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Self-similar solutions to Smoluchowski coagulation equation

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The aim of this talk is to present recent contributions in collaboration with Marco A. Fontelos: the study of self-similarity in coagulation and fragmentation models. Smoluchowski's coagulation equation is a mean field model describing cluster growth by binary aggregation that has been used in a very wide set of applications, ranging from physical chemistry to astrophysics and population dynamics. Let the function $\mathrm{c}\,(x,t)$ represent the mean amount of $x-$mass polymers per unit volume at a given time. Then, the evolution of c is expressed by the non linear, integrodifferential equation:

$$\partial_t \mathrm{c}\,(x,t) = \frac{1}{2}\int_0^x K\,(x-y,y)\,\mathrm{c}\,(x-y,t)\,\mathrm{c}\,(y,t)\;\mathrm{d}y - \mathrm{c}\,(x,y)\int_0^\infty K\,(x,y)\,\mathrm{c}\,(y,t)\;\mathrm{d}y.$$

The dynamical properties depend on the integration kernel $K\,(x,y)$, which determines the reactivity between couples of masses. It is known that, for certain kernels such as $K_* = xy$, a singularity in finite time occurs: the solution develops a heavy tail in finite time and the total mass is no longer conserved. This phenomenon is called gelation and represents the formation of a cluster with infinite density that drains mass from the coagulating system.

We will consider homogeneous kernels $K\,(x,y) = (xy)^\lambda$ with $\lambda \leq 1$ and present some results about self-similar solutions both in singular and non singular cases. Such self-similar solutions depend on a free exponent that cannot be determined from dimensional considerations -self-similar solution of the second kind-; instead it must be determined imposing the behavior at the origin and infinity.

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