Study of lineshapes of the $X(3872)$: Unveiling novel possible scenarios

J. A. Oller

Departamento de Física, Universidad de Murcia.
Murcia (Spain)

- [I] arXiv:1612.08420 [hep-ph] in collaboration with **X.W. Kang**
- New material added

Partially funded by MINECO (Spain) and EU, project FPA2016-77313-P

Bad Honnef, Germany, March, 2017

Overview

Introduction

The $X(3872)$ was first observed by Belle in $B^{\pm} \to K^{\pm} J/\psi \pi^{+}\pi^{-}$
PRL91,262001(2003)

Quantum number $J^{PC} = 1^{++}$ LHCb PRL110,222001(2013)

- It mass is extremely close to the $D^0 \bar{D}^{*0}$ threshold PDG 2016

$$M_X - M_{D^0} - M_{D^{*0}} = -0.12 \pm 0.19 \text{ MeV}$$
$$\Gamma_X < 1.2 \text{ MeV}$$

- Interplay of explicit mesonic and underlying degrees of freedom

It is a very interesting laboratory to put into practice general results of S-matrix theory when crossed channel dynamics is integrated out
Oller, Oset PRD60,074023 (1999); Meißner,Oller, NPA679,671(2001)

- To be applied in this case or in other near-threshold XYZ states

Perturbating on pion-exchanges:

$1/r^3$ “massless” OPE potential

$$\frac{(M_{D^{*0}} - M_{D^0})^2 - m_{\pi^0}^2}{m_{\pi^0}^2} \simeq 0.1 \to 0$$

Intrinsic momentum scale

$$\Lambda = 8\pi f_\pi^2/2\mu g^2 \simeq 350 \text{ MeV} \gg \sqrt{2\mu|E_b|} \ (\lesssim 30 \text{ MeV})$$

$g = 0.6 \simeq g_A/2, \ f_\pi = 92.4$

XEFT Fleming et al. PRD76,034006(2007)

Braaten *et al.*, PRD82,014013(2010)

Pavon, PRD85,114037(2012);

Supported by the full Faddeev solution Baru *et al.*,PRD84,074029(2011)

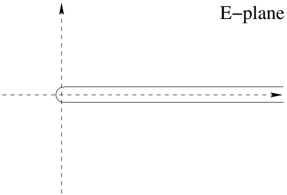
Scattering Amplitude $t(E)$

Dispersion Relation for the inverse of $t(E)$

$$\text{Im}\, t(E)^{-1} = -ik$$

One subtraction is needed

$$\oint dz \frac{t(z)^{-1}}{(z-E)(z-C)}$$



The only other structure apart from the threshold that can give rise to a strong distortion in $t(E)^{-1}$ is a pole at M_Z

$$t(E) = \frac{1}{\frac{\lambda}{E-M_Z} + \beta - ik}$$

CDD pole Castillejo,Dalitz,Dyson, PR,101,453(1956)

The general formula for a partial-wave without crossed-channel dynamics was deduced in: Oller, Oset PRD60,074023 (1999)

Final State Interactions (FSI)

- We follow the formalism of Meißner,Oller, NPA679,671(2001)

$$d(E) = \frac{1}{1 + \frac{E-M_Z}{\lambda}(\beta - ik)} = \frac{\lambda}{E - M_Z} t(E)$$

- It drives FSI in a production amplitude $\Gamma(s)$:

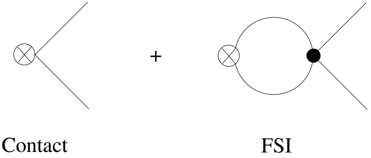
$$\Gamma(E) = d(E)R(E) \ ,$$

$R(E)$ has no right-hand cut

$$\text{Im}\Gamma(E) = \theta(E)k\, t(E)\Gamma(E)^*$$

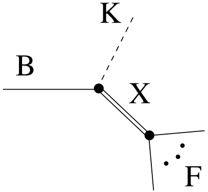
Diagrammatically (point-like source):

$$\Gamma(E) = v_1 - v_1(\delta_1 - ik)t(E)$$



Partial-decay width formulae

$$T_F = -\frac{\mathcal{V}_L \mathcal{V}_X}{Q^2 - P_X^2}$$



$$\Gamma_{B\to KF} = \frac{1}{2M_B} \int (2\pi)^4 \delta(P - Q - p_k) \frac{d^3 p_K}{(2\pi)^3 2E_K} d\mathcal{F} \frac{|\mathcal{V}_L|^2 |\mathcal{V}_X|^2}{|Q^2 - P_X^2|^2}$$

$$\frac{d\Gamma_{B\to KF}}{dE} = \frac{\Gamma_{B\to KX}(Q^2)\,\Gamma_{X\to F}(Q^2)}{2\pi|E - E_X + i\Gamma_X/2|^2} \; ,$$

$$\frac{d\Gamma_{B\to KF}}{dE} = \Gamma_{B\to KX}(Q^2)\Gamma_{X\to F}(Q^2)\frac{|d(E)|^2}{2\pi|\alpha|^2} \; .$$

$D^0\bar{D}^{*0}$ invariant-mass distributions

- Measured by Belle PRD81,031103(2010) and Babar PRD77,011102(2008) from $D^0\bar{D}^0\pi^0 + D^0\bar{D}^0\gamma$
- As a function of the invariant mass distribution of $D^0\bar{D}^{*0}$:

The proper equation can be obtained by splitting the three-body phase space, introducing an intermediate $\bar{D}^{*0}(\delta + \mathcal{E} + \mathbf{p}^2_{D^*}/2M_{D^*}, \mathbf{p}_{D^*})$

$$1 = \int (2\pi)^4 \delta(\mathbf{p}_{D^*} - \mathbf{p}_D - \mathbf{p}_\pi)\delta(\delta + \mathcal{E} + \frac{\mathbf{p}^2_{D^*}}{2M_{D^{*0}}} - \frac{\mathbf{p}^2_D}{2M_{D^0}} - \frac{\mathbf{p}^2_\pi}{2M_{\pi^0}})\frac{d^3p_{D^*}}{(2\pi)^3}\frac{d\mathcal{E}}{2\pi}$$

$$\delta = M_{D^{*0}} - M_{D^0} - M_{\pi^0}$$

$$\mathcal{E}' = E - \mathcal{E} = \frac{\mathbf{p}^2_{\bar{D}}}{2\mu}$$

Finite width of the D^{*0}, $\Gamma_* \approx 65$ keV

$$\frac{d\Gamma_{B\to KD^0\bar{D}^{*0}}}{d\mathcal{E}'} = \frac{\Gamma_B \mathcal{B}\sqrt{\mathcal{E}'}}{\sqrt{2}\pi\sqrt{E_X + \sqrt{E_X^2 + \Gamma_*^2/4}}}\int_{-\infty}^{+\infty} dE \frac{\Gamma_*}{(\mathcal{E}' - E)^2 + \frac{\Gamma_*^2}{4}} \frac{\Gamma_X |d(E)|^2}{2\pi|\alpha|^2}$$

