Lectures on Nuclear and Hadron Physics

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Outline Lecture II

Introduction

NN interaction is important for nuclear structure, nuclear reactions, nuclear matter, neutron star, nucleosinthesys, etc ...

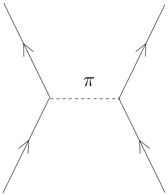
Application of Chiral Perturbation Theory (ChPT) to NN
S. Weinberg, PLB **251** (1990) 288; NPB **363** (1991) 3; Phys. Lett. B **295** (1992) 114.

Weinberg's counting: Calculate the two-nucleon irreducible graphs in ChPT (the NN potential V_{NN}) and then solve the Lippmann-Schwinger (LS) equation

$$T_{NN}(\mathbf{p}',\mathbf{p}) = V_{NN}(\mathbf{p}',\mathbf{p}) + \int d\mathbf{p}'' V_{NN}(\mathbf{p}',\mathbf{p}'')\frac{m}{\mathbf{p}^2-\mathbf{p}''^2+i\epsilon}T_{NN}(\mathbf{p}'',\mathbf{p})$$

Ordóñez, L. Ray and U. van Kolck, PRL **72** (1994) 1982; PRC **53** (1996) 2086.

In 1935 H. Yukawa introduced the pion as the carrier of the strong nuclear force



The pion mass was inferred from the range of strong nuclear forces

This was estimated from the radius of the atomic nucleus

Relativistic-Quantum-Mechanical argument:

Heisenberg uncertainty principle: $\Delta t \Delta E \geq \hbar$

Relativity: Velocity of light is the Maximum velocity c

$$\Delta t \Delta E = \frac{\Delta \ell}{c} \Delta E \geq \hbar$$
$$\Delta E = \frac{\hbar c}{\Delta \ell}$$

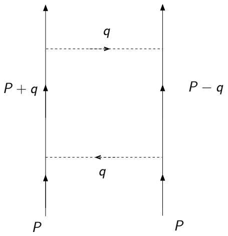
$\Delta \ell \sim 2$ fm $\quad$ (1 fm $= 10^{-15}$ m)

$$m_\pi \sim \frac{\hbar c}{2 \text{ fm}} \sim 100 \text{ MeV}$$

$m_\pi = 138$ MeV

• A typical three-momentum cut-off $\Lambda \sim 600$ MeV (fine tuned to data) is used in order to regularize the Lippmann-Schwinger equation because chiral potentials are singular.
E.g. The tensor part of One-Pion Exchange (OPE) diverges as $1/r^3$ for $r \to 0$

• NN scattering is nonperturbative: Presence of bound states (deuteron) in $^3S_1$ and anti-bound state in $^1S_0$.
Spectroscopic notation $^{2S+1}L_J$



$$\int d^4q\,(q^0+i\epsilon)^{-1}(q^0-i\epsilon)^{-1}(q^2+m_\pi^2)^{-2}\,P(q)$$

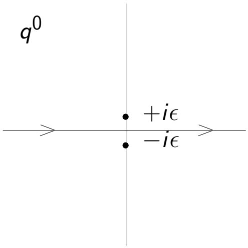
Infrared enhancement
$1/|\mathbf{q}| \to 1/|\mathbf{q}| \times m/|\mathbf{q}|$.
$q^0 \to q^0 - \mathbf{q}^2/(2m)$, non-relativistic nucleon propagator

Extreme non-relativistic propagator (or Heavy-Baryon propagator)

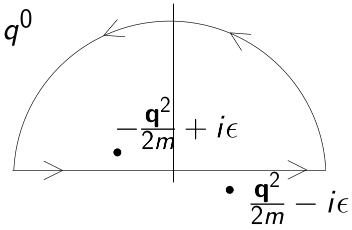
$$\frac{1}{q^0 + i\epsilon}$$

Non-relativistic propagator

$$\frac{1}{q^0 - \frac{\mathbf{q}^2}{2m} + i\epsilon}$$



"Pinch" singularity
The integration contour cannot be deformed



$$\int dq^0\,(q^0 - \frac{\mathbf{q}^2}{2m} + i\epsilon)^{-1}(q^0 + \frac{\mathbf{q}^2}{2m} - i\epsilon)^{-1} = -2\pi i\frac{m}{\mathbf{q}^2}$$

- V_{NN} is calculated up to next-to-next-to-next-to-leading order (N^3LO) and applied with great phenomenological success

D. R. Entem and R. Machleidt, PLB **254** (2002) 93; PRC **66** (2002) 014002; PRC **68** (2003) 041001

Epelbaum, Glöckle, Meißner, Nucl. Phys. A **747** (2005) 362

- **On the cut-off dependence**

Chiral counterterms introduced in V_{NN} following naive chiral power counting are not enough to reabsorb the dependence on cut-off when solving the LS equation

Nogga, Timmermans and van Kolck, PRC **72** (2005) 054006
Pavón Valderrama and Arriola, PRC **74** (2006) 054001; PRC **74** (2006) 064004

Kaplan, Savage, Wise NPB **478** (1996) 629

▷ In Nogga *et al.* one counterterm is promoted from higher to lower orders in $^3P_0$, $^3P_2$ and $^3D_2$ and then stable results for $\Lambda \to \infty$ are obtained.

