Meson-meson interactions from $U(3)$ chiral perturbation theory

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Outline

1. Background
2. Setup of the analytical calculation
   - Relevant chiral lagrangian & perturbative calculation
   - Unitarization : N/D method
3. Preliminary numerical results
   - Fit results
   - Poles from the unitarized amplitudes
4. Conclusion & Outlook

Background

In the chiral limit $m_u = m_d = m_s = 0$ the QCD Lagrangian is invariant under $U_L(3) \otimes U_R(3)$ symmetry

$SU_L(3) \otimes SU_R(3) \to SU_V(3)$ is Spontaneously Broken. Goldstone bosons appear π, K, η

$U_V(1) \equiv U_{L+R}$ Conserved Baryon Number

$U_A(1) \equiv U_{L-R}$ Neither Conserved nor Goldstone Boson
Puzzle:
Goldstone mode: η_0 mass would be $< \sqrt{3} m_\pi$ Weinberg PRD'75 but η' is much heavier, $M_{\eta'} \sim 1$ GeV

The ninth axial singlet current has an anomalous divergence Adler PR'69 Fujikawa PRD'80

$$J_5^{\mu\,(0)} = \bar{q}\gamma_\mu\gamma_5 q$$
$$\partial_\mu J_5^{\mu\,(0)} = \frac{g^2}{16\pi^2}\frac{1}{N_c}\,Tr_c(G_{\mu\nu}\widetilde{G}^{\mu\nu})$$

Large N_c QCD 't Hooft NPB'74, Witten NPB'79 $N_c \to \infty$, $g^2 N_c \to constant$

$U_L(N_F) \otimes U_R(N_F) \to U_{L+R}(N_F)$ Coleman, Witten PRL'80
Entire Nonet of Goldstone bosons results

Explicit breaking of chiral symmetry due to quark masses and to the $U_A(1)$ anomaly are treated perturbatively

Combined power expansion in light quark masses and $1/N_c$

This formalism is set up in
Di Vecchia, Veneziano NPB'80
Rosenzweig, Schechter, Trahern PRD'80
Witten Ann.Phys.'80
The Leading Order in $1/N_c$ and the Derivative Expansion has been worked out

Herrera-Siklody, Latorre, Pascual, Taron NPB'97
Generalization of Gasser, Leutwyler Ann.Phys.'84, NPB'85 from $SU_L(3) \otimes SU_R(3)$ to $U_L(3) \otimes U_R(3)$

Generating functional in the presence of external sources $\mathcal{Z}[l, r, s, p, \theta]$. Chiral Lagrangian to $\mathcal{O}(p^4)$ and all orders in $1/N_c$

Bilinear Quark operators (currents) and the Topological Charge operator coupled to external sources:

$$\mathcal{L} = \mathcal{L}_{QCD} + \bar{q}_L \gamma_\mu \ell^\mu(x) q_L + \bar{q}_R \gamma_\mu r^\mu(x) q_R - \bar{q}_R (s(x) + ip(x)) q_L$$
$$- \bar{q}_L (s(x) - ip(x)) q_R - \frac{g^2}{16\pi^2} \frac{\theta(x)}{N_c} Tr_c(G_{\mu\nu} \widetilde{G}^{\mu\nu})$$

$$g_L = I + i(\beta - \alpha) \; , \; g_R = I + i(\beta - \alpha)$$
$$\theta(x) \to \theta(x) - 2\langle \alpha(x) \rangle$$

The extra term in the fermionic determinant due to the anomaly is compensated

The non-abelian anomaly cannot be compensated. The Wess Zumino Witten term has to be added by hand
Wess and Zumino PLB'71 , Witten NPB'83
Adler, PR'69, Bardeen, PR'69 , Adler and Bardeen, PR'69

$$D_\mu U \to g_R (D_\mu U) g_L^\dagger$$

The combination

$$X(x) = \langle \log U(x) \rangle + i\theta(x) \equiv i \frac{\sqrt{2} N_F}{f} \eta_0 + i\theta(x)$$