$$\mathcal{B} = \frac{\Gamma_{B\to KX}\Gamma_{X\to D^0\bar{D}^{*0}}\left(\Gamma_{D^{*0}\to D^0\pi^0} + \Gamma_{D^{*0}\to D^0\gamma}\right)}{\Gamma_B\Gamma_X\Gamma_*} = \frac{\Gamma_{B\to KX}\Gamma_{X\to D^0\bar{D}^{*0}}}{\Gamma_B\Gamma_X}$$

$$\frac{d\Gamma_{B\to KD^0\bar{D}^{*0}}}{d\mathcal{E}'} = \frac{\Gamma_B \mathcal{B}\sqrt{\mathcal{E}'}}{\sqrt{2}\pi\sqrt{E_X+\sqrt{E_X^2+\Gamma_*^2/4}}}\int_{-\infty}^{+\infty} dE \frac{\Gamma_*}{(\mathcal{E}'-E)^2+\frac{\Gamma_*^2}{4}} \frac{\Gamma_X |d(E)|^2}{2\pi|\alpha|^2}$$

$$\mathcal{B} = \frac{\Gamma_{B\to KX}\Gamma_{X\to D^0\bar{D}^{*0}}}{\Gamma_B\Gamma_X}$$

$$\frac{d\hat{M}(E)}{dE} = \frac{\Gamma_X |d(E)|^2}{2\pi|\alpha|^2}$$

$$\int_{-\infty}^{+\infty} dE \frac{d\hat{M}(E)}{dE} = N$$

$N=1$ for bound state, narrow resonance

Mass distribution of the state

But not for a virtual state

Finite width of the D^{*0}, $\Gamma_* \approx 65$ keV
Its importance was first emphasized in Braaten,Lu PRD76,094028(2007)

Moving to the pole position of the D^{*0}, $\delta - i\frac{\Gamma_*}{2}$

$$k(E) = \sqrt{2\mu E} \to \sqrt{2\mu(E + i\frac{\Gamma_*}{2})}$$

Alvarez-Ruso, Oller, Alarcon, PRD80,054011(2009)

This is appropriate because

$$\lambda = \frac{\Gamma_*}{2\delta} = 4.5 \times 10^{-3} \ll 1$$

Hanhart, Kalashnikova, Nefediev, PRD81,094028(2010)

$$t(E) = \frac{1}{\frac{\lambda}{E - M_Z} + \beta - i\sqrt{2\mu(E + i\Gamma_*/2)}}$$

1st Riemann Sheet (RS): $\mathrm{Arg}(\mathrm{E} + \mathrm{i}\Gamma_*/2) \in [0, 2\pi[$
2nd RS: $\mathrm{Arg}(\mathrm{E} + \mathrm{i}\Gamma_*/2) \in [2\pi, 4\pi[$

Interference effects Voloshin, PLB579,316(2004), $J^{PC} = 1^{++}$,

$$X(3872) = \frac{1}{\sqrt{2}}(D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})$$

$$\begin{aligned} X(3872) &\to D^0\bar{D}^{*0} \to D^0(\bar{D}^0\pi^0) \\ &\to \bar{D}^0 D^{*0} \to \bar{D}^0(D^0\pi^0) \end{aligned}$$

Studied in detail in Hanhart, Kalashnikova, Nefediev, PRD81,094028(2010). Very modest effects for above threshold production ($\lambda \ll 1$) and virtual states. Suppressed for $X \to 0$ (bound states) Voloshin, PLB579,316(2004)

Event distributions

- $D^0\bar{D}^{*0}$

$$N_i(E_i) = \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_0^{\infty} dE_{\rm exp} R(E', E_{\rm exp}) \sqrt{E_{\rm exp}} \times \left\{ Y_D \int_{-\infty}^{\infty} dE \frac{\Gamma_*}{(E_{\rm exp}-E)^2+\Gamma_{*0}^2/4} \frac{d\hat{M}(E)}{dE} + \widetilde{\rm cbg}_D \right\}$$

- $J/\psi\pi^-\pi^-$

$$N_i(E_i) = \left\{ Y_J \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_{-\infty}^{\infty} dE R(E', E) \frac{d\hat{M}(E)}{dE} + \widetilde{\rm cbg}_J \Delta \right\}$$

- Inclusive $p\bar{p} \to J/\psi\pi^+\pi^-$

$$N_i = \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_{-\infty}^{\infty} dE R_{p\bar{p}}(E', E) \left(Y_p \frac{d\hat{M}(E)}{dE} + \zeta + \varrho E \right)$$

7 free parameters:

3 Yields: $Y_{D,J,p}$.

4 Constants for combinatorial backgrounds: $\widetilde{\rm cbg}_{D,J}$, ζ, ϱ

$$t(E) = \frac{1}{\frac{\lambda}{E-M_Z} + \beta - i\sqrt{2\mu(E + i\Gamma_*/2)}}$$

3 free parameters in $t(E)$

Too many

We study interesting different scenarios with less free parameters

Case 1. Single shallow bound pole

$$t(E) = \frac{1}{-\gamma - ik}$$

1 free parameter in $t(E)$ Braaten, Lu, PRD76,094028(2007)

$$\gamma = -\beta + \frac{\lambda}{M_Z} \; , \; \frac{E}{M_Z} \to 0$$

- 8 free parameters

$$\gamma = 21.36^{+0.58}_{-0.53} \text{ MeV} \to a = 9.24^{+0.23}_{-0.23} \text{ fm}$$

- Bound State:

$$E_R = -0.24^{+0.01}_{-0.01} - i\,0.0325 \text{ MeV}, \; g^2 = 16.7^{+0.5}_{-0.4} \text{ GeV}^2, \; X = 1.0$$

Case 2. Double virtual-state pole

- We impose resonance poles at $E_R \pm i G_R/2$, $E_R < 0$ and $G_R \to 0^+$. This is appropriate for S waves Hanhart,Pelaez,Rios, PLB,739,375(2014)

- We deduce:

$$\lambda = \sqrt{\frac{\mu}{2|E_R|}}(E_R - M_Z)^2 \ ,$$
$$\beta = \sqrt{\frac{\mu}{2|E_R|}}(M_Z - 3E_R) \ .$$

- Secular equation for the pole positions

$-i\kappa$ is the momentum of the double virtual state

$$(k + i\kappa)^2 (k - i\frac{\kappa^2 - 2\mu M_Z}{2\kappa}) = 0$$
$$E_R = -\frac{\kappa^2}{2\mu}$$

- We fit 147 points:

  Belle (2010) $B \to K D^0 \bar{D}^{*0}$: 50
  Belle (2008) $B^+ \to K^+ J/\psi \pi^+ \pi^-$: 40
  BaBar (2008) $B^+ \to K^+ J/\psi \pi^+ \pi^-$: 20
  CDF (2009) inclusive $p\bar{p} \to J/\Psi \pi^+ \pi^-$: 37

