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Large Derived Morita Theory

Pedro Nicolás[1], **Manuel Saorín**[2]

Some interesting properties of rings can not be conveniently understood by regarding just the rings themselves, but they can be studied by using the category of the representations of the rings, namely, the so-called category of modules. Morita Theory says that two module categories, $\mathsf{Mod}\, A$ and $\mathsf{Mod}\, B$, are equivalent if and only if there exists an A-module P such that:

a) it is finitely generated and projective, b) it generates $\mathsf{Mod}\, A$, c) $\mathsf{End}_A(P) \cong B$.

It turns out that certain interesting properties of rings can not be properly understood within the module category, but they can be well studied by using a larger, more sophisticated, category, namely the derived category. It does not make sense to speak of exactness in derived categories. In particular, they don't have short exact sequences. Instead, they have *triangles*, they are *triangulated categories* [1]. When dealing with this kind of categories, it seems natural to consider not just ordinary rings but differential graded algebras. In this case, after Happel [2], Rickard [4] and Keller [3], we have a 'Derived' Morita Theory: two derived categories, cdA and cdB, are triangle equivalent if and only if there exists an object $T \in cdA$ such that:

a) T is compact,
b) T generates $\mathcal{D}A$,
c) $\mathsf{REnd}_A(T) \cong B$.

At this point there are some natural questions:

i) What if T is not compact?
ii) What if T does not generate $\mathcal{D}A$?
iii) What if we replace equivalence of categories for something weaker but still interesting?

We shall answer these questions.

Referencias

[1] A. Neeman: *Triangulated categories*. Annals of Mathematics studies. Princeton University Press, 2001.

[2] D. Happel: On the derived category of a finite dimensional algebra, *Comment. Math. Helvetici* **62** (1987), 339–389.

[3] B. Keller: Deriving DG categories, *Ann. Scient. Ec. Norm. Sup.* **27**(1) (1994), 63–102.

[4] J. Rickard: Morita theory for derived categories, *J. London Math. Soc.* **39** (1989), 436–456.

[1]Departamento de Didáctica de las Ciencias Matemáticas y Sociales
Universidad de Murcia
Facultad de Educación, Campus de Espinardo, 30100
pedronz@um.es

[2]Departamento de Matemáticas
Universidad de Murcia
Facultad de Educación, Campus de Espinardo, 30100
msaorinc@um.es