is invariant and any of its functions Witten NPB'79, Leutwyler PLB'96

Leading Order Lagrangian

$$\begin{aligned}\mathcal{L}_{0+2} = & -W_0(X) + W_1(X)\langle D_\mu U^\dagger D^\mu U \rangle + W_2(X)\langle U^\dagger \chi + \chi^\dagger U \rangle \\ & + iW_3(X)\langle U^\dagger \chi - \chi^\dagger U \rangle + W_4(X)\langle U^\dagger D_\mu U \rangle \langle U^\dagger D^\mu U \rangle \\ & + W_5(X)\langle U^\dagger (D_\mu U)\rangle D^\mu \hat{\theta} + W_6(X) D_\mu \hat{\theta} D^\mu \hat{\theta}\end{aligned}$$

$$W_4(0) = 0 \ , \ \ W_1(0) = W_2(0) = \frac{f^2}{4}$$

$1/N_c$ counting for couplings Leutwyler PLB'96

$$G(X) = g(\frac{X}{N_c})\, N_c^{2-N(Tr_F)-N(\hat{\theta})}$$

for Arbitrary Number of Flavors. $f \sim \sqrt{N_c} \rightarrow$ Each Loop Meson suppressed by N_c^{-1}

$$M^2_{\eta_0}\Big|_{U_A(1)} = -\frac{2N_F}{f^2} W_0''(0) \propto \frac{1}{N_c}$$

$$\mathcal{L}_4 = \sum_{i=0}^{57} \beta_i O_i$$

Herrera-Siklody *et al.* NPB'97

Current status along this line:

- Lagrangians up to $\mathcal{O}(p^4)$ have been thoroughly studied.
- The $\eta-\eta'$ mixing and parts of the η,η' decays have been calculated at one loop level.

Variant:

These Lagrangians were employed by Borasoy *et al* PRD'01, EPJA'01, NPA'02
One-loop calculation in Infrared Regularization for $\eta' \to \eta\pi\pi$ decay

It is stressed that $M_{\eta'}$ is large

But $M_{\eta'}$ also appears from vertices and there is nothing like 'baryon number conservation' that acts in the meson-baryon sector

Proliferation of free parameters. Poor predictive power.

δ-expansion: $p^2 \sim m_q \sim 1/N_c \sim \delta$
For the low energy implications of $U(3)$ theory Leutwyler PLB'96, Kaiser and Leutwyler EPJC'00

$\mathcal{L}_{\delta^0}$: B, F, M_0^2

$\mathcal{L}_{\delta}$: 10 operators

Loops are suppressed by $p^2/f^2 \sim \mathcal{O}(\delta^2)$

- We aim a the complete calculation of the meson-meson scattering within $U(3)$ χPT at one loop level and also study the various resonances by unitarizing the χPT amplitudes
- We include explicit exchange of tree level resonances instead of local counterterms

- Are the results stable under the inclusion of the η_0?
- Influence on the running of the pole positions with Large N_c
- Chiral symmetry restoration. Scalar and Pseudoscalar Spectrum.

Shifman, Vainshtein PRD'08

$$\Pi_S(Q) - \Pi_P(Q) \sim \frac{g^2\langle\bar{q}q\rangle^2}{Q^4} \; , \; j_S = \bar{q}q \; , \; j_P = \bar{q}\gamma_5 q$$

$$\Pi_{S,P}(Q) = -i\int d^4x\, e^{iqx}\, T\langle j_{S,P}(x) j_{S,P}(0)\rangle$$

Bernard, Duncan, LoSecco, Weinberg PRD'75

$$\int_0^\infty ds\left[\Pi_S^{(0)}(s) - \Pi_P^{(3)}(s)\right] = 0 = \int_0^\infty ds\left[\Pi_S^{(3)}(s) - \Pi_P^{(0)}(s)\right]$$

In the chiral limit Moussallam EPJC'99, HEP'00

$$\int_0^\infty \mathrm{Im}\Pi_{SS}^{0-3}(s)ds = 0 = \int_0^\infty \mathrm{Im}\Pi_{PP}^{0-3}(s)ds$$