- 9 free parameters

$$E_R = -0.28^{+0.06}_{-0.04} \;,\; M_Z = -8.91^{+0.07}_{-1.26} \text{ MeV}$$

$$-2\log L + 2\log L_{max} = 10.7 \text{ for } 138 \text{ dof } (\Delta\chi^2 = n_\sigma \sqrt{2 \cdot 138} \approx 17 n_\sigma)$$

Etkin *et al.*, PRD25,1786(1982)

- Outcome for scattering parameters:

$$\beta = -335 \text{ MeV} \;,\; M_Z = -8.91 \text{ MeV} \;,\; \lambda = 3094 \text{ MeV}^2$$

$$\gamma = 1/a = -12 \text{ MeV} \;,\; a = -16.45 \text{ fm} \;,\; r = -7.96 \text{ fm}$$

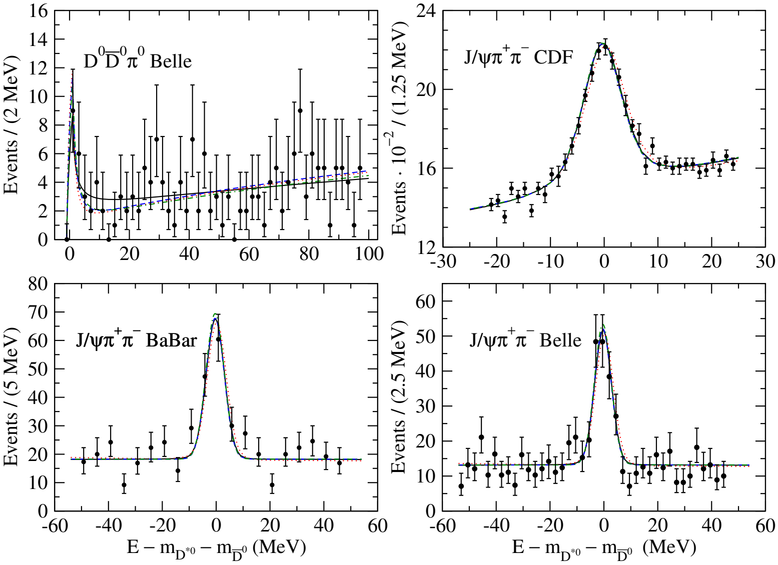
$$E_R \neq -\gamma^2/2\mu = -0.075 \text{ MeV}$$

- Poles

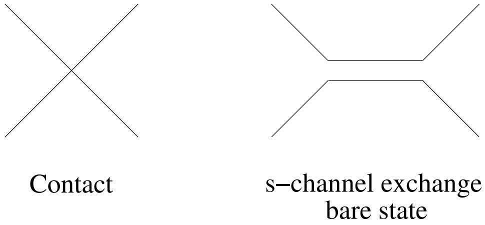
$$E_V = -0.15 - i0.13 \;,\; -0.41 + i0.12 \text{ MeV}$$

$$E_B = -75.4 - i0.025 \text{ MeV}$$

Case 2: Red dotted line



Contact interaction plus s-channel exchange of bare resonances



[BHKKN] Baru,Hanhart,Kalashnikova,Kudryavtsev,Nefediev EPJA,44,93(2010) *Interplay of quark and meson degrees of freedom in a near-threshold resonance*

[ABK] Artoisenet,Braaten,Kang, PRD,82,014013(2010) *Using line shapes to discriminate between binding mechanisms for the* $X(3872)$

[BHKKN]

$$D(E) = E - E_f - \frac{(E-E_f)^2}{(E-M_Z)^2} + \frac{i}{2} g_f k$$

$$t(E) = \frac{g_f}{8\pi^2 \mu D_F(E)}$$

$$t(E) = \frac{1}{4\pi^2\mu} \frac{E - E_f + \frac{1}{2} g_f \gamma_V}{(E-E_f)(\gamma_V + ik) + \frac{i}{2} g_f \gamma_V k}$$

$$g_f = \frac{2\lambda}{\beta^2}$$

$$E_f = M_Z - \frac{\lambda}{\beta}$$

$$\gamma_V = -\beta$$

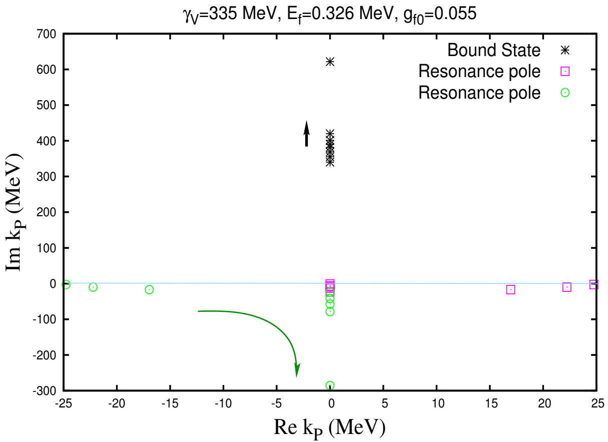
$\gamma_V = 1/a_V$, a_V scattering length in pure contact-interaction theory.

For $|M_Z| \gg |E_f|$ one recovers the standard Flatté approximation

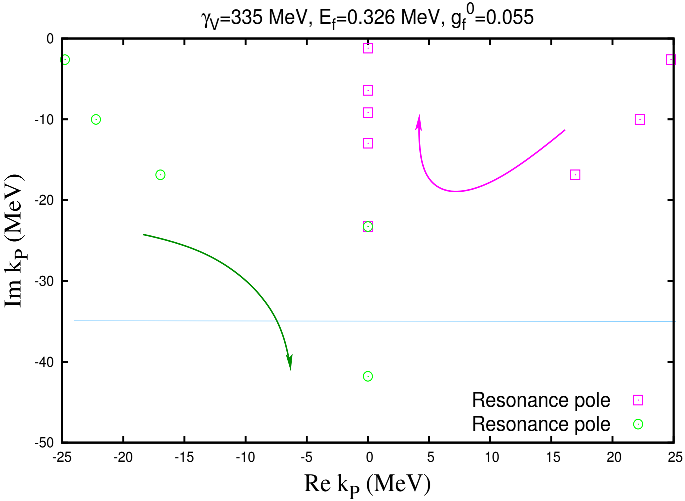
[BHKKN] gives quantitative analyses for $|\gamma_V|$ between $20 - 55$ MeV

Case 2:

$\gamma_V = -\beta = 335$ MeV, much larger value
$E_f = 0.236$ MeV, $\sqrt{2\mu E_f} = \pm 25.1$ MeV
$g_f = 0.055$. Plots with increasing g_f



Detail of the near-threshold region



Spectral density function $w(E)$

Baru,Haidenbauer,Hanhart,Kalashnikova,Kudryavtsev, PLB,586,53(2004)

