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Application of Homological Algebra for error detection in topological computations

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The effective homology method [4] is a technique for the computation of homology groups of complicated spaces, implemented in the Computer Algebra system Kenzo [1]. This method has made it possible to determine some homology and homotopy groups which were not known before.

Given a group G, its classifying space, that is to say, the Eilenberg-MacLane space $K(G, 1)$ [2], has trivial homotopy groups: $\pi_1(K(G, 1)) \cong G$ and $\pi_n(K(G, 1)) = 0$ for each $n \neq 1$. But when applying the suspension functor Σ, the new homotopy groups $\pi_*(\Sigma K(G, 1))$ are in general unknown. In the work [3], several groups $\pi_n(\Sigma K(G, 1))$ are obtained for some particular cases of G and n, making use of different results and techniques from group theory and homotopy theory. More concretely, the main results of the article by Mikhailov and Wu are descriptions of the groups $\pi_4(\Sigma K(A, 1))$ and $\pi_5(\Sigma K(A, 1))$ when A is any finitely generated Abelian group; as applications, they also determine $\pi_n(\Sigma K(G, 1))$ with $n = 4$ and 5 for some non-Abelian groups, as $G = \Sigma_3$ the 3-th symmetric group and $G = SL(\mathbb{Z})$ the standard linear group, and $\pi_4(\Sigma K(A_4, 1))$ for A_4 the 4-th alternating group.

The goal of this work consists in the development of an algorithm based on the effective homology method to determine $\pi_*(\Sigma K(G, 1))$ for a *general* group G, which has allowed us to obtain as particular computations some theoretical results of [3]. Thanks to our programs, groups $\pi_4(\Sigma K(A, 1))$ and $\pi_5(\Sigma K(A, 1))$ have been computed for several finitely generated Abelian groups A, reproducing the same results obtained in [3]. We have also *experimentally* determined some new groups $\pi_6(\Sigma K(A, 1))$ (which do not appear in [3]) and $\pi_n(\Sigma K(G, 1))$ for other non-Abelian groups G not considered in that article. Moreover, our experiments have made it possible to detect an error in Mikhailov and Wu's paper. The authors state in Theorem 5.4: *Let A_4 be the 4-th alternating group. Then $\pi_4(\Sigma K(A_4, 1)) = \mathbb{Z}_4$.*

Our programs produce a different result, namely $\pi_4(\Sigma K(A_4, 1)) = \mathbb{Z}_{12}$. The authors of the paper inadvertently forgot the 3-primary component, as they have admitted in a private communication.

Referencias

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