Jamin,Oller,Pich, NPB'00 I=1/2 S-wave meson-meson scattering
It was not a full one-loop calculation for the kernel

Beisert and Borasoy, PRD'03 studied S-wave meson-meson scattering from $\mathcal{L}_{\delta^0}$ and $\mathcal{L}_{\delta}$
The interaction kernel is calculated at tree level
Local terms instead of resonance exchanges

Another framework is non-relativistic effective field theory Kubis and Schneider EPJC'09 studied cusp effects in $\eta' \to \eta\pi\pi$ similarly to $K \to 3\pi$ Colangelo,Gasser,Kubis,Rusetksy PLB'06

Relevant Chiral Lagrangian

$$\mathcal{L}^{(0)} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{3} M_0^2 \ln^2 \det U$$

where

$$U = e^{i\frac{\sqrt{2}\Phi}{2F}}$$

$$\Phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} \end{pmatrix}$$

In the calculation, we use

$$\begin{aligned} \eta &= C_\theta\, \eta_8 - S_\theta\, \eta_1 \,, \\ \eta' &= S_\theta\, \eta_8 + C_\theta\, \eta_1 \,, \end{aligned}$$

with

$$\theta = f(m_\pi, m_K, M_0, L_i, \ldots) \text{ or } f(m_\pi, m_K, M_0, c_d, c_m, \ldots)$$

Throughout the present discussion, θ is chosen as

$$\sin\theta = -0.32, \text{ i.e. } \theta \simeq -18.5^o \,.$$

L_i's generically correspond to the higher order local operators.

At $\mathcal{O}(\delta)$ one has $\mathcal{O}(N_c p^4)$ and $\mathcal{O}(p^2)$ operators:

$$\mathcal{L}^{(\delta)} = L_2 \langle u_\mu u_\nu u^\mu u^\nu \rangle + (2L_2 + L_3) \langle u_\mu u^\mu u_\nu u^\nu \rangle + \ldots \\ + F^2 \widetilde{\Lambda}_1 \langle u_\mu \rangle \langle u^\mu \rangle + F^2 \widetilde{\Lambda}_2 \ln(\det U) \langle \chi_- \rangle + \ldots$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger \quad , \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$\mathcal{O}(p^4)$

$$\mathcal{L}^{(\delta^2)} = (L_1 - L_2/2) \langle u_\mu u^\nu \rangle^2 + L_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle + \ldots \\ + L_{18} i D_\mu X \langle D^\mu U^\dagger \chi - D^\mu U \chi^\dagger \rangle + L_{25} i X \langle U^\dagger \chi U^\dagger \chi - \chi^\dagger U \chi^\dagger U \rangle$$

[Kaiser and Leutwyler, '00] [Herrera-Siklody, *et al.*, '97]

and $c_d, c_m, \ldots$ correspond to the resonance operators:

$$\begin{aligned}\mathcal{L}_S = \quad & c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle \\ & + \widetilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \widetilde{c}_m S_1 \langle \chi_+ \rangle + \hat{c}_d \langle S_9 u_\mu \rangle \langle u_\mu \rangle\end{aligned}$$

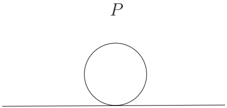
$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

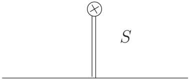
[Ecker, *et al.*, '89]

In the current discussion, we assume the resonance saturation and exploit the above resonance operators to calculate the meson-meson scattering.

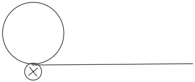
Perturbative calculation of the scattering amplitudes

The relevant Feynman diagrams for the wave function renormalization and mass renormalization are:





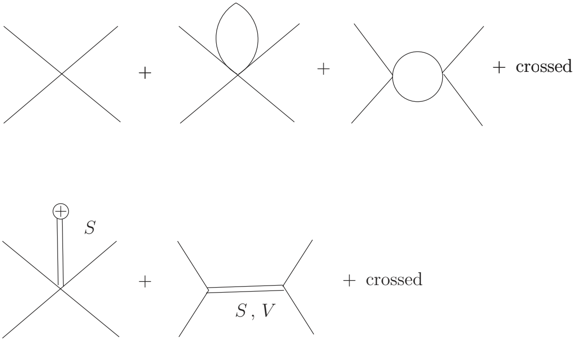
$\Rightarrow \; Z_P \, , \, m_P$



$\Rightarrow \; F_P$

We expressed all the amplitudes in terms of physical masses and $F_\pi = 92.4$ MeV