Bare state $|\psi_0\rangle$

$$|\psi_0\rangle = \int d\mathbf{k} c_0(k)|\mathbf{k}\rangle$$

$$\omega(E) = 4\pi\mu k|c_0(E)|^2\theta(E)$$

$$W = \int_0^\infty dE\,\omega(E)$$

Bound states $|B_i\rangle$

$$W = 1 - \sum_i Z_i$$

$$Z_i = |\langle\psi_0|B_i\rangle|^2$$

One recovers results in Weinberg, PR,137,B672(1965); clear connection with the pole-counting rule of Morgan NPA,543,632(1962)

It provides a nice smooth transition from the clear bound states and narrow resonances ($g_f \to 0$)

Still, conceptually, it is not fully settled as a quantitative estimate of compositeness for resonances

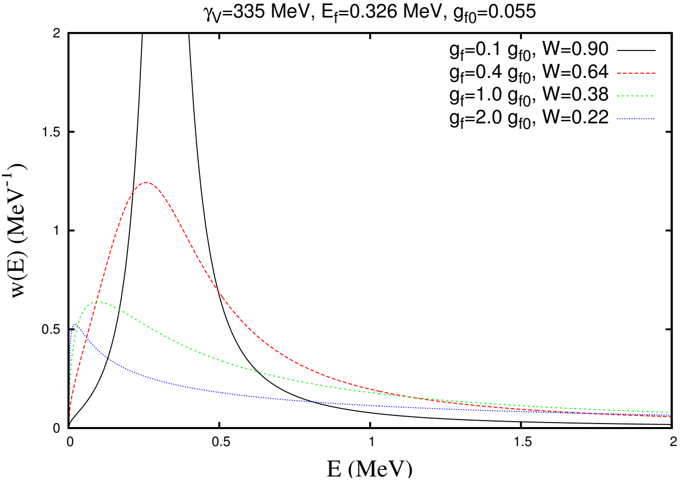
A cutoff $E_+ = 1$ MeV is introduced to cover the $X(3872)$ energy region

$$W = \int_0^{E_+} dE\, \omega(E)$$

For [BHKKN] and our own parameterization $\omega(E)$ reads:

$$\begin{aligned}\omega(E) &= \theta(E)\frac{\gamma_V(E_f - M_Z)\,k/\pi}{|\gamma_V(E - E_f) + i(E - M_Z)k|^2}\\ &= \theta(E)\frac{\lambda k/\pi}{|\lambda + (\beta - ik)(E - M_Z)|^2}\end{aligned}$$

$$W \to \frac{\sqrt{8E_+/\mu}}{\pi g_f} \quad \text{for } g_f \gg 1$$



$W = 0.38$ for our fit $\rightarrow$ sub-leading bare component.

To be expected because $M_Z = -8.91$ MeV is relatively far away from threshold.
$\omega(E) \propto \lambda / M_Z^2$, connection with analysis in Morgan NPA,543,632(1962)

[ABK]

$$\Delta = M_{D^+} + M_{D^{*-}} - M_{D^0} - M_{D^{*0}}$$
$$\kappa_2 = \sqrt{2\mu\Delta}$$

$$t(E) = \frac{(-\gamma_1 - \gamma_0 + 2\kappa_2)(E - \nu + g^2\gamma_0) + g^2\gamma_0^2}{D(E)}$$
$$D(E) = (2\gamma_1\gamma_0 - (\gamma_0 + \gamma_1)(\kappa_2 - ik) - i2\kappa_2 k)(E - \nu + g^2\gamma_0) + g^2\gamma_0^2(-2\gamma_1 + \kappa_2 - ik)$$

γ_0, γ_1: isoscalar and isovector scattering lengths for the pure zero-range contact interactions.

$$\lambda = \frac{2g^2\gamma_0^2(\gamma_1 - \kappa_2)^2}{(\gamma_0 + \gamma_1 - 2\kappa_2)^2}$$
$$M_Z = \nu - \frac{g^2\gamma_0(\gamma_1 - 2\kappa_2)}{\gamma_0 + \gamma_1 - 2\kappa_2}$$
$$\beta = \frac{-2\gamma_0\gamma_1 + (\gamma_0 + \gamma_1)\kappa_2}{\gamma_0 + \gamma_1 - 2\kappa_2} = -1/a_V$$

- [ABK] takes $g = 0.4$ from quark models + $^3P_0$ model, CCC decay models ($\nu > 50$ MeV, Flatté limit)

- [ABK] argues that $|\gamma_1| \gg \kappa_2 = 125$ MeV

Expected scale for $|\gamma_1|$ is 400 MeV ($1/r^3$ pion-exchange potential)

[ABK] secular equation for $|\gamma_1| \gg \kappa_2$ and $\gamma \to 0$:

$$\frac{1}{\gamma_0} = \frac{g^2}{\nu} + \frac{2}{\kappa_2}$$

Fine-tuning resonance position ($\gamma_0 \gg \kappa_2/2$)

$$\nu_F \approx -\frac{g^2 \kappa_2}{2}$$

- Our fit with $\gamma_1 = 400$ MeV corresponds to:

$$\begin{array}{ll} \gamma_0 = 295 \text{ MeV} & g = 0.22 \\ \nu = -4.3 \text{ MeV} & \nu_F = -3 \text{ MeV} \end{array}$$

- Our type of fits are not discussed in [ABK]

[ABK] fits 28 points of $D^0\bar{D}^{*0}$ event distributions with 7 free parameters
Fixed: $g = 0.4$

- [ABK] estimates inverse scattering length $\gamma = 28^{+12}_{-20}$ MeV from the mass and with of the $X(3872)$ in $J\Psi\pi^+\pi^-$. Wrong if there is a CDD close to threshold. E.g. cases 3.a-b below

- A fit is presented with fixed: $g = 0.4$, $\gamma = 28$ MeV, $\nu = -10$, $\gamma_1 = 250$ MeV ($\gamma_0 = -204$ MeV)

- The main point of [ABK] is to discern between binding mechanics in two limit and fine-tuned cases:

Zero-range ($\gamma_{0,1}$) or near-threshold resonance quarkonium

- But within such scenarios there are still interesting issues left over: type of pole (simple, double, triplet), precise location, compositeness, further discussions for large $|1/a|$, etc.