▷ Higher order counterterms could be also treated perturbatively

N/D Method

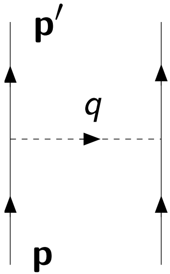
Chew and Mandelstam, Phys. Rev. **119** (1960) 467

A *NN* partial wave amplitude has two type of cuts:

Unitarity or Right Hand Cut (RHC)

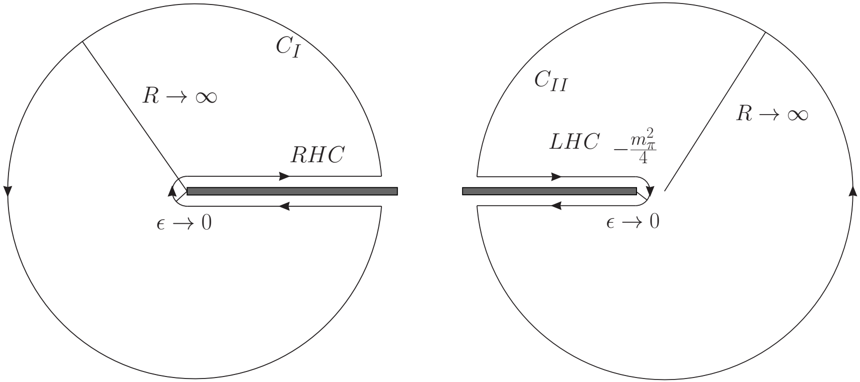
$$\mathbf{p}^2 > 0$$

Left Hand Cut (LHC)



$$\frac{1}{(\mathbf{p}-\mathbf{p}')^2 + m_\pi^2}$$

$$\mathbf{p}^2 = -\frac{m_\pi^2/2}{1-\cos\theta} \rightarrow \mathbf{p}^2 \in]-\infty, -m_\pi^2/4]$$



$$T_{J\ell S}(A) = \frac{N_{J\ell S}(A)}{D_{J\ell S}(A)}$$

$N_{J\ell S}(A)$ Only LHC

$D_{J\ell S}(A)$ Only RHC

Elastic Partial Waves

$$\Im D_{J\ell S}(A) = -N_{J\ell S}(A)\frac{m\sqrt{A}}{4\pi}\ ,\ A > 0$$
$$\Im N_{J\ell S}(A) = D_{J\ell S}(A)\,\Im T_{J\ell S}(A)\ ,\ A < -m_\pi^2/4$$
$$A \equiv |\mathbf{p}|^2$$

$$\oint_{C_I} dz \frac{D_{J\ell S}(z)}{(z-A)(z-D)} = 2\pi i \frac{D_{J\ell S}(A) - D_{J\ell S}(D)}{A-D}$$
$$= \int_0^\infty dq^2 \frac{\left[D_{J\ell S}(q^2+i\epsilon) - D_{J\ell S}(q^2-i\epsilon)\right]}{(q^2-A+i\epsilon)(q^2-D+i\epsilon)}$$

Schwartz's reflection principle:
If $f(z)$ is real along an interval of the real axis and is analytic then:
$f(z^*) = f(z)^*$

$$D_{J\ell S}(q^2+i\epsilon)-D_{J\ell S}(q^2-i\epsilon)=2i\Im D(q^2+i\epsilon)$$

$$D_{J\ell S}(A)=1-\frac{A-D}{\pi}\int_0^\infty dq^2\frac{\rho(q^2)N_{J\ell S}(q^2)}{(q^2-A)(q^2-D)}$$
$$N_{J\ell S}(A)=N_{J\ell S}(D)+\frac{A-D}{\pi}\int_{-\infty}^{-m_\pi^2/4}dk^2\frac{\Delta_{J\ell S}(k^2)D_{J\ell S}(k^2)}{(k^2-A)(k^2-D)}$$

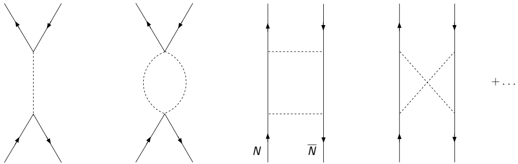
$$\rho(A)=m\sqrt{A}/4\pi\ ,\ A>0$$
$$\Delta(A)=\Im T_{J\ell S}(A)\ ,\ A<-m_\pi^2/4$$

$$D_{J\ell S}(A) = 1 - N_{J\ell S}(D)\frac{A-D}{\pi}\int_0^{+\infty} dq^2 \frac{\rho(q^2)}{(q^2-A)(q^2-D)}$$
$$- \frac{A-D}{\pi^2}\int_0^{+\infty} dq^2 \frac{\rho(q^2)}{q^2-A}\int_{-\infty}^{L} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{(k^2-q^2-i\epsilon)(k^2-D)}$$