Scattering amplitudes consist of



Partial wave amplitude and its unitarization

Partial wave projection:

$$T^I_J(s) = \frac{1}{2(\sqrt{2})^N} \int_{-1}^{1} dx \, P_J(x) \, T^I[s, t(x), u(x)] \, ,$$

where $P_J(x)$ denote the Legendre polynomials and $(\sqrt{2})^N$ is a symmetry factor to account for the identical particles, such as $\pi\pi, \eta\eta, \eta'\eta'$.

This defines the Unitary Normalization Oller,Oset NPA'97

$$\mathrm{Im}\, T^I_{J\,mn} = \sum_k \theta(s - s^k_{th}) \rho_k \, T^I_{J\,ik} \, T^{I\,*}_{J\,kj} \tag{1}$$

Identical and non-identical particle states are treated in the same way

The N/D method Chew, Mandelstam PR'60 is employed approximately for unitarizing T_J:

$$T_J = \frac{N}{D}\,,$$

where

$$\begin{aligned}
\mathrm{Im}D &= N\,\mathrm{Im}\,T_J^{-1} = -\rho N\,, \quad \text{for } s > 4m^2\,,\\
\mathrm{Im}D &= 0\,, \quad \text{for } s < 4m^2\,,\\
\mathrm{Im}N &= D\,\mathrm{Im}\,T_J\,, \quad \text{for } s < 0\,,\\
\mathrm{Im}N &= 0\,, \quad \text{for } s > 0\,,
\end{aligned}$$

due to the fact that the unitarity condition for the elastic channel $s > 4m^2$ is

$$\mathrm{Im}\,T_J^{-1} = -\rho\theta(s - s_{th})\,,$$

with $\rho = \sqrt{1 - 4m^2/s}/16\pi = q/8\pi\sqrt{s}$.

One can now write the dispersion relations for N and D:

$$D(s) = \widetilde{a}^{SL}(s_0) - \frac{s-s_0}{\pi}\int_{4m^2}^{\infty}\frac{N(s')\ \rho(s')}{(s'-s)(s'-s_0)}ds'$$

$$N(s) = \int_{-\infty}^{0}\frac{D(s')\ \mathrm{Im}\, T_J(s')}{s'-s}ds'\,.$$

It can be greatly simplified if one imposes the perturbative solution for $N(s)$ in terms of the left hand cut (LHC) discontinuity Oller, Oset PRD'99, Oller PLB'00

No LHC:

$$N(s) = 1$$

$$D(s) = a^L + \sum_i \frac{R_i}{s-s_i} + g(s) = Q(s)^{-1} + g(s)$$

$$g(s) = \frac{a^{SL}(s_0)}{16\pi^2} + \frac{s-s_0}{\pi}\int_{4m^2}^{\infty}\frac{\rho(s')}{(s'-s)(s'-s_0)}ds'\,.$$

$$T_J(s) = \frac{Q(s)}{1+g(s)Q(s)} ,$$

Including LHC (e.g. in our perturbative calculation there is LHC from crossed exchange of resonances and crossed loops)

$$Q(s) \to N(s)$$

$N(s)$ only has LHC

$$T_J(s) = \frac{N(s)}{1+g(s)\ N(s)} ,$$

Matching with $T_J(s)|_{\chi PT} = T_2 + T_R + T_L$ up to one-loop at $\mathcal{O}(p^4)$:

$$T_2 + T_R + T_L = N(s) - N(s)g(s)N(s) + \mathcal{O}(\hbar^2)$$

$$N(s) = T_2 + T_R + T_L + N(s)g(s)N(s)$$

The generalization to the inelastic case is straightforward:

$$T_J(s) = [1 + g(s) \cdot N(s)]^{-1} \cdot N(s)\,,$$

For $IJ = 00$ channel we have 5 channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$

$$N_0^0(s) = \begin{pmatrix} N_{\pi\pi\to\pi\pi} & N_{\pi\pi\to K\bar{K}} & N_{\pi\pi\to\eta\eta} & N_{\pi\pi\to\eta\eta'} & N_{\pi\pi\to\eta'\eta'} \\ N_{\pi\pi\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\eta\eta} & N_{K\bar{K}\to\eta\eta'} & N_{K\bar{K}\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta} & N_{K\bar{K}\to\eta\eta} & N_{\eta\eta\to\eta\eta} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta'} & N_{K\bar{K}\to\eta\eta'} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta'\to\eta\eta'} & N_{\eta\eta'\to\eta'\eta'} \\ N_{\pi\pi\to\eta'\eta'} & N_{K\bar{K}\eta'\eta'} & N_{\eta\eta\to\eta'\eta'} & N_{\eta\eta'\to\eta'\eta'} & N_{\eta'\eta'\to\eta'\eta'} \end{pmatrix}$$

$$g_0^0(s) = \begin{pmatrix} g_{\pi\pi} & 0 & 0 & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 & 0 & 0 \\ 0 & 0 & g_{\eta\eta} & 0 & 0 \\ 0 & 0 & 0 & g_{\eta\eta'} & 0 \\ 0 & 0 & 0 & 0 & g_{\eta'\eta'} \end{pmatrix}$$

For $IJ = 1\,0$ there are 3 channels: $\pi^0\eta$, $K\bar{K}$ and $\pi^0\eta'$

$$N(s)^1_0 = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to K\bar{K}} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\pi\eta'} \\ N_{\pi\eta\to\pi\eta'} & N_{K\bar{K}\to\pi\eta'} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix}$$

$$g(s)^1_0 = \begin{pmatrix} g_{\pi\eta} & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 \\ 0 & 0 & g_{\pi\eta'} \end{pmatrix}$$

For $IJ = 1/2\ 0$ there are tree channels: $K\pi$, $K\eta$ and $K\eta'$

$$N(s)_0^{1/2} = \begin{pmatrix} N_{K\pi\to K\pi} & N_{K\pi\to K\eta} & N_{K\pi\to K\eta'} \\ N_{K\pi\to K\eta} & N_{K\eta\to K\eta} & N_{K\eta\to K\eta'} \\ N_{K\pi\to K\eta'} & N_{K\eta\to K\eta'} & N_{K\eta'\to K\eta'} \end{pmatrix}$$

$$g(s)_0^{1/2} = \begin{pmatrix} g_{K\pi} & 0 & 0 \\ 0 & g_{K\eta} & 0 \\ 0 & 0 & g_{K\eta'} \end{pmatrix}$$

For $IJ = 3/2\ 0$:

$$N(s)_0^{3/2} = N_{K\pi\to K\pi}$$

$$g(s)_0^{3/2} = g_{K\pi}$$

For $IJ = 2\ 0$:

$$N(s)_0^2 = N_{\pi\pi\to\pi\pi}\ ,$$

$$g(s)_0^2 = g_{\pi\pi}$$

Preliminary numerical results

By performing the χ^2 fit, we get

$$c_d = (17.0^{+3.0}_{-2.7})\,\mathrm{MeV} \qquad c_m = (41^{+18}_{-18})\,\mathrm{MeV}$$
$$\widetilde{c}_d = (14.5^{+1.1}_{-0.9})\,\mathrm{MeV} \qquad \widetilde{c}_m = (17.7^{+4.5}_{-3.7})\,\mathrm{MeV}$$
$$G_V = (55.9^{+2.5}_{-2.4})\,\mathrm{MeV} \qquad M_{S_1} = (1050^{+29}_{-37})\,\mathrm{MeV}$$
$$M_{S_8} = (1400^{+32}_{-30})\,\mathrm{MeV} \qquad a_{SL} = (-0.90^{+0.03}_{-0.04})$$
$$c_1 = (1.33^{+0.09}_{-0.11})$$

with $\chi^2/\mathrm{d.o.f} = 684/(285-9) \simeq 2.4$. We have used $\Delta\chi^2/\sqrt{2\chi^2} \leq 2$ to get the errors Etkin *et al.* PRD'82