Pole positions with $\Gamma_* = 65$ keV $\neq 0$:

$\rho = \Gamma_*/|E_R| = 0.23$, $\rho^{1/2} = 0.48$, $8E_R/M_Z = 0.25$

$$k_V = -i\kappa \left\{ 1 + \frac{1}{2}\rho^{1/2}(\pm 1 \mp i) \right\}$$

$$k_B = -i\left(\frac{\mu M_Z}{\kappa} - \frac{\kappa}{2}\right) - \kappa \frac{4E_R\Gamma_*}{M_Z^2}$$

Energy: $E = k^2/2\mu - i\Gamma_*/2$

$$E_V = E_R\left(1 + \rho^{1/2}(\pm 1 \mp i)\right)$$

$$E_B = -\frac{-1}{2\mu}\left(\frac{\mu M_Z}{\kappa} - \frac{\kappa}{2}\right)^2 - i\frac{\Gamma_*}{2}(1 - \frac{8E_R}{M_Z})$$

Non-analytic in $\rho \rightarrow$ stronger effects Hyodo, PRC90,055208(2014); Hanhart,Pelaez,Rios,PLB739,375(2014)

$$E_V = -0.15 - i0.13 \;,\; -0.41 + i0.12 \text{ MeV}$$

$$E_B = -75.4 - i0.025 \text{ MeV}$$

With $\Gamma_* = 0$: $E_V = E_R = -0.28$, $E_B = -75.32$ MeV

Limitation of [BHKKN] and [ABK]

- They predict only $\lambda \geq 0$

[BHKKN]	[ABK]
$\lambda = \frac{\gamma_V^2}{2} g_f$	$\lambda = \frac{2g^2\gamma_0^2(\gamma_1-\kappa_2)^2}{(\gamma_0+\gamma_1-2\kappa_2)^2}$

- Positive effective range r, v_3, v_5, etc, cannot be reproduced with $\lambda \geq 0$:

$$r = -\frac{\lambda}{\mu M_Z^2} < 0$$

$$v_3 = -\frac{\lambda}{8\mu^3 M_Z^4} < 0$$

- $\omega(E) \geq 0 \to \lambda \geq 0$:

$$\omega(E) = \theta(E)\frac{\lambda k/\pi}{|\lambda + (\beta - ik)(E - M_Z)|^2}$$

Constant contact term plus one s-channel bare-pole exchange picture collapses for $\lambda < 0$

Case 4. Triple virtual-state pole

Secular equation for case 2, double virtual pole at $E_R = -\kappa^2/2\mu < 0$:

$$(k + i\kappa)^2 (k - i\frac{\kappa^2 - 2\mu M_Z}{2\kappa}) = 0$$

We impose the third pole at $-i\kappa$:

$$M_Z = -3E_R$$

- 8 free parameters

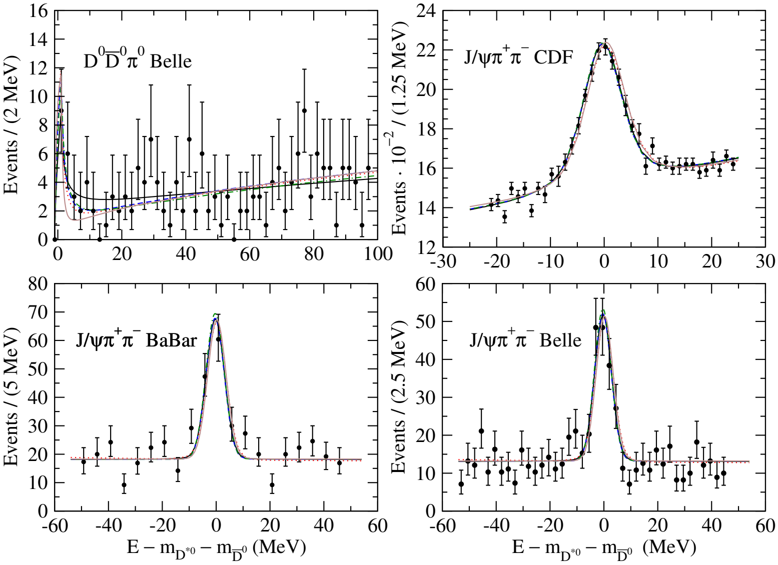
$$E_R = -0.43 \pm 0.07 \text{ MeV}$$
$$-2\log L + 2\log L_{max} = 23.6 \text{ for } 139 \text{ dof } (17\, n_\sigma)$$

- Outcome for scattering parameters:

$$\beta = 86.5 \text{ MeV}\ ,\ M_Z = 1.3 \text{ MeV}\ ,\ \lambda = 99.2 \text{ MeV}^2$$
$$\gamma = -9.65 \text{ MeV}\ ,\ a = -20.5 \text{ fm}\ ,\ r = -12.2 \text{ fm}$$

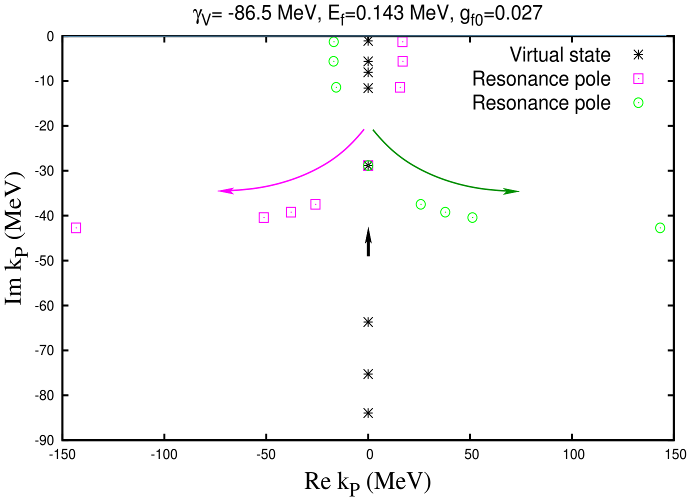
Both a and r and negative and large

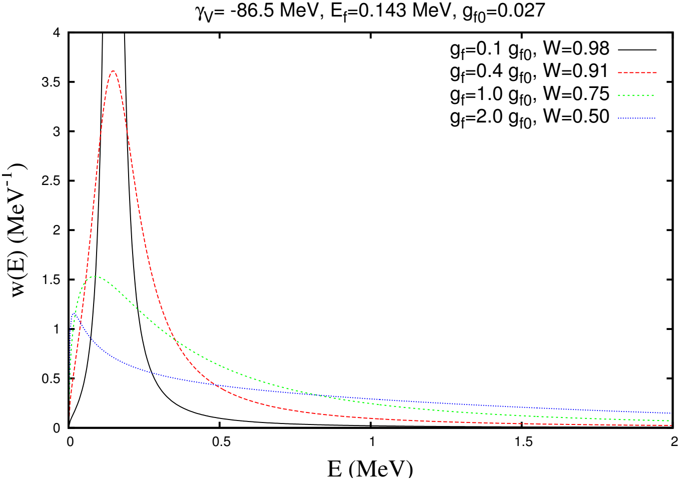
Case 4: Brown solid line



This pole trajectory is not an explicit example considered in [BHKKN]

$\gamma_V = -\beta = -86.5$ MeV, larger in absolute value
$E_f = 0.143$ MeV, $\sqrt{2\mu E_f} = \pm 16.7$ MeV
$g_f = 0.027$. Plots with increasing g_f





$W = 0.75$ for our fit, bare component is important.

This is in agreement with the fact that now $M_Z = 1.3$ MeV, closer to threshold.
Connection with analysis in Morgan NPA,543,632(1962)

[ABK]

- [ABK] takes $g = 0.4$ from quark models + $^3P_0$ model
- [ABK] argues that $|\gamma_1| \gg \kappa_2 = 125$ MeV

- Our fit has larger sensitivity on γ_1 than case 2.

