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Overview of algebraic reliability

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Algebraic reliability is the use of concepts and techniques of commutative algebra in the analysis of the reliability of coherent systems. A system S with m components is said to be coherent if the improvement of any of its components does not degrade the performance of the whole system. Coherent systems are ubiquitous in industry, communications or biology.

To any coherent system S with m components we associate a monomial ideal in a polynomial ring with m variables, such that each of the variables corresponds to one of the components. The algebraic properties of this ideal have their counterparts in features of the system and vice versa. In particular, the numerator of the (multigraded) Hilbert series of the ideal gives us the reliability of the system. Furthermore, if we compute the Hilbert series in terms of the ranks of any free resolution of the ideal, we can obtain upper and lower bounds for the reliability, , which are of paramount importance in the analysis of the system. These bounds are tighter than the usual Bonferroni bounds and among them, the ones obtained by using the minimal free resolution of the ideal are the most efficient ones [1, 2, 3].

More applications of the algebra of monomial ideals to the study of coherent systems include the use of the Hilbert *function* for the design of robust systems and networks [4, 5]. Recent developments in this topic include the study of percolation in trees and the analysis of multiple failures and signature analysis in coherent systems by means of the lcm-filtration of the system ideal [6, 7].

Referencias

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