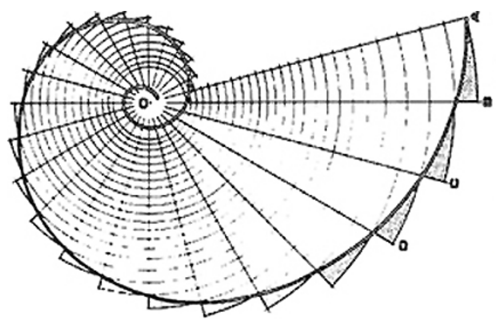
III

CONGRESO DE JÓVENES INVESTIGADORES

de la Real Sociedad Matemática Española

Universidad de Murcia, 7-11 Septiembre, 2015



SESIÓN

ÁLGEBRA NO CONMUTATIVA Y MÉTODOS CATEGÓRICOS

Financiado por:

Fundación Séneca-Agencia de Ciencia y Tecnología de la Región de Murcia, 19625/OC/14, con cargo al Programa “Jiménez de la Espada de Movilidad, Cooperación e Internacionalización”; plan propio de investigación de la Universidad de Murcia; Departamento de Matemática Aplicada de la Universidad Politécnica de Cartagena.

The monoidal structure on strict polynomial functors

Cosima Aquilino[1], **Rebecca Reischuk**[1]

The category of strict polynomial functors inherits an internal tensor product from the category of divided powers. This monoidal structure also yields a tensor product for the category of modules over the Schur algebra $S_k(n;d)$, indeed the two categories are equivalent for $n \geq d$. To investigate this monoidal structure, we consider the category of modules over the symmetric group algebra kS_d which admits a tensor product coming from its Hopf algebra structure. Based on work by Schur, there exists a functor F going from the category of strict polynomial functors to the category of modules over the symmetric group. We show that the functor F is monoidal.

This is joint work with Rebecca Reischuk.

[1]Fakultaet fuer Mathematik
Universitaet Bielefeld
Universitaet Str. 25, 33615 Bielefeld
caquilin@math.uni-bielefeld.de

A counterexample in the theory of braces

David Bachiller[1]

The search for solutions of the Yang-Baxter equation has motivated the study of many algebraic structures. The most commonly used are quantum groups and Hopf algebras. In the last years, a new algebraic structure, called left braces, has been introduced for its relations with a specific class of solutions, the non-degenerate involutive set-theoretic ones (see [2]).

A left brace is a set B with two binary operations, $+$ and $\cdot$, such that $(B, +)$ is an abelian group, $(B, \cdot)$ is a group, and these two operations are related by the following property: for every $a, b, c \in B$,

$$a \cdot (b + c) + a = a \cdot b + a \cdot c.$$

A difficult open problem is the classification of finite left braces. For this, it would be useful to know which groups appear as multiplicative groups of left braces. In [1], it is proved that, for a finite left brace B, $(B, \cdot)$ is always solvable. In this talk, we show that the converse is not true, by presenting a finite p-group which is not the multiplicative group of any left brace. We include in the talk all the necessary background about left braces necessary to understand this counterexample. We also try to explain the connection of this algebraic structure with other topics in algebra.

Referencias

[1] P. Etingof, T. Schedler, A. Soloviev: Set-theoretical solutions to the quantum Yang-Baxter equation, *Duke Math. J.* **100** (no. 2) (1999), 169–209.

[2] W. Rump: Braces, radical rings, and the quantum Yang-Baxter equation, *J. Algebra* **307** (no. 1) (2007), 153–170.