$\gamma_1 = 800$ MeV:

$$\gamma_0 = 33 \text{ MeV} \quad g = 0.18$$
$$\nu = 2.3 \text{ MeV} \quad \frac{2}{\kappa_2} + \frac{g^2}{\nu} \simeq 2\frac{g^2}{\nu} \approx \frac{1}{\gamma_0}$$

$\gamma_1 = -800$ MeV:

$$\gamma_0 = 5.6 \text{ MeV} \quad g = 1.4$$
$$\nu = 12.7 \text{ MeV} \quad \frac{g^2}{\nu} \simeq \frac{1}{\gamma_0}$$

Both fine-tuning in γ_0 and in ν plays an important role.

Pole positions with $\Gamma_* = 65$ keV $\neq 0$:

$\rho = \Gamma_*/|E_R| = 0.15$, $\rho^{1/3} = 0.53$

$$k_1 = -i\kappa - \kappa\rho^{1/3}$$
$$k_{2,3} = -i\kappa\left(1 \pm \frac{\sqrt{3}}{2}\rho^{1/3}\right) + \frac{1}{2}\kappa\rho^{1/3}$$

Energy: $E = k^2/2\mu - i\Gamma_*/2$

$$E_1 = E_R\left(1 - i2\rho^{1/3}\right)$$
$$E_{2,3} = E_R\left(1 \pm \frac{\sqrt{3}}{2}\rho^{1/3}\right)^2 + iE_R\left(1 \pm \frac{\sqrt{3}}{2}\rho^{1/3}\right)\rho^{1/3}$$

Non-analytic in $\rho \to$ even stronger effects

$$E_V = -0.35 + i0.45\ ,\ -0.87 - i0.31\ ,\ -0.07 - i0.17\ \text{MeV}$$

With $\Gamma_* = 0$: $E_V = E_R = -0.43$ MeV

Triple & Double poles

• The triple-pole situation goes beyond the "general" treatment on pole trajectories as a function of a strength parameter given in Hanhart,Pelaez,Rios, PLB739,375(2014)

Now, assuming that there is at least one resonance pole, and that it is not too far away from threshold, we are now in the position of writing down the most general expression for the S-matrix in the vicinity of that pole or its conjugate partner

This is not right when three (or more) poles coalesce

Case 3. Near-threshold bound and virtual states

$$\sigma_1 = M_{D^0} - M_{D^{*0}} \ , \ \sigma_2 = M_{D^+} - M_{D^{*+}}$$
$$\Delta = \sigma_2 - \sigma_1$$

- Conclusions in [BHKKN]:

However, a near-threshold t-matrix zero could exist only if several requirements are met. First, one needs the direct interaction in the mesonic channel to be strong enough to support a bound or virtual state. Second, a nearby bare quark state should exist, with a weak coupling to the mesonic channel ... Without such special arrangements the effective-range formulae are valid
... one can see then that $M_Z \ll \Delta$ only if all three poles are located very close to the threshold.

- **This conclusion is wrong. Why?**

Let q_i be the roots of secular equation

$$M_Z = -\frac{1}{2\mu}(q_1(q_2+q_3)+q_2q_3)$$

[BHKKN] requires that all $|q_i| \ll \kappa_2$ for $|M_Z| \ll \Delta$

But, even if $|q_1| \gg |q_2|, |q_3|$ with $q_2+q_3 \approx \mathcal{O}(|q_{2,3}|^2/|q_1|)$ then $|M_Z| \ll \Delta$

- Here I develop an explicit example

- We impose resonance poles at $M_R \pm iG_R/2$, $G_R \to 0^+$, *both* in $D^0\bar{D}^{*0}$ and D^+D^{*-}

As in case 2, but now for each of the two channels separately

- Pole position is independent of the two-meson threshold → Dominant bare component in the resonance.

Connection with pole-counting criterion Morgan NPA,543,632(1962)

- $D^0\bar{D}^{*0}$: $E_R = M_R - \sigma_1$,

$$\widetilde{\lambda} = \sqrt{\frac{\mu}{2|E_R|}}(E_R - M_Z^{(1)})^2 \ ,$$

$$\widetilde{\beta} = \sqrt{\frac{\mu}{2|E_R|}}(M_Z^{(1)} - 3E_R) \ .$$

- $D^+\bar{D}^{*-}$: $E_R - \Delta = M_R - \sigma_2$

$$\widetilde{\lambda} = \sqrt{\frac{\mu}{2(\Delta + |E_R|)}}(E_R - \Delta - M_Z^{(2)})^2 \ ,$$

$$\widetilde{\beta} = \sqrt{\frac{\mu}{2(\Delta + |E_R|)}}(M_Z^{(2)} - 3(E_R - \Delta)) \ .$$

- We equate expressions for $\widetilde{\lambda},\ \widetilde{\beta} \longrightarrow M_Z^{(i)} \longrightarrow \widetilde{\lambda},\ \widetilde{\beta}$
- $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering: Leading isospin-breaking effects due to threshold branch-point singularity, $k_i(E)$.

$$t(E) = \frac{1}{\frac{2\widetilde{\lambda}}{E - M_Z} + 2\widetilde{\beta} - ik_2 - ik_1}$$

For $|E| << \Delta$, $-ik_2(E) \approx \kappa_2$

$$\kappa_2 = \sqrt{2\mu\Delta}\ ,\ \kappa_R = \sqrt{2\mu|E_R|}$$

$$\alpha = \sqrt{\frac{\kappa_2}{\kappa_R}} = \left(\frac{\Delta}{|E_R|}\right)^{1/4} \simeq \left(\frac{8}{0.4}\right)^{1/4} = 2.1$$

1st solution, case 3.a)

$$\lambda = 2\widetilde{\lambda} = \frac{8\mu\alpha^2}{\kappa_R(1+\alpha)^2}\left(E_R + \sqrt{|E_R|(\Delta - E_R)}\right)^2$$

$$\beta = 2\widetilde{\beta} + \kappa_2 = \frac{4\mu}{\kappa_R(1+\alpha)}\left(-E_R + \alpha\sqrt{|E_R|(\Delta - E_R)}\right) + \kappa_2$$

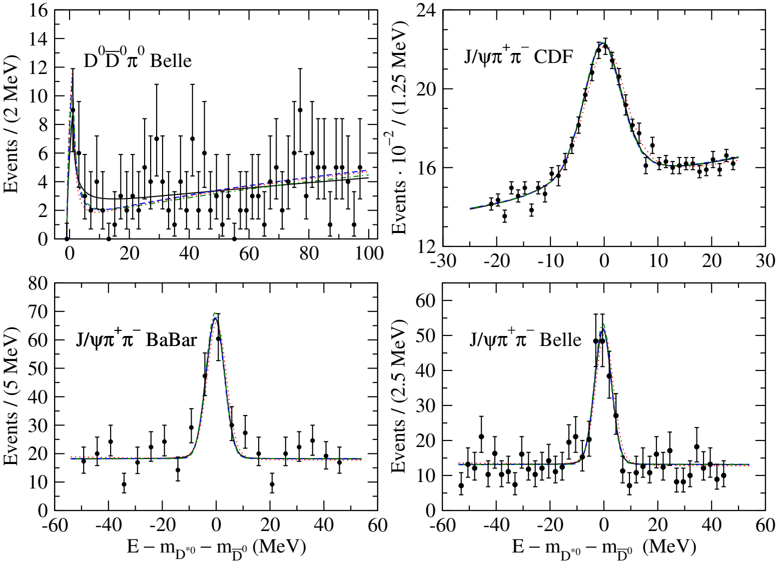
2nd solution, case 3.b): $\alpha \leftrightarrow -\alpha$