We choose $D=0$ because $N_{J\ell S}=(0)\quad \ell \geq 1$

Only one subtraction constant is left for the S-wave
$N_{J\ell S} = -4\pi a_s/m$

$\Delta_{J\ell S}(A)$ is amenable to a chiral perturbative calculation
A. Lacour, U.-G. Meißner, J.A.O., Annals Phys. **326** (2011) 241
The discontinuity along the LHC implies to put pions on-shell
From $NN \to NN$ the crossed process is $N\bar{N} \to N\bar{N}$
Low energies. There is no infrared enhancement ($N\bar{N}$ is very far off-shell)



$$D_{J\ell S}(A) = 1 - AN_{J\ell S}(0)g(A,0) + \frac{A}{\pi}\int_{-\infty}^{L} dk^2 \frac{\Delta_{J\ell S}(k^2)D_{J\ell S}(k^2)}{k^2} g(A,k^2)$$

$$g(A,k^2) = \frac{1}{\pi}\int_0^{+\infty} dq^2 \frac{\rho(q^2)}{(q^2-A)(q^2-k^2)}$$

$$N_{J\ell S}(A) = N_{J\ell S}(0) + \frac{A}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2)D_{J\ell S}(k^2)}{(k^2-A)k^2}$$

$$T_{J\ell S} = \frac{N_{J\ell S}}{D_{J\ell S}}$$

$\ell \geq 2$

Theorem A partial wave should vanish as $|\mathbf{p}|^{2\ell}$ when $|\mathbf{p}| \to 0$ (threshold)

$$N_{J\ell S}(A) = \frac{A}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{(k^2 - A)k^2}$$

$$N_{J\ell S}(A) \to A \quad (A \to 0)$$

One-pion exchange case

$$T^{1\pi}_{J\ell S} = \frac{A}{\pi}\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2)}{k^2(k^2-A)}$$

$T^{1\pi}_{J\ell S}(A) \to A^\ell$ for $A \to 0$.

Constraints:

$$\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta^{1\pi}_{J\ell S}(k^2)}{k^{2\lambda}} = 0 \quad \lambda = 2, 3, \ldots, \ell \ , \ell \geq 2 \ .$$

For $^1D_2$, $J = 2$, $\ell = 2$ and $S = 0$

$$\Delta_{J\ell S} = \pi \frac{g_A^2 m_\pi^2 (3m_\pi^4 + 12 m_\pi^2 \mathbf{p}^2 + 8|\mathbf{p}|^4)}{128 F_\pi^2 |\mathbf{p}|^6}$$

Generalization of constraints

Imposing right behavior at threshold:

$$T_{J\ell S}(A) = A^{\ell} \frac{N_{J\ell S}(A)}{D_{J\ell S}(A)}$$

$$N_{J\ell S}(A) = \frac{1}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^{2\ell}(k^2 - A)}$$

$$D_{J\ell S}(A) = 1 - \frac{A}{\pi} \int_0^{\infty} dq^2 \frac{\rho(q^2) q^{2(\ell-1)} N_{J\ell S}(q^2)}{q^2 - A}$$

$$= 1 + \frac{A}{\pi^2} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^{2\ell}}$$

$$\times \int_0^{\infty} dq^2 \frac{\rho(q^2) q^{2(\ell-1)}}{(q^2 - A)(q^2 - k^2)}$$

To end with a convergent integral

$$N_{J\ell S}(A) \to 1/A^{\ell} \quad \text{for} \quad A \to \infty$$

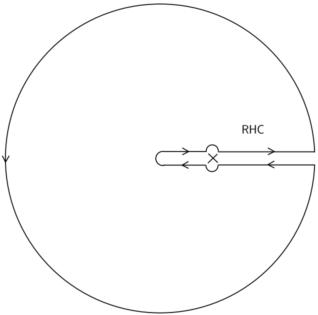
Geometric series

$$\frac{1}{k^2 - A} = -\frac{1}{A}\frac{1}{1 - k^2/A} = -\frac{1}{A}\sum_{m=0}^{\ell-2}\left(\frac{A}{k^2}\right)^m + \left(\frac{k^2}{A}\right)^{\ell-1}\frac{1}{k^2 - A}$$

Constraints

$$\int_{-\infty}^{L} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^{2\lambda}} = 0 \quad , \quad \lambda = 2, 3, \ldots, \ell \quad , \quad \ell \geq 2$$

Castillejo-Dalitz-Dyson (CDD) poles



Poles of $D_{J\ell S}$ along the real axis

They correspond to zeroes of $T_{J\ell S}$

Two parameters: residue and pole position, $\gamma/(A-B)$

L. Castillejo, R. H. Dalitz, F. J. Dyson, Phys. Rev. **101** (1956) 453

They could be included quite arbitrarily for a given $\Delta_{J\ell S}$

They correspond to Adler zeroes or preexisting resonances

E. Oset and J.A.O., Phys. Rev. D **60** (1999) 074023

We add CDD poles at **infinite** in order to satisfy the constraints

$$\sum_{i=1}^{\ell-1} \frac{A}{B_i} \frac{\gamma_i}{A - B_i} \rightarrow \frac{A \sum_{n=0}^{\ell-2} c_n A^n}{(A-B)^{\ell-1}} \qquad \text{with } B \rightarrow \infty$$

$$T_{J\ell S}(A) = A^{\ell} \frac{N_{J\ell S}(A)}{D_{J\ell S}(A)}$$

$$N_{J\ell S}(A) = \frac{1}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^{2\ell}(k^2 - A)}$$

$$D_{J\ell S}(A) = 1 + \frac{A}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^2} g(A, k^2)$$

$$+ \frac{A \sum_{n=0}^{\ell-2} c_n A^n}{(A-B)^{\ell-1}}$$

$$\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{J\ell S}(k^2) D_{J\ell S}(k^2)}{k^{2\lambda}} = 0 \ , \ \lambda = 2, 3, \ldots, \ell \ , \ \ell \geq 2$$