[1]Departamento de Matemáticas
Universitat Autònoma de Barcelona
Edifici C, Facultat de Ciències, 08193 Bellaterra (Barcelona), España
dbachiller@mat.uab.cat

On some local cohomology spectral sequences

Alberto F. Boix[1]

The goal of this talk is to introduce a formalism to produce spectral sequences. More precisely, on one hand we obtain a collection of spectral sequences which involve the left derived functors of the colimit functor on a finite poset; in particular, these spectral sequences not only recover and extend the Mayer-Vietoris spectral sequence of local cohomology modules stated in full generality by Lyubeznik in 2007, but also produce as new result the Mayer-Vietoris long exact sequence of local cohomology modules with respect to pairs of ideals, which were introduced by Takahashi, Yoshino and Yoshizawa in 2009. On the other hand, carrying over another collection of spectral sequences which involve in their second page the right derived functors of the limit functor on a finite poset, we recover Hochster's decomposition of local cohomology modules obtained by Brun, Bruns and Römer in 2007. Finally, in all the introduced spectral sequences we also provide sufficient conditions to ensure their degeneration at their second page and, in this case, we study the corresponding extension problems that this degeneration produces.

The content of this talk is based in a work in progress with Josep Àlvarez Montaner and Santiago Zarzuela.

[1]Department of Economics and Business
Universitat Pompeu Fabra
Jaume I Building, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain.
alberto.fernandezb@upf.edu

Generalizaciones de Álgebras de Sabinin

Daniel de la Concepción Sáez[1], **Abdenacer Makhlouf**[2]

Es ampliamente conocida la relación entre los grupos de Lie y sus espacios tangentes en el elemento neutro. Esta relación se puede extender de la siguiente manera:

Hay variedades diferenciales donde en un entorno U de un punto $e \in U$ se puede definir una estructura de lazo, para el que $e \in U$ es el elemento neutro. El espacio tangente a la variedad en e hereda así una estructura algebraica, que es la estructura de un álgebra de Sabinin. La definición de estas álgebras, llamadas hyperálgebras en sus inicios, puede consultarse en [6] en su forma original.

En esta charla se presenta una traslación de resultados clásicos de álgebras de Sabinin, que aparecen en [4] y [2], a generalizaciones de las mismas que se obtienen variando los conceptos naturales de conmutatividad y asociatividad.

En concreto, explicamos la construcción de herramientas categóricas y su aplicación en la construcción de envolventes universales de álgebras de color y álgebras de tipo Hom. Estas clases de álgebras tienen sus orígenes en [3] y [5], respectivamente. El contenido de la charla aparece en las prepublicaciones [7], inspirada en resultados de super-álgebras que se pueden encontrar en [1], y [8], realizada junto con Abdenacer Makhlouf.

Referencias

[1] Y. Bahturin, A. A. Mikhalev, V. M. Petrogradsky, M. V. Zaicev: *Infinite-dimensional Lie superalgebras*. de Gruyter Expositions in Mathematics, City, 1992.

[2] J.M. Pérez-Izquierdo: Algebras, hyperalgebras, nonassociative bialgebras and loops, *Advances in Mathematics* **208** (2) (2007), 834–876.

[3] M. Scheunert: Generalized Lie algebras, *J. Math. Phys.* **20** (1979), 712–720.

[4] J.M. Pérez-Izquierdo, G. Benkhart, S. Madariaga: Hopf algebras with triality, *Trans. Amer. Math. Soc.* **265** (2013), 1001–1023.

[5] J. Hartwig, D. Larsson, S. Silvestrov: Deformations of Lie algebras using σ-derivations, *J. Algebra* **295** (2006), 95–108.

[6] L.V. Sabinin, P.O. Mikheev: Infinitesimal theory of local analytic loops, *Soviet Math. Dokl.* **36** (3) (1988), 545–548.

[7] D. de-la-Concepción: Universal enveloping algebras for Malcev color algebras, Preprint (2015).

[8] D. de-la-Concepción, A. Makhlouf: Varieties of Hom-Sabinin algebras and related subclasses, Preprint.

