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Optimal solvability conditions on a fractional parabolic problem. A weighted Harnack inequality

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In this talk we will analyze the influence of the Hardy potential in the following fractional parabolic problem,

$$\begin{cases} u_t + (-\Delta)^s u &= \lambda \dfrac{u}{|x|^{2s}} + cf \text{ in } \Omega \times (0,T), \\ u(x,t) &> 0 \text{ in } \Omega \times (0,T), \\ u(x,t) &= 0 \text{ in } (\mathbb{R}^N \setminus \Omega) \times [0,T), \\ u(x,0) &= u_0(x) \text{ if } x \in \Omega, \end{cases}$$

where $N > 2s$, $s \in (0,1)$, $c, \lambda > 0$, and $u_0 \geq 0$, $f \geq 0$ are in a suitable class of functions. We will see that the best constant $\Lambda_{N,s}$ in the fractional Hardy inequality provides the threshold between existence and nonexistence of positive solutions to this problem, in the spirit of the results obtained by Baras and Goldstein in [2]. Moreover, we will find the optimal summability conditions on u_0 and f in order to have solvability. These results will require to prove a weak Harnack inequality for a weighted operator that naturally arises from the Hardy potential.

Referencias

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[2] P. Baras, J.A. Goldstein: The heat equation with a singular potential, *Trans. Amer. Math. Soc.* **2**84 no. 1 (1984), 121–139.

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