- Bound state at E_R required $\rightarrow M_Z = E_R + \lambda/(\beta + \kappa_R)$

- 8 free parameters

	Case 3.a	Case 3.b
E_R (MeV)	$-0.40^{+0.08}_{-0.01}$	$-0.39^{+0.03}_{-0.02}$ MeV
$-\frac{2}{N_{\text{dof}}} \log \frac{L}{L_{max}}$	7.4 & $17 n_\sigma$	6.4 & $17 n_\sigma$

Case 3.a, Blue dashed; Case 3.b, Green-Dashed Dotted Lines



Case 3a

- Scattering parameters

$\lambda = 254.6$ MeV^2, $M_Z = 0.33$ MeV, $\beta = 316.4$ MeV

Bound state:

$E_B = -0.41$ MeV, $g^2 = 1.51$ GeV^2, $X = 0.07$, $Z = 1 - X = 0.93$

Virtual state:

$E_V = -0.57$ MeV, $g^2 = 2.63$ GeV^2

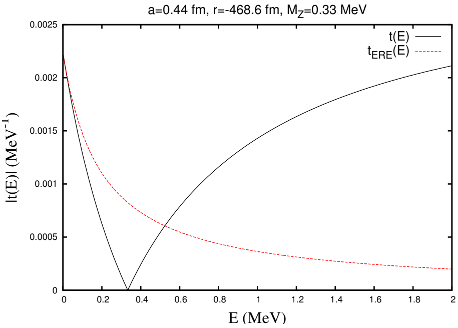
Note: $M_Z < |E_B|$, $|E_V|$

$$a = 0.44 \text{ fm} \ , \ r = -469 \text{ fm} \ , \ v_2 = -364 \text{ fm}^3 \ , \ v_3 = -56 \text{ fm}^5$$

Failure of ERE if $M_Z < |E_{B,V}|$

used e.g. in Hanhart,Kalashnikova,Kudryavtsev,Nefediev,PRD76,034007(07); Braaten, Lu, PRD76,094028(07); Braaten, Stapleton, PRD81,014019('10); Zhang,Meng,Zheng,PLB680,453(09); Guo-Ying,Wen-Sheng,Qiang CPC,39,093101(15)

$$t(E)=\left(\frac{\lambda}{E-M_Z}+\beta-ik\right)^{-1}\to t_{ERE}(E)=\left(-\frac{1}{a}+\frac{1}{2}r^2k^2+v_2k^4+v_3k^6+\mathcal{O}(k^8)-ik\right)^{-1}$$



Pole position

Light bound-virtual poles:

$k_{B,V}=i28.1,\ -i33.1$ MeV

r: $k_{B,V}=i19.0,\ -i19.9$ MeV

v_2: 4 Poles
$k_{B,V}=\pm 8.9+i20.2,\ \pm 9.6-i20.2$ MeV

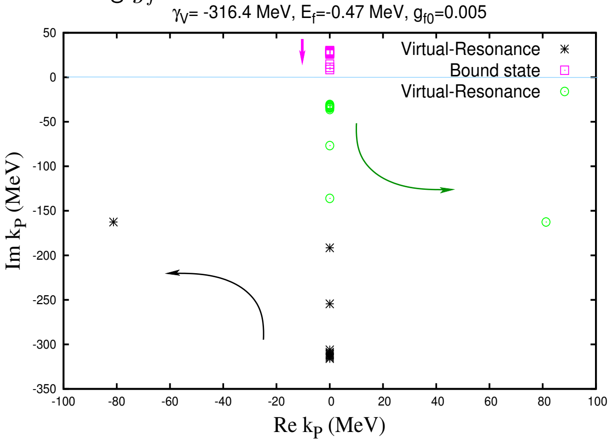
v_3: $k_{B,V}=i21.3,\ -i22.0$ MeV

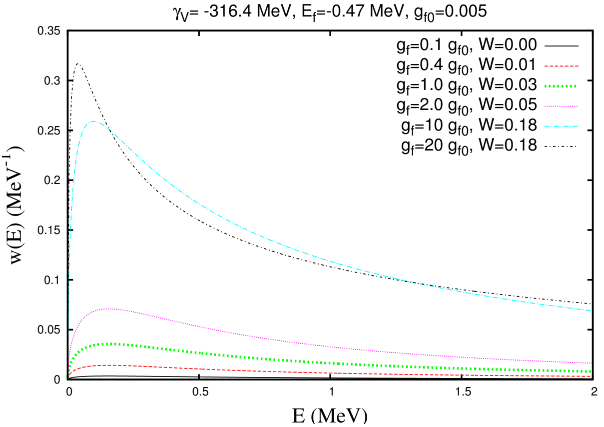
No convergent pattern
It would also spoil the application of XEFT
Fleming, van Kolck, Mehen, Hammer, Braaten,...
PRD76,034006(07)

This pole trajectory $k_1 \approx -k_2$, $|k_3| \gg |k_{1,2}|$ is not considered in [BHKKN]. γ_V between 20-55 MeV

$\gamma_V = -\beta = -316.4$ MeV, Large number in absolute value
$E_f = -0.47$ MeV, $\sqrt{2\mu E_f} = \pm i30.2$ MeV
$g_f = 0.005$. 10 times smaller than for case 2.

Plot with increasing g_f





$X \approx W = 0.03$, $Z \approx 1 - W = 0.97$, bare component is overwhelming in the bound state, $E_B = -0.41^{+0.08}_{-0.03}$ MeV.

This is in agreement with the fact that now $M_Z = 0.33$ MeV is "on top" of threshold.