Coupled Waves

$$S_{JIS} = I + i\frac{|\mathbf{p}|m}{4\pi}T_{IJS}$$
$$S_{JIS}\cdot S_{JIS}^{\dagger} = S_{JIS}^{\dagger}\cdot S_{JIS} = I$$

$$S_{JIS} = \begin{pmatrix} \cos 2\epsilon\, e^{i2\delta_1} & i\sin 2\epsilon\, e^{i(\delta_1+\delta_2)} \\ i\sin 2\epsilon\, e^{i(\delta_1+\delta_2)} & \cos 2\epsilon\, e^{i2\delta_2} \end{pmatrix} \quad , \quad |\mathbf{p}|^2 \geq 0$$

ϵ is the mixing angle

$$\mathrm{Im}\, T_{ii}^{-1}(A) = -\rho\Big[1 + \frac{\frac{1}{2}\sin^2 2\epsilon}{1-\cos 2\epsilon\,\cos 2\delta_i}\Big]^{-1} \equiv -\nu_i(A)\ .$$

$$T_{ii}(A) = A^{\ell_i}\frac{N_i(A)}{D_i(A)}$$

$$N_i(A) = N_i(0) + \frac{A}{\pi}\int_{-\infty}^{L} dk^2 \frac{\Delta_i(k^2)D_i(k^2)}{k^2(k^2-A)} \quad , \quad \ell = 0$$

$$N_i(A) = \frac{1}{\pi}\int_{-\infty}^{L} dk^2 \frac{\Delta_i(k^2)D_i(k^2)}{k^{2\ell}(k^2-A)} \quad , \quad \ell > 0$$

$$D_i(A) = 1 - \frac{A}{\pi}\int dk^2\, k^{2(\ell-1)}\frac{\nu_i(k^2)N_i(k^2)}{k^2-A}$$

Constraints

Threshold behavior

$$\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_i(k^2) D_i(k^2)}{k^{2\lambda}} = 0 \quad , \quad \lambda = 2, 3, \ldots, \ell \ , \ \ell \geq 2$$

$$D_i(A) = 1 - N_i(0) \frac{A}{\pi} \int_0^\infty dk^2 \frac{\nu_i(k^2)}{(k^2 - A) k^2} + \frac{A}{\pi^2} \int_{-\infty}^{L} dk^2 \frac{\Delta_i(k^2) D_i(k^2)}{k^2} \int_0^\infty dq^2 \frac{\nu_i(q^2)}{(q^2 - A)(q^2 - k^2)} + \frac{A \sum_{n=0}^{\ell_i - 2} c_n A^n}{(A - B)^{\ell_i - 1}}$$

For the mixing amplitude: $\overline{\ell} = (\ell_1 + \ell_2)/2$

$$T_{12}(A) = A^{\overline{\ell}} \frac{N_{12}(A)}{D_{12}(A)}$$

$$N_{12}(A) = \frac{1}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{12}(k^2) D_{12}(k^2)}{k^{2\overline{\ell}}(k^2 - A)} \ ,$$

$$D_{12}(A) = 1 + \frac{A \sum_{n=0}^{\overline{\ell}-2} c_n A^n}{(A-B)^{\overline{\ell}-1}} + \frac{A}{\pi^2} \int_{-\infty}^{L} dk^2 \frac{\Delta_{12}(k^2) D_{12}(k^2)}{k^2} \int_0^\infty dq^2 \frac{2\rho(q^2)\sin(\delta_1 + \delta_2)}{(q^2 - A)(q^2 - k^2)\sin 2\epsilon}$$

Constraints

$$\int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{12}(k^2) D_{12}(k^2)}{k^{2\lambda}} = 0 \ , \quad \lambda = 2, 3, \ldots, \overline{\ell} \ , \quad \overline{\ell} \geq 2$$

One proceeds in a coupled-iterative way:

1. $\nu_1 = \nu_2 = \rho$ (elastic case): $T_{11}(A)$ and $T_{22}(A) \longrightarrow \delta_1,\ \delta_2$
2. Calculate $T_{12}(A)$ and then $\longrightarrow \epsilon \longrightarrow \nu_1(A),\ \nu_2(A)$
3. Repeat the process

The deuteron case

Bound state in the $^3S_1$-$^3D_1$

A bound state is a pole in $T_{11}(A)$, $T_{12}(A)$ and $T_{22}(A)$

It should be located in exactly the same position, $A = k_D^2$

We impose it

Extra subtraction in $D_{12}(A)$ and $D_{22}(A)$ so that:

$D_{12}(k_D^2) = 0$, $D_{22}(k_D^2) = 0$

$$D_{ij}(A) = 1 - \frac{A}{k_D^2} + \frac{A(A - k_D^2)}{\pi} \int_{-\infty}^{-m_\pi^2/4} dk^2 \frac{\Delta_{ij}(k^2) D_{ij}(k^2)}{k^{2\ell_{ij}}} g_{ij}^d(A, k^2)$$

$$g_{ij}^d(A, k^2) = \frac{1}{\pi} \int_0^\infty dq^2 \frac{\nu_{ij}(q^2) q^{2\ell_{ij}}}{q^2(q^2 - A)(q^2 - k^2)(q^2 - k_D^2)}$$

k_D^2 is taken from the $^3S_1$ partial wave
It changes in every iteration until convergence is reached

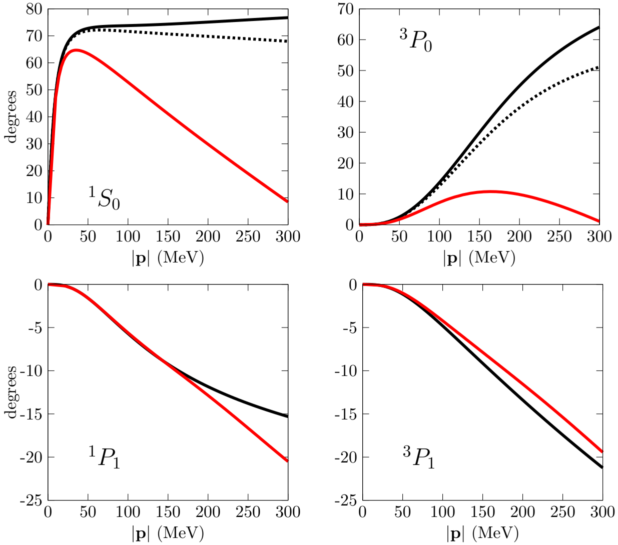
We take the position of the deuteron from the $^3S_1$ partial wave because this bound state has around 95% probability of S-wave

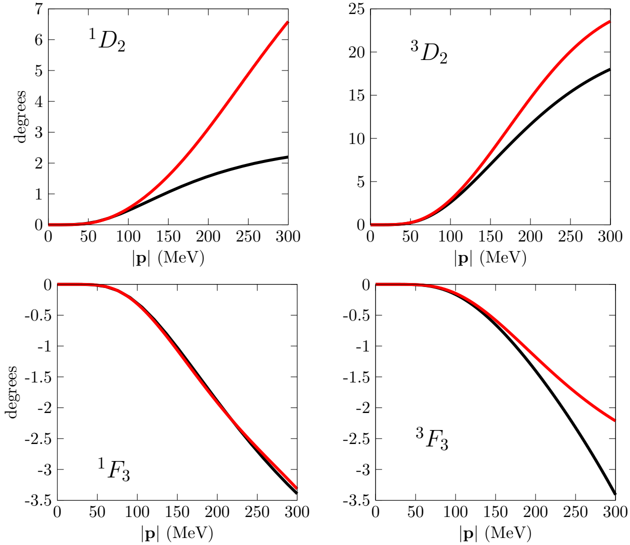
The final deuteron pole position is almost the same as the original one from $^3S_1$

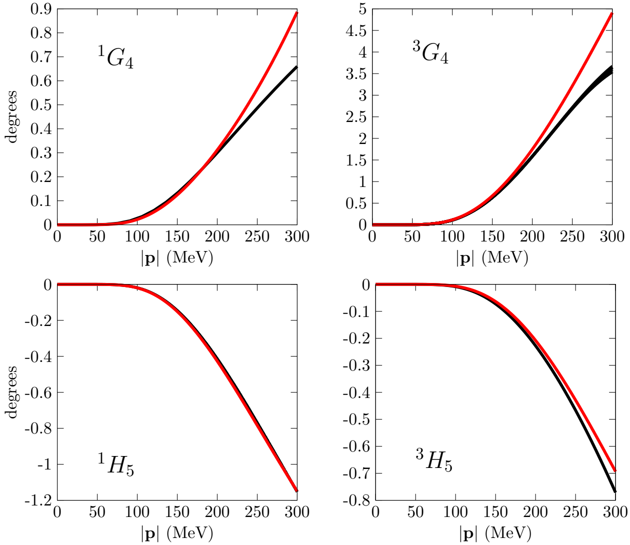
Our method provides NN partial waves that are independent of regulator, e.g. cut-off

Comparison with exp. phase shifts

Elastic case


degrees
$^1S_0$
$^3P_0$
$|\mathbf{p}|$ (MeV)
degrees
$^1P_1$
$^3P_1$
$|\mathbf{p}|$ (MeV)


$^1D_2$
$^3D_2$
$^1F_3$
$^3F_3$
degrees
degrees
$|\mathbf{p}|$ (MeV)
$|\mathbf{p}|$ (MeV)
$|\mathbf{p}|$ (MeV)
$|\mathbf{p}|$ (MeV)
0
1
2
3
4
5
6
7
0
5
10
15
20
25
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
50
100
150
200
250
300


$^1G_4$
$^3G_4$
$^1H_5$
$^3H_5$
degrees
degrees
|**p**| (MeV)
|**p**| (MeV)
|**p**| (MeV)
|**p**| (MeV)

Nogga, Timmermans, van Kolck, Phys. Rev. C 72 (2005) 054001

obtained cut-off independent results when solving the Lippmann-Schwinger equation with the OPE chiral potential.