[1]Departmento de Matemáticas y Computación
Universidad de La Rioja
Avda. de la Paz, 93 26006 Logroño (España)
daniel-de-la.concepcion@unirioja.es

[2]Laboratoire de Mathématiques, Informatique et Applications
Université de Haute Alsace
4, rue des Frères Lumière 68093 Mulhouse cedex (France)
Abdenacer.Makhlouf@uha.fr

Nuevos ejemplos de álgebras de Hopf co-Frobenius

Laura Martín Valverde[1], **Juan Cuadra Díaz**[1]

Sweedler introdujo en [5] la noción de *integral* para un álgebra de Hopf a partir de la invarianza de la integral de Haar sobre un grupo compacto. Las álgebras de Hopf con integral no nula se llaman *co-Frobenius*, pues cumplen una condición dual a la que define un álgebra Frobenius. Estas álgebras de Hopf se pueden ver como versiones algebraicas de los grupos cuánticos compactos introducidos por Woronowicz en [6]. A lo largo de los años se ha puesto de manifiesto que el concepto de álgebra de Hopf co-Frobenius está ligado a varias condiciones de finitud. En [3] Andruskiewitsch y Dăscălescu investigan la condición de ser finitamente generada sobre el zócalo de Hopf. Demuestran que un álgebra de Hopf que lo sea es co-Frobenius y se preguntan si el recíproco es cierto. Andruskiewitsch, Cuadra y Etingof responden negativamente a esta pregunta en [2] con la construcción de una familia de ejemplos que no encaja en ninguno de los patrones conocidos hasta entonces. Su método de construcción consiste en dualizar y explotar levantamientos de rectas cuánticas sobre grupos abelianos. Está inspirado en la famosa clasificación de las álgebras de Hopf punteadas de Andruskiewitsch y Schneider [4].

En esta charla usaremos dicho método para construir nuevos ejemplos sobre ciertos grupos no abelianos, diédricos y cuaterniónicos, y presentaremos un esquema general en el que encajan tanto los ejemplos de [2] como estos. Los resultados que se expondrán forman parte de mi tesis doctoral que está siendo realizada en la Universidad de Almería bajo la dirección del profesor Juan Cuadra Díaz.

Referencias

[1] N. Andruskiewitsch y J. Cuadra: On the structure of (co-Frobenius) Hopf algebras. *J. Noncommut. Geom.* **7** (2013), 83–104.

[2] N. Andruskiewitsch, J. Cuadra y P. Etingof: On two finiteness conditions for Hopf algebras with nonzero integral. *Ann. Sc. Norm. Super. Pisa Cl. Sci* . (5) **XIV** (2015), 1–40. En prensa. ArXiv:1206.5934.

[3] N. Andruskiewitsch y S. Dăscălescu: Co-Frobenius Hopf algebras and the coradical filtration. *Math. Z.* **243** (2003), 145–154.

[4] N. Andruskiewitsch y H.-J. Schneider: On the classification of finite-dimensional pointed Hopf algebras. *Ann. of Math.* (2) **171** (2010), 375–417.

[5] M.E. Sweedler: Integrals for Hopf algebras. *Ann. of Math.* (2) **89** (1969), 323–335.

[6] S.L. Woronowicz: Compact matrix pseudogroups. *Comm. Math. Phys.* **111** (1987), 613–665.

[1]Departamento de Matemáticas
Universidad de Almería
Ctra. Sacramento s/n, La Cañada de San Urbano, 04120 Almería
lmartinvalverde@gmail.com, jcdiaz@ual.es

Tilting t-structures

Francesco Mattiello[1], **Luisa Fiorot**[1], **Alberto Tonolo**[1]

We discuss a generalization of the results of Happel, Reiten and Smalø [1] concerning the notion of *tilted t-structure* and derived equivalences. The motivating example is the t-structure on the derived category of left R-modules over a ring R generated by a n-tilting module $_RT$.

Referencias

[1] D. Happel, I. Reiten, S.O. Smalø: *Tilting in abelian categories and quasitilted algebras*. Memoirs of the American Mathematical Society, Amer. Math. Society, 1996.