$G_V = 55$ MeV Gasser and Leutwyler '85

c_1 is defined by

$$\frac{d\sigma}{dE_{\pi\eta}} = p_{\pi\eta}|c_1\, T_{K\bar{K}\to\pi\eta}|^2$$

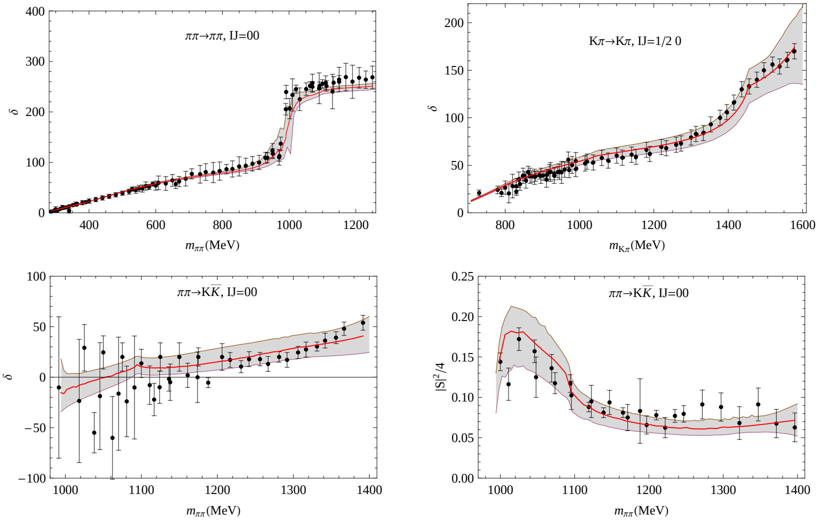
Other more constraint fits of similar quality are also possible by taking $c_d = c_m$ and $\widetilde{c}_d = \widetilde{c}_m$

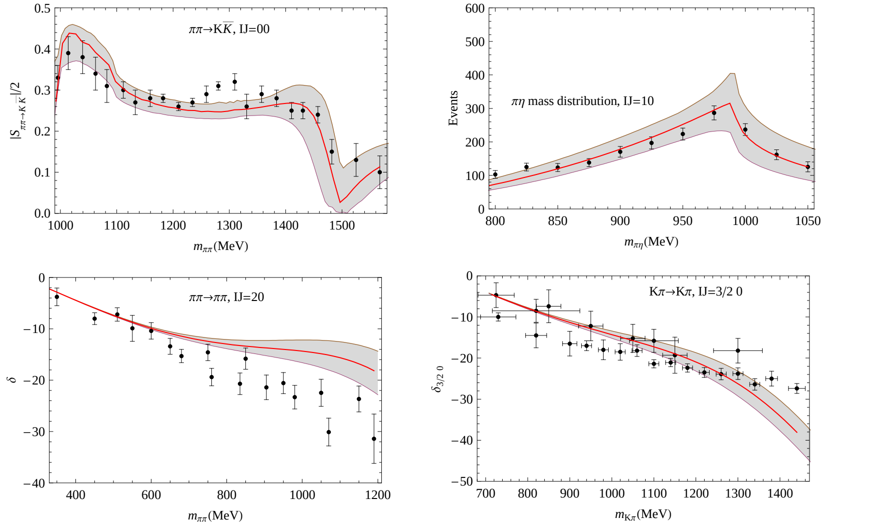
Short distance constraints requiring vanishing of I=1/2 scalar form factors for $s \to \infty$ Jamin, Oller, Pich NPB'00,'02

$$\sum_{i=1}^{N} c_d c_m = f^2/4 \quad , \quad \sum_{i=1}^{N} \frac{c_{m,i}}{M_{S_i}^2}(c_{m,i} - c_{d,i}) = 0 \tag{2}$$

The latter is fulfilled taking $c_{m,i} = c_{d,i} = 0$.
The former requires the contribution of the higher octet of scalar resonances.