They accomplish it by promoting higher order counterterms to lower orders in the calculation of the potential for the ${}^3P_0$ and ${}^3D_2$ partial waves
The chiral counting is violated

Their results are very close to ours, except for the ${}^3P_0$ where they agree better with data by including the new counterterm

$$T = V + VgT = V + VgV + \dots$$

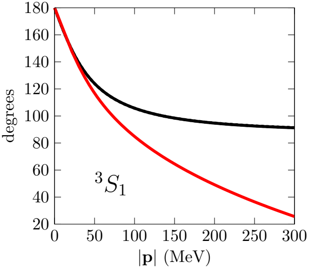
When iterating once the Lippmann-Schwinger equation the term VgV generates a two-pion source of LHC.

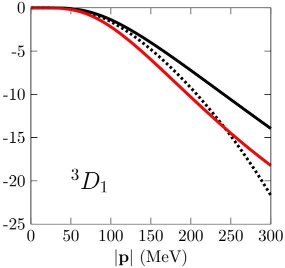
It diverges, and then a counterterm is required in order to renormalize it.

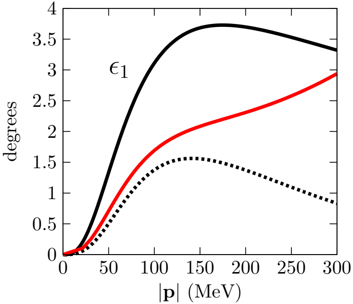
This source of discontinuity along the LHC is included in our theory at next-to-leading order (together with the rest of two-pion exchange contributions to the chiral NN potential)

New subtraction constants are required in order to cope with divergent dispersion integrals and make them finite

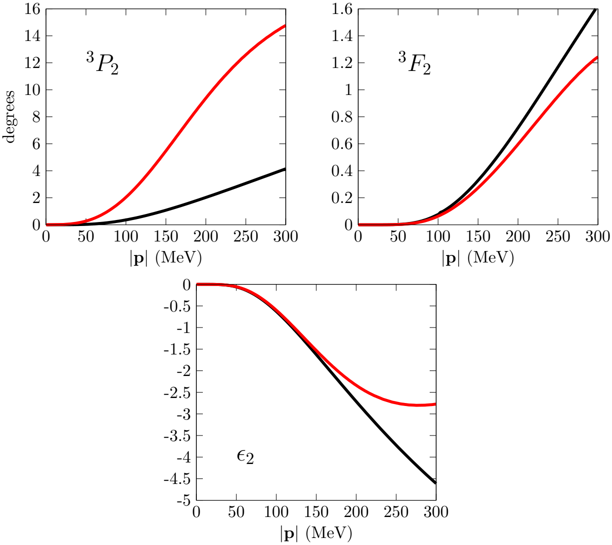
Coupled case

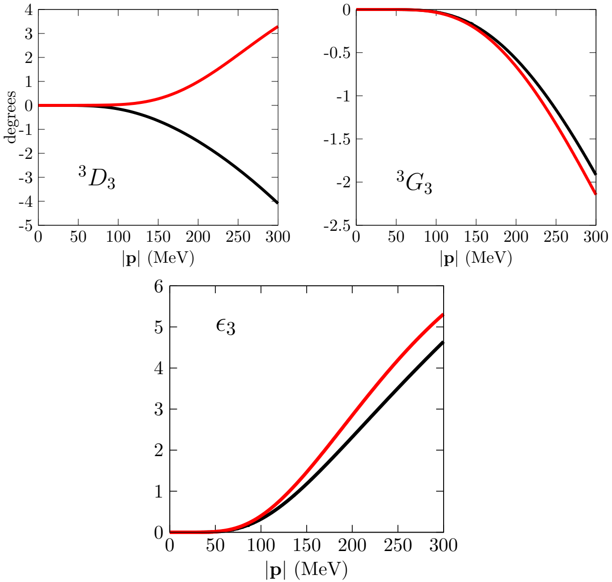






Solid: Full result
Dashed: without imposing deuteron pole in $3D_1$ and $^3S_1$-$^3D_1$


$^3P_2$
degrees
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300
$|\mathbf{p}|$ (MeV)
$^3F_2$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
$|\mathbf{p}|$ (MeV)
ϵ_2
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
0
50
100
150
200
250
300
$|\mathbf{p}|$ (MeV)


degrees
$^3D_3$
$^3G_3$
ϵ_3
|p| (MeV)
|p| (MeV)
|p| (MeV)
4
3
2
1
0
-1
-2
-3
-4
-5
0
-0.5
-1
-1.5
-2
-2.5
6
5
4
3
2
1
0
0
50
100
150
200
250
300

No LHC

$$T = \frac{N}{D}$$
$$\Im N = 0$$
$$\Im D = -\rho N \quad \text{along RHC}$$

N has no LHC and we can reabsorb it in D

$$T = \frac{1}{D/N} \equiv \frac{1}{\mathcal{D}}$$

$$\Im \mathcal{D} = -\rho$$
$$\mathcal{D}(A) = \mathcal{D}(D) - \frac{A-D}{\pi} \int_0^\infty dk^2 \frac{\rho(k^2)}{(k^2 - A - i\epsilon)(k^2 - D)} + \sum_i \frac{A-D}{B_i - D} \frac{\gamma_i}{A - B_i}$$