[1]Department of Mathematics
University of Padova
35121, Padova (PD)
mattiell@math.unipd.it

Purity and Enriched Categories

Sinem Odabaşı[1]

For any commutative ring R, R-Mod and R-mod denote the category of R-modules and finitely presented R-modules, respectively. Then R may be viewed as an additive category having just one object $\underline{R}$ with morphism group $\mathrm{Hom}(\underline{R}, \underline{R}) := R$. Then R-Mod is just the category $\mathrm{Add}(\underline{R}, \mathrm{Ab})$ of additive abelian group valued functors. Conversely, for a small additive category $\mathcal{A}$, $\mathrm{Add}(\mathcal{A}, \mathrm{Ab})$ can be seen as a generalization of a ring.

This comparison between modules and functors plays an important role in (Relative) Homological Algebra and Representation Theory. Among them, it helps us to handle the pure-exact structure in R-Mod as the usual exact structure of some subcategories of S-Mod, for some ring S with enough idempotents. These correspondences are precisely given by functors $\mathrm{Hom}(-,-)$ and $- \otimes -$. In [1], it was shown that the Hom functor would continue doing its duty for any additive category $\mathcal{A}$ whenever $\mathcal{A}$ is locally finitely presentable.

In this talk, we claim to work on the second case, i.e., the link between purity and functor categories through the tensor functor $- \otimes -$ when a category $\mathcal{V}$ has a symmetric closed monoidal structure $\otimes$. For that, we are needed to deal with not only additive but also $\mathcal{V}$-enriched functors. Then we see that the theory can be developed for Grothendieck and locally finitely presentable base categories.

Referencias

[1] W. Crawley-Boovey: Locally finitely presented categories, *Comm. Algebra* **22**(1994), 1641–1674.

[2] E. Enochs, S. Estrada and S. Odabaşı: Pure injective and absolutely pure sheaves, Accepted in *P. Edinburgh Math. Soc.*, arXiv:1307.1729v2.

[3] S. Estrada, J. Gillespie and S. Odabaşı: Pure exact structures and the pure derived category of a scheme, arXiv: 1408.2846v1.

Recollements of (derived) module categories

Chrysostomos Psaroudakis[1], **Jorge Vitória**[2]

Recollements of abelian, resp. triangulated, categories are exact sequences of abelian, resp. triangulated, categories where the inclusion functor as well as the quotient functor have left and right adjoints. They appear quite naturally in various settings and are omnipresent in representation theory. Recollements which all categories involved are module categories (abelian case) or derived categories of module categories (triangulated case) are of particular interest. In the abelian case, the "standard"example is the recollement induced by the module category of a ring R with an idempotent element e, and in the triangulated case the "standard"example is given as the derived counterpart of the previous recollement of module categories when the ideal ReR is stratifying. The latter recollement is called stratifying. The aim of this talk is twofold. First, we classify, up to equivalence, recollements of abelian categories whose terms are equivalent to module categories. Then, we provide necessary and sufficient conditions for a recollement of derived categories of module categories over rings to be equivalent with a stratifying one and we discuss applications. This is joint work with Jorge Vitória.

[1]Department of Mathematical Sciences
Norwegian University of Science and Technology
7491 Trondheim, Norway
chrysostomos.psaroudakis@math.ntnu.no

[2]Dipartimento di Infortmatica - Settore di Matematica
Universitá Degli Studi di Verona
Strada le Grazie 15 - Ca' Vignal, I-37314, Verona, Italy
jorge.vitoria@univr.it

Quasicrossed product on graded quasialgebras

José M. Sánchez[1], **Helena Albuquerque**[2], **Elisabete Barreiro**[2]

G-graded quasialgebras were introduced by H. Albuquerque and S. Majid about a decade ago [1]. This class of algebras includes several types of algebras. Indeed, the class of associative algebras fits into this concept, as well as some notable nonassociative algebras such as deformed group algebras (for example, Cayley algebras and Clifford algebras).

Inspired by the theory of graded rings and graded algebras [2, 3, 4, 5], we show an extension of the notion of crossed product to the setting of graded quasialgebras.