3 free parameters less$\longrightarrow$ 6 free parameters.


ππ→ππ, IJ=00
δ
$m_{\pi\pi}$(MeV)
Kπ→Kπ, IJ=1/2 0
δ
$m_{K\pi}$(MeV)
ππ→K$\bar{K}$, IJ=00
δ
$m_{\pi\pi}$(MeV)
ππ→K$\bar{K}$, IJ=00
$|S|^2/4$
$m_{\pi\pi}$(MeV)



ππ→KK̄, IJ=00
|S_{ππ→KK̄}|/2
m_{ππ}(MeV)
πη mass distribution, IJ=10
Events
m_{πη}(MeV)
ππ→ππ, IJ=20
δ
m_{ππ}(MeV)
Kπ→Kπ, IJ=3/2 0
δ_{3/2 0}
m_{Kπ}(MeV)

Poles from the unitarized amplitudes

- $IJ = 00$

$$
\begin{aligned}
M_\sigma &= 451^{+3}_{-5}\,\mathrm{MeV}\,, \quad \Gamma_\sigma/2 = 249^{+5}_{-7}\,\mathrm{MeV}\,,\\
|g_{\sigma\pi\pi}| &= 3.08^{+0.02}_{-0.02}\,\mathrm{GeV}\,,\\
|g_{\sigma K\bar{K}}|/|g_{\sigma\pi\pi}| &= 0.50,\\
|g_{\sigma\eta\eta}|/|g_{\sigma\pi\pi}| &= 0.09
\end{aligned}
$$

with $g^2_{\sigma\pi\pi} = \frac{1}{2\pi i}\oint_{|s-s_\sigma|=R} T^{\mathrm{II}}(s)\,ds\,.$

- $IJ = 00$

$$
\begin{aligned}
M_{f_0} &= 1003^{+13}_{-17}\,\mathrm{MeV}\,, \quad \Gamma_{f_0}/2 = 24^{+12}_{-14}\,\mathrm{MeV}\,,\\
|g_{f_0\pi\pi}| &= 1.9^{+0.4}_{-0.4}\,\mathrm{GeV}\\
|g_{f_0 K\bar{K}}|/|g_{f_0\pi\pi}| &= 2.0\\
|g_{f_0\eta\eta}|/|g_{f_0\pi\pi}| &= 1.4
\end{aligned}
$$

- $IJ = 1/2\,0$

$$M_\kappa = 685^{+22}_{-17}\,\text{MeV}\,, \quad \Gamma_\kappa/2 = 273^{+18}_{-10}\,\text{MeV}\,,$$
$$|g_{\kappa K\pi}| = 4.5^{+0.2}_{-0.2}\,\text{GeV}$$
$$|g_{\kappa K\eta}|/|g_{\kappa K\pi}| = 0.57$$
$$|g_{\kappa K\eta'}|/|g_{\kappa K\pi}| = 0.50$$

- $IJ = 1\,0$:

$$M_{a_0}^{\text{IV}} = 1043\,\text{MeV}\,, \; \Gamma_{a_0}/2 = 62\,\text{MeV}\,,$$
$$|g_{a_0\pi\eta}| = 3.9\,\text{GeV}\,,$$
$$|g_{a_0 K\bar{K}}|/|g_{a_0\pi\eta}| = 1.54$$
$$|g_{a_0\pi\eta'}|/|g_{a_0\pi\eta}| = 0.04$$

where $a_{SL}^{\pi\eta\to\pi\eta} = -1.4$ has been used.

