CONGRESO DE JÓVENES INVESTIGADORES
Real Sociedad Matemática Española
Universidad de Murcia, del 7 al 11 de Septiembre de 2015

The Simultaneous (Strong) Metric Dimension of Graph Families

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In a graph $G = (V, E)$, a vertex $v \in V$ is said to *distinguish* two vertices x and y if $d_G(v, x) \neq d_G(v, y)$, where $d_G(x, y)$ is the length of a shortest path between u and v. Likewise, a vertex x is said to *strongly distinguish* two different vertices u and v if there exists a shortest $u - x$ path containing v or there exists a shortest $v - x$ path containing u, *i.e.* $d_G(u, x) = d_G(u, v) + d_G(v, x)$ or $d_G(v, x) = d_G(u, v) + d_G(u, x)$. A set $S \subset V$ is said to be a (*strong*) *metric generator* for G if any pair of different vertices of G is (strongly) distinguished by some element of S. A minimum (strong) metric generator is called a (*strong*) *metric basis*, and its cardinality the (*strong*) *metric dimension* of G, denoted by $\dim(G)$ ($\dim_s(G)$) [1, 2, 3].

Given a family $\mathcal{G} = \{G_1, G_2, ..., G_k\}$ of (not necessarily edge-disjoint) connected graphs $G_i = (V, E_i)$ on a common vertex set V (the union of whose edge sets is not necessarily the complete graph), we define a *simultaneous* (*strong*) *metric generator* for $\mathcal{G}$ as a set $S \subset V$ such that S is simultaneously a (strong) metric generator for each G_i. We say that a minimum simultaneous (strong) metric generator for $\mathcal{G}$ is a *simultaneous* (*strong*) *metric basis* of $\mathcal{G}$, and its cardinality the *simultaneous* (*strong*) *metric dimension* of $\mathcal{G}$, denoted by $Sd(\mathcal{G})$ ($Sd_s(\mathcal{G})$) or explicitly by $Sd(G_1, G_2, ..., G_k)$ ($Sd_s(G_1, G_2, ..., G_k)$) [4, 5].

We investigate the properties of these simultaneous resolvability parameters. In particular, we determine their general bounds and give exact values or tight bounds for specific families. Additionally, we analyse the relations between both notions of simultaneous resolvability. Finally, we show that computing the simultaneous (strong) metric dimension is NP-hard, even for families composed by graphs whose individual (strong) metric dimension is easily computable.

Referencias

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