By collecting basic definitions and properties related to graded quasialgebras we introduce the notion of quasicrossed product, including some examples and the relationship with quasicrossed system. We show that the quasicrossed system corresponding to the deformed group algebra obtained from the Cayley-Dickson process applied to a deformed group algebra is related to the quasicrossed system corresponding to the initial algebra. Finally we obtain results about simple quasicrossed products.

Referencias

[1] H. Albuquerque, S. Majid: Quasialgebra structure of the octonions, *J. Algebra*, **220** (1999), 188–224.

[2] Yu. A. Drozd, V. V. Kirichenko: *Finite dimensional algebras*, translated from the 1980 Russian original by Vlastimil Dlab. Springer-Verlag Co., Berlin, 1994.

[3] G. Karpilovsky: *Clifford theory for group representations*. North-Holland Mathematics Studies, 156. Notas de Matemática, 125. North-Holland Publishing Co., Amsterdam, 1989.

[4] G. Karpilovsky: *Induced modules over group algebras*. North-Holland Mathematics Studies, 161. North-Holland Publishing Co., Amsterdam, 1990.

[5] C. Năstăsescu, F. van Oystaeyen: *Graded ring theory*. North-Holland Mathematical Library, 28. North-Holland Publishing Co., Amsterdam-New York, 1982.

[1]Department of Mathematics
University of Cadiz
Campus de Puerto Real, 11510, Puerto Real, Cadiz, Spain.
txema.sanchez@uca.es

[2]CMUC, Department of Mathematics
University of Coimbra
Apartado 3008, 3001-454 Coimbra, Portugal.
lena@mat.uc.pt, mefb@mat.uc.pt

The Structure Theorem for quasi-Hopf bimodules: from quasi-antipodes to preantipodes.

Paolo Saracco[1]

It is known that the Fundamental Theorem of Hopf modules can be used to characterize Hopf algebras: a bialgebra H over a field is a Hopf algebra (i.e. it is endowed with a so-called antipode) if and only if every Hopf module M over H can be decomposed in the form $M^{\mathrm{co}H} \otimes H$, where $M^{\mathrm{co}H}$ denotes the space of coinvariant elements in M. A partial extension of this equivalence to the case of quasi-bialgebras is due to Hausser and Nill: if a quasi-bialgebra admits a quasi-antipode, then it is possible to define a suitable space of coinvariants such that every quasi-Hopf bimodule could be decomposed in the same way.

The main aim of this talk is to show how this characterization could be fully extended to the framework of quasi-bialgebras by introducing the notion of (the) preantipode for a quasi-bialgebra and by proving a Structure Theorem for quasi-Hopf bimodules. As a consequence some previous results, as the Fundamental Theorem of Hopf modules and the Hausser-Nill theorem for quasi-Hopf algebras, can be deduced from our Structure Theorem.

This talk is based on the paper [P. Saracco, *On the Structure Theorem for Quasi-Hopf Bimodules*, preprint. (arXiv:1501.06061)].

[1]Department of Mathematics “G. Peano”
University of Turin
Via Carlo Alberto, 10 - 10123 Torino
p.saracco@unito.it

Partial Augmentations Of Torsion Units In The Integral Group Ring of PSL$(2, p)$

Mariano Serrano[1], **Ángel del Río**[1]

Let G be a finite group and let $\mathbb{Z}G$ be the integral group ring of G. Denote by $V(\mathbb{Z}G)$ the group of units of augmentation 1 in $\mathbb{Z}G$. If G is an abelian group, it is well known that the torsion units of $V(\mathbb{Z}G)$ are the elements of G. However, if G is not abelian one can produce other torsion units by conjugating the elements of G with other units. Therefore, it is natural to ask under which conditions the torsion units of $V(\mathbb{Z}G)$ are conjugate to an element of G. In this context Hans Zassenhaus proposed the following conjectures:

(ZC1) Every torsion element of $V(\mathbb{Z}G)$ is conjugate to an element of G in $\mathbb{Q}G$.

(ZC2) Every subgroup of $V(\mathbb{Z}G)$ with the same order as G is conjugate to G in $\mathbb{Q}G$.