In nuclear physics the *NN* scattering lengths of S-waves are much larger than $1/m_\pi$ (the range of the interaction)

This allows the Pionless EFT

U. van Kolck, Nucl. Phys. **A645** (1999) 273
E. Epelbaum, H.-W. Hammer, U.-G. Meißner, Rev. Mod. Phys. **81** (2009) 1773

Valid at very low energies $k \sim 1/a \ll m_\pi$

No LHC, only contact interactions

One recovers the solution given by the N/D method in this case when solving a Lippmann-Schwinger equation (once it is renormalized)

To neglect the LHC or mimic it by a polynomial of low degree is a good phenomenological approximation in many cases, particularly for resonant scattering,

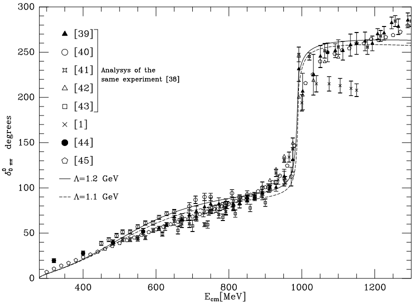
Some early examples:

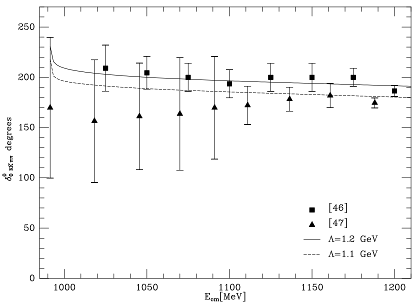
Meson-Meson scattering Oset, J.A.O., Nucl. Phys. A 620 (1997) 438
Oset, J.A.O., Phys. Rev. D 60 (1999) 074023

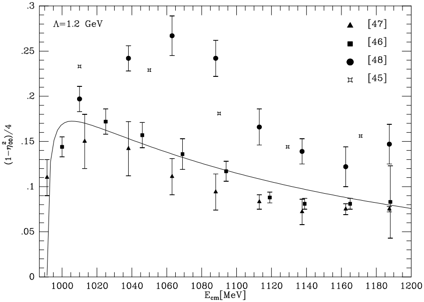
$$T = \left[R^{-1} + g\right]^{-1}$$
$$R_{ij} = \frac{a_{ij}s + b_{ij}}{F_\pi^2}$$

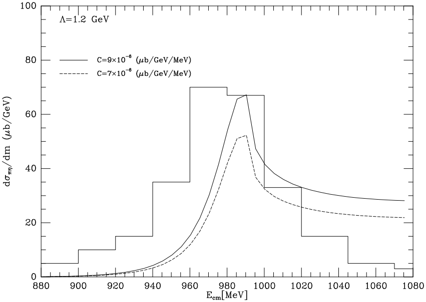
S-wave $\pi\pi$, $K\bar{K}$ and $\pi\eta$ scattering
Good phenomenological success. The resonances reproduced were the σ, $f_0(980)$ and $a_0(980)$

Only one free parameter, a cut-off of natural value, around the scale $\Lambda = 4\pi f_\pi$






Λ=1.2 GeV
[47]
[46]
[48]
[45]
(1−η²₀₀)/4
E_cm[MeV]
.3
.25
.2
.15
.1
.05
0
1000
1020
1040
1060
1080
1100
1120
1140
1160
1180
1200


Λ=1.2 GeV
C=9×10⁻⁶ (μb/GeV/MeV)
C=7×10⁻⁶ (μb/GeV/MeV)
dσ_ππ/dm (μb/GeV)
E_cm[MeV]
100
80
60
40
20
0
880
900
920
940
960
980
1000
1020
1040
1060
1080

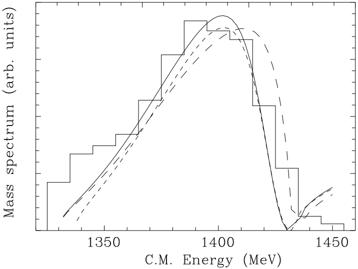
Meson-Baryon scattering Kaiser, Siegel, Weise, Nucl. Phys. A 594 (1995) 325

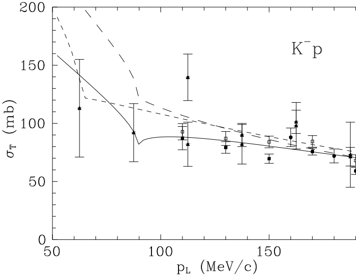
Oset, Ramos, Nucl. Phys. A 635 (1998) 99

$$T = \left[R^{-1} + g\right]^{-1}$$
$$R_{ij} = C_{ij} \frac{E + E'}{4F_\pi^2}$$

Good phenomenological success. The most prominent resonance is the $\Lambda(1405)$

Only one free parameter, a natural sized three-momentum cut-off





Early review on Unitary CHPT

Oset, Ramos, J.A.O., Prog. Part. Nucl. Phys. 45 (2000) 157

$\bar{K}N$ scattering

The interest on $\bar{K}N$ scattering plus coupled channels has been renewed due to precise experiments designed to measure the energy and width of the ground state of kaonic hydrogen K^-p

G. Beer *et al.* (DEAR Coll.), Phys. Rev. Lett. 94 (2005) 212302

M. Bazzi *et al.* (Siddhartha Coll.), arXiv:1105.3090

It is like a standard hydrogen atom but with the electron replaced by the K^-.