- $IJ = 1\,1$ (Not Fitted):

$$M_\rho = 752\,\mathrm{MeV}\,,\ \Gamma_\rho/2 = 56\,\mathrm{MeV}\,,\ |g_{\rho\pi\pi}| = 2.2\,\mathrm{GeV}$$

- $IJ = 1/2\,1$ (Not Fitted):

$$M_{K^*} = 879\,\mathrm{MeV}\,,\quad \Gamma_{K^*}/2 = 19\,\mathrm{MeV}\,,\ |g_{K^*\pi K}| = 1.6\,\mathrm{GeV}$$

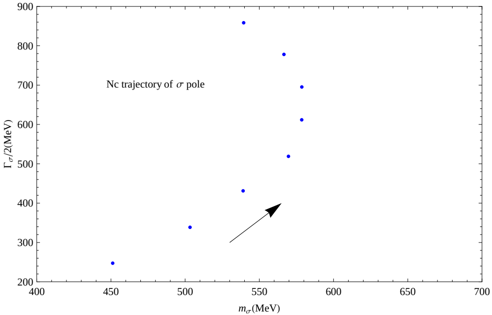
Runing of pole position with N_c

We solve for M^2_η, $M^2_{\eta'}$, mixing angle θ at leading order $\mathcal{L}_{\delta 0}$ and vary them with N_c

$$M_0^2 \sim 1/N_c$$
$$c_d \sim c_m \sim G_V \sim F \sim \sqrt{N_c}$$
$$M_V^2 \sim M^2_{S_8} \sim M^2_{S_1} \sim \mathcal{O}(N_c^0)$$

PRELIMINARY CURVES

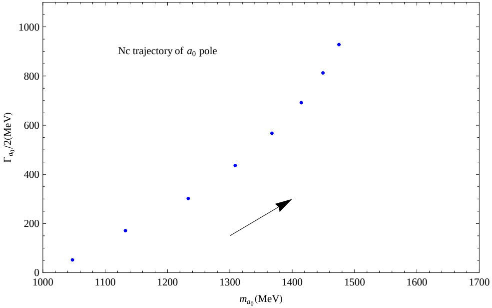
Step of $+1$ in N_c for every dot: σ



Oller, Oset PRD'99 $M_S^2 \propto f^2 \propto N_c$ For $M_S^2 \gtrsim 4m^2 \; (\log s/m^2)$

Peláez *et al* '04,'06,...,'10

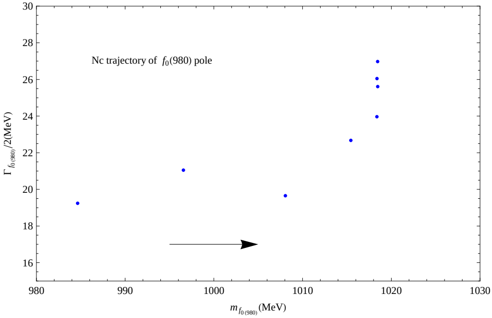
Step of $+1$ in N_c for every dot: $a_0(980)$



$$(M - i\Gamma/2)^2 = (a - ib)N_c$$

$$M^2 = \frac{aN_c}{2}\left(\sqrt{1+\frac{4b^2}{a^2}}+1\right) \; , \; \Gamma^2 = \frac{aN_c}{2}\left(\sqrt{1+\frac{4b^2}{a^2}}-1\right)$$

Step of +1 in N_c for every dot: $f_0(980)$ Singlet Bare Mass: $M_{S_1} = 1050$



N_c	$\mathrm{Re}\sqrt{s_P}(MeV)$	$\mathrm{Im}\sqrt{s_P}(MeV)$			
20	1026.7	10.1	60	1041.4	1.9
30	1033.2	5.5	90	1044.30	1.1
40	1037.3	3.6			
50	1039.8	2.6			

Conclusion & Outlook

- A complete one loop calculation of all meson-meson scattering amplitudes within $U(3)$ χPT has been worked out.
- They include the explicit exchange of tree level scalar and vector resonances in the s-,t- and u-channels.
- All of the scalar channels have been studied by unitarizing the perturbative amplitudes using an approach based on the N/D method while treating perturbatively the crossed channel dynamics.
- More resonances than included are generated. Dynamical generation of resonances from the self-interactions between the lightest pseudoscalars.

- N_C dependence considered of various resonance quantities, such as the pole positions and the residuals.
- Peculiar trajectories indicating dynamical generation of lightest scalar resonances.
- Bare pole present in the $f_0(980)$
- Ready for the study of all the vector channels.