(ZC3) Every finite subgroup of $V(\mathbb{Z}G)$ is conjugate to a subgroup of G in $\mathbb{Q}G$.

Clearly, (ZC1) and (ZC2) are particular cases of (ZC3). Moreover, there are several counterexamples for (ZC2) and (ZC3), see for example [5, 6]. However, (ZC1) is still an open cuestion. One of the mail tools to study (ZC1) is the Luthar-Passi-Hertweck Method, known as the HeLP Method. One of the latest results on (ZC1), due to Hertweck, proves (ZC1) for A_6 using partially the HeLP Method [3]. However, Hertweck discovered the limitations of the method which is not enough to stablish (ZC1) for $A6$. Recently, Margolis and Bachle [1] have proved (ZC1) for PSL(2,19) and PSL(2,23). Moreover, it is also known that (ZC1) holds for PSL(2,p) for all prime $p \leq 17$ [2, 4].

(ZC1) is strongly related to the partial augmentations of torsion units. The HeLP Method provides constraints on the partial augmentations of torsion units. We study this constraints for the group $\mathrm{PSL}(2,p)$ for an odd prime p. Using this, we obtain all the possible partial augmentations of some torsion unit according to the HeLP Method. As an application, we prove (ZC1) for $\mathrm{PSL}(2,\ 31)$ and we obtain the limitations of the HeLP Method for $\mathrm{PSL}(2,29)$.

Referencias

[1] A. Bachle and L. Margolis: Rational conjugacy of torsion units in integral group rings of non-solvable groups, *http://arxiv.org/abs/1305.7419* (2014).

[2] M. Hertweck: Partial augmentations and Brauer character values of torsion units in group rings, *http://arXiv:math/0612429v2* (2007).

[3] M. Hertweck: Zassenhaus conjecture for A_6, *Proc. Indian Acad. Sci. Math. Sci.* **118** 2 (2007), 189–195.

[4] W. Kimmerle and A. Konovalov: Recent Advances on Torsion Subgroups of Integral Group Rings, *To appear in Proc. Groups St Andrews 2013* (2013).

[5] L. Klingler: Construction of a counterexample to a conjecture of Zassenhaus, *Comm. Algebra* **19** 8 (1991), 2303–2330.

[6] K. W. Roggenkamp: Observations on a conjecture of Hans Zassenhaus, *London Math. Soc. Lecture Note Ser.* **160** (1991), 427–444.

[1]Department of Mathematics, University of Murcia
30100 (Murcia) Spain
mariano.serrano@um.es
adelrio@um.es

Support tilting modules and ring epimorphisms for hereditary rings

Jorge Vitória[1], **Lidia Angeleri Hügel**[2], **Frederik Marks**[3]

Tilting modules are important tools in the representation theory of algebras: they provide an insight into the whole category of modules and its derived category. In the module category, they generate torsion pairs with good approximation properties; in the derived category they are known to be related with localisations and, sometimes, equivalences.Ê

Epimorphisms in the category of unital rings are useful to study certain subcategories of the module category and, although seemingly unrelated, they have close ties to tilting theory. This link is particularly visible when considering epimorphisms of hereditary rings.Ê

In this talk, we will discuss the relevance of the two concepts in representation theory and clarify the relation between them in the case of hereditary rings. This is joint work with Lidia Angeleri Hügel and Frederik Marks.

[1]Dipartimento di Informatica
Università degli Studi di Verona
Strada le Grazie 15 - Ca'Vignal, 37134 Verona - Italy
jorge.vitoria@univr.it

[2]Dipartimento di Informatica
Università degli Studi di Verona
Strada le Grazie 15 - Ca'Vignal, 37134 Verona - Italy
lidia.angeleri@univr.it

[3]Institut für Algebra und Zahlentheorie
Universität Stuttgart
Pfaffenwaldring 57, 70569 Stuttgart - Germany
marks@mathematik.uni-stuttgart.de