The radius is of course much smaller by a factor $m_e/m_K \sim 0.5/500 = 10^{-3}$

Most of the energy of the ground state of K^-p is due to electromagnetic origin

$$E_{1s} = E_{1s}^{em} + \epsilon_{1s}$$

Due to the smallness of the radius the kaon feels the strong interaction with the proton (nuclear size)

Shift in the energy as well as a Width due to the strong and electromagnetic decays to open channels

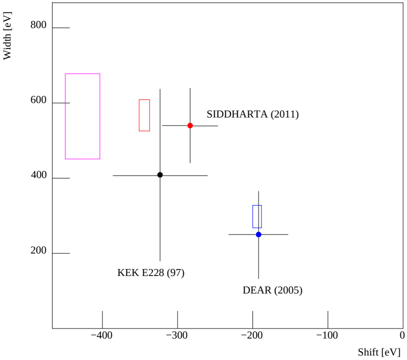
$$K^- p \to \begin{cases} \pi^0\Lambda, \ \pi^\mp\Sigma^\pm \ \text{[strong]} \\ \Sigma\pi\gamma \ , \ \Sigma\pi e^+ e^- \ , \ \Sigma\gamma \ , \ < 1\% \end{cases}$$

Relation between $\epsilon_{1s} = \Delta E - \frac{i}{2}\Gamma$ and $T_{\bar{K}N}(0)$:
Deser formula Deser *et al.*, Phys. Rev. 96 (1954) 774

$$\Delta E - \frac{i}{2}\Gamma = -\frac{\alpha^3 \mu_c^3}{2\pi M_{K^+}} T_{\bar{K}N}(0)$$

Corrections are given in Meißner, Raha, Rusetsky, Eur. Phys. J. C 35 (2004) 349

From atomic physics one learns about strong interactions at threshold



The measurement is difficult due to the large amount of background.

The last measurement by SIDDHARTHA confirms KEK ('97) instead of DEAR('05)

A factor 2 of uncertainty in the scattering length of $K^- p$

Several studies making use of Unitary CHPT:

Borasoy, Nissler, Weise Phys. Rev. Lett. 94 (2005) 213401; Eur. Phys. J. A 25 (2005) 79

Borasoy, Nissler, Weise, Phys. Rev. C 74 (2006) 055201

Prades, Verbeni, J.A.O., Phys. Rev. Lett. 95 (2005) 172502; 96 (2006) 199202

Borasoy, Meißner, Nissler, Phys. Rev. C74 (2006) 055201

J.A.O., Eur. Phys. J. A28 (2006) 63

Bouzas, Eur. Phys. J. A43 (2010) 351

These are studies based at most on next-to-leading order Baryon CHPT

A large amount of experimental data was fitted

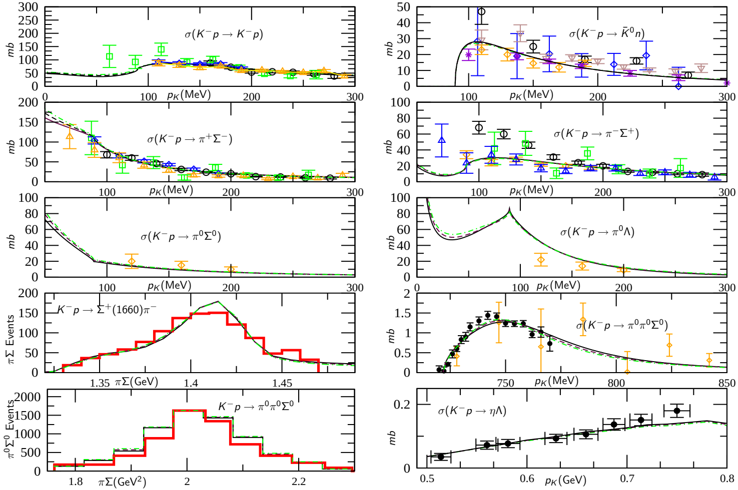


Figure: J.A.O., Eur. Phys. J. A28 (2006) 63

All the resonances listed in the PDG with $J^P = \frac{1}{2}^-$ $S = -1$ are reproduced
$\Lambda(1405)$, $\Lambda(1670)$, $\Lambda(1800)$, $\Sigma(1480)$, $\Sigma(1620)$ and $\Sigma(1750)$

DEAR measurement is also reproduced

In the future we want to study the new SIDDHARTHA measurement and going to N^{2}LO

This is already done by our group for πN in
Alarcón, Martín-Camalich, Alvarez-Ruso,J.A.O., Phys. Rev. C 83 (2011) 055205; Another one forthcoming
For SU(3) see the threshold study of
Mai, Bruns, Kubis, Meißner, Phys. Rev. D80 (2009) 094006