PDG)	Γ (PDG/BESII)
$\sigma \equiv f_0(600)$	$456 \pm 6 - i\ 241 \pm 7$		
$f_0(980)$	$983 \pm 4 - i\ 25 \pm 4$	980 ± 10	$40 - 100$
$f_0(1370)$	$1466 \pm 15 - i\ 158 \pm 12$	$1200 - 1500$	$200 - 500$
$f_0(1500)$	$1602 \pm 15 - i\ 44 \pm 15$	1505 ± 6	109 ± 7
$f_0(1710)$	$1690 \pm 20 - i\ 110 \pm 20$	1724 ± 7	137 ± 8
$f_0(1790)$	$1810 \pm 15 - i\ 190 \pm 20$	1790^{+40}_{-30}	270^{+30}_{-60}

$I = 1/2$ Poles (MeV)	Effect	M (PDG)	Γ (PDG)
$708 \pm 6 - i\ 313 \pm 10$	$\kappa \equiv K_0^*(800)$	–	–
$1435 \pm 6 - i\ 142 \pm 8$	$K_0^*(1430)$	1414 ± 6	290 ± 21
$1750 \pm 20 - i\ 150 \pm 20$	$K_0^*(1950)$	–	–

Spectroscopy. Couplings

$f0(1370)$ $(I=0)$			$K_0^*(1430)$ $(I=1/2)$		
Coupling	bare	final	Coupling	bare	final
$g_{\pi^+\pi^-}$	3.9	3.59 ± 0.18	$g_{K\pi}$	5.0	4.8
$g_{K^0\bar{K}^0}$	2.3	2.23 ± 0.18	$g_{K\eta}$	0.7	0.9
$g_{\eta\eta}$	1.4	1.70 ± 0.30	$g_{K\eta'}$	3.4	3.8
$g_{\eta\eta'}$	3.7	4.00 ± 0.30			
$g_{\eta'\eta'}$	3.8	3.70 ± 0.40			

- Bare coupling are those of $S_8^{(1)}$, with $M_8^{(1)} = 1.3$ GeV.
- The first preexisting scalar octet is a pure one, not mixed with the nearby $f_0(1500)$ and $f_0(1710)$

Spectroscopy. Couplings

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

Coupling (GeV)	$f0(1500)$	$f_0(1710)$
g_{ss}	11.5 ± 0.5	13.0 ± 1.0
g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

This pattern suggests a suppression in $u\bar{u}$ and $d\bar{d}$ production.
With a pseudoscalar mixing angle $\sin\beta = -1/3$ for η and η':

$$\begin{aligned} g_{\eta'\eta'} &= \frac{2}{3}g_{ss} + \frac{1}{3}g_{nn} + \frac{2\sqrt{2}}{3}g_{ns} \ , \\ g_{\eta\eta'} &= -\frac{\sqrt{2}}{3}g_{ss} + \frac{\sqrt{2}}{3}g_{nn} + \frac{1}{3}g_{ns} \ , \\ g_{\eta\eta} &= \frac{1}{3}g_{ss} + \frac{2}{3}g_{nn} - \frac{2\sqrt{2}}{3}g_{ns} \ . \end{aligned}$$

If chiral suppression M.S.Chanowitz, PRL95('05)172001; 98('07)149104 operates:
The coupling of the glueball to $\bar{q}q$ is proportional to m_q
$|g_{ss}| \gg |g_{nn}|$
$|g_{ns}| \gg |g_{nn}|$ expected from OZI rule

Consider $K\bar{K}$ coupling

- Valence quarks: K^0 corresponds to $\sum_{i=1}^{3} \bar{s}_i u^j/\sqrt{3}$, and analogously $\bar{K}^0$
- Production of color singlet $s\bar{s}$ requires combination of color wave functions of K^0, $\bar{K}^0$
- Decompose $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^j$
- Only $s\bar{s}u\bar{u}$ contributes (factor $1/3$) and $s\bar{s}s\bar{s}$ has an extra factor two compared to $s\bar{s}u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$

The $f_0(1710)$ resonance corresponds to the lightest scalar glueball. We find a really good agreement with the mechanism of chiral suppression from QCD proposed by Chanowitz with our exhaustive multichannel hadronic calculation.

It is a pure glueball not mixed with any other resonance

$f_0(1370)$ is almost purely octet states

$f_0(1500)$ is made up of the sum of the poles that give rise to the $f_0(1370)$ and $f_0(1710)$ together with a strong cusp effect from the $\eta\eta'$ threshold.