Connection with analysis in Morgan NPA,543,632(1962)

- Compositeness for a bound state in the form $X = g^2 \frac{dG(E)}{dE}$

Hyodo,Jido,Hosaka, PRC85,015201(2012); Aceti,Oset, PRD86,014012(2012); Sekihara,Hyodo,Jido, PTEP2015,063D04(2015); Guo,Oller, PRD93, 096001 (2016)

$$X = -\, i g_k^2$$

g_k^2 is the residue in the k variable Kang,Guo,Oller, PRD94,014012(2016)

Cases 2 and 3.a-b have a bound state (k_1) and two virtual states

$$X = i g_k^2 = \frac{(k_1+k_2)(k_1+k_3)}{(k_1-k_2)(k_1-k_2)} = \frac{(|k_1|-|k_2|)(|k_1|-|k_3|)}{(|k_1|+|k_2|)(|k_1|+|k_3|)} \leq 1$$

Case 3.a, bound state: $X = 0.069^{+0.005}_{-0.002}$, compares well with $W = 0.03$ ($W = 0.09$ if $\omega(E)$ is integrated up to ∞)

- For virtual state $|X|$ is not bounded , Guo,Oller,PRD93, 096001(2016)

Cases 3: Virtual state k_3, $|k_3| \gg |k_2| > |k_1|$

$$X \approx 1 + 2\frac{|k_2|-|k_1|}{|k_3|} + \ldots > 1$$

Case 3b

- Scattering parameters

$\lambda = 1880.2$ MeV^2, $M_Z = 2.87$ MeV, $\beta = 546.8$ MeV

Bound state:

$E_B = -0.40$ MeV, $g^2 = 3.58$ GeV^2, $X = 0.16$, $Z = 1 - X = 0.84$

Virtual state:

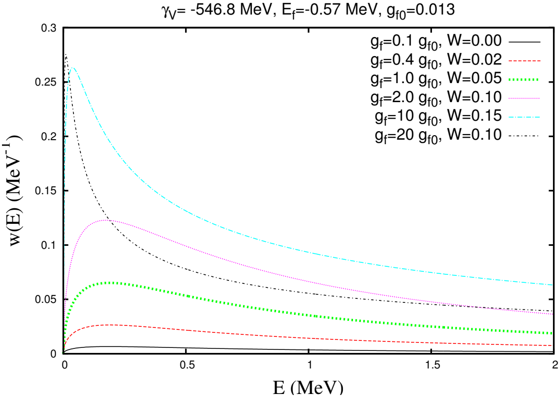
$E_V = -0.85$ MeV, $g^2 = 6.70$ GeV^2

$$a = 1.81 \text{ fm} \; , \; r = -46.7 \text{ fm} \; , \; v_2 = -4.2 \text{ fm}^3 \; , \; v_3 = -0.08 \text{ fm}^5$$

$M_Z > |E_B|$. Weinberg's relation for elementariness Z applies Weinberg, PR137,B672 (1965)

$Z = 1 - X = \frac{2(-1+a\kappa_B)}{-2+a\kappa_B} = 0.86$ in good agreement

$X = 0.16$ is larger than in case 3a ($X = 0.07$).
This is in agreement with the fact that now $M_Z = 2.8 \gg 0.33$ MeV



$X \approx W = 0.05$, $Z \approx 1 - W = 0.95$, bare component is overwhelming

Some disagreement between W and our determination or Weinberg's formula

When integrating up $\infty \to W = 0.16$ and the agreement is restored

[ABK]

- [ABK] takes $g = 0.4$ from quark models + $^3P_0$ model
- [ABK] argues that $|\gamma_1| \gg \kappa_2 = 125$ MeV
- [ABK] takes $\gamma = 1/a = 28^{+12}_{-20}$ MeV from $X(3872)$ mass

For cases 3.a-b γ is much bigger:

Case 3a: $\gamma = 448$ MeV **Case 3b**: $\gamma = 109$ MeV

One cannot estimate a from the mass of the resonance

$$\gamma \gg \sqrt{2\mu|E_B|}$$

[ABK] fine-tuning condition to obtain a small scattering length cannot be applied here in these cases:

$$\frac{1}{\gamma_0} \neq \frac{g^2}{\nu} + \frac{2}{\kappa_2}$$

- Our fit has larger sensitivity on γ_1 than case 2.

	γ_1 MeV	γ_0 MeV	ν MeV	g
Case 3.a	800	-41	-1.6	0.21
Case 3.a	-800	-165	-0.82	0.09
Case 3.b	800	-99	-2.3	0.21
Case 3.b	-800	-402	-1.3	0.12

ν is small. Bare state is nearby threshold.

Conclusions

- Study of line shapes of $X(3872)$. Keep an open mind.

- Possibility of a near threshold zero of $t(E)$ (CDD pole). Connection with increasing elementariness Z.

- Three shallow poles are not necessary to have a near-threshold CDD.

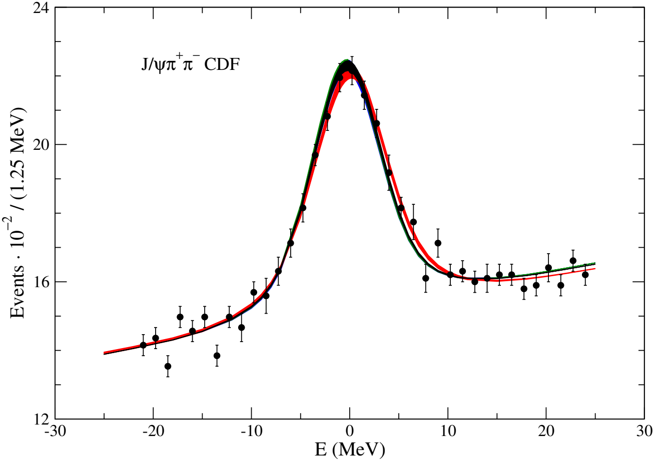
- Double virtual-state pole. Triple virtual-state pole.

- Non-analytic dependence on $\Gamma_*/E_R \to$ larger effects.

- Virtual- and bound-state poles simultaneously. $M_Z \to 0$, $X \to 0$.

- It is not necessary a large scattering length to produce a shallow pole.

CDF: $J/\psi\pi^+\pi^-$ with error bands



Limit $\alpha \to \infty$

$$\alpha = \left(\frac{\Delta}{|E_R|} \right)^{1/4} \xrightarrow[|E_R| \to 0]{} \infty$$

Relevant for the $X(3872)$.

Formulas get simpler

$$M_Z \to \frac{4}{3}\sqrt{\Delta |E_R|} = 1.8 \text{ MeV}$$
$$\beta \to 3\kappa_2 = 374.7 \text{ MeV}$$
$$\lambda \to 4\Delta\kappa_R = 898 \text{ MeV}$$

Poles:
$$\begin{aligned} \kappa_B &= \kappa_R & E_B &= E_R = -0.4 \text{ MeV} \\ \kappa_V &= \tfrac{13}{9}\kappa_R & E_V &= -0.83 \text{ MeV} \\ \kappa_V &= 3\kappa_2 & E_V &= -9\Delta = -73 \text{ MeV} \end{aligned}$$

Limit $\alpha \to \infty$, $\Gamma_* \neq 0$

- Virtual states

$$E_V = E_R(\frac{13}{9})^2 + i\frac{\Gamma_*}{11} \approx -0.84 \text{ MeV}$$
$$E_V = -9\Delta - i\frac{\Gamma_*}{2} \approx -73 - i0.033 \text{ MeV}$$

- Bound state:

$$E_B = E_R \approx -0.4 \text{ MeV}$$

The light E_V and E_B do not have imaginary part
$X \to 0$, no width from the constituents in the bound state

Analytic corrections in $\Gamma_*/E_R \to$ smaller effects.