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Small values of the hyperbolicity constant in graphs

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If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a *geodesic triangle* $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1 x_2]$, $[x_2 x_3]$ and $[x_3 x_1]$ in X. The space X is δ-*hyperbolic* (in the Gromov sense) if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. We denote by $\delta(X)$ the sharpest hyperbolicity constant of X, i.e., $\delta(X) := \text{ínf}\{\delta \geq 0 : X \text{ is } \delta\text{-hyperbolic}\}$. In the study of any parameter on graphs it is natural to study the graphs for which this parameter has small values. In this work we study the graphs (with every edge of length k) with small hyperbolicity constant, i.e., the graphs which are like trees (in the Gromov sense). In this work we obtain simple characterizations of the graphs G with $\delta(G) = k$ and $\delta(G) = \frac{5k}{4}$ (the case $\delta(G) < k$ is known). Also, we give a necessary condition in order to have $\delta(G) = \frac{3k}{2}$ (we know that $\delta(G)$ is a multiple of $\frac{k}{4}$). Although it is not possible to obtain bounds for the diameter of graphs with small hyperbolicity constant, we obtain such bounds for the effective diameter if $\delta(G) < \frac{3k}{2}$. This is the best possible result, since we prove that it is not possible to obtain similar bounds if $(G) \geq \frac{3k}{2}$.

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