CONGRESO DE JÓVENES INVESTIGADORES
Real Sociedad Matemática Española
Universidad de Murcia, del 7 al 11 de Septiembre de 2015

Units of group rings, the Bogomolov multiplier, and the fake degree conjecture

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Let J be a finite dimensional nilpotent algebra over a finite field $\mathbb{F}_q$. Then the set $G = 1 + J$ becomes a finite group. The groups constructed in this way are called *algebra groups*. The group G acts by conjugation on J and this induces a G-action on the dual space $J^* = Hom_{\mathbb{F}_q}(J, \mathbb{F}_q)$, called the coadjoint action. Consider the list of integers obtained by taking the square roots of the sizes of the coadjoint orbits of G on J^*. We will see that these numbers are q-powers, the sum of their squares is $|G|$ and the length of the list is the number of conjugacy classes of G. This precisely resembles the list of degrees of the irreducible characters of G. Indeed, if $J^p = 0$, there exists an explicit expression that gives a bijective correspondence between the characters of G and the orbits of J^*. I.M. Isaacs conjectured that this was true for any algebra group $G = 1 + J$. This is the so called "Fake degree conjecture". The study of this conjecture will lead us to study the abelianizations of groups of the form $1 + I_\pi$ where I_π is the augmentation ideal of the group ring $\mathbb{F}_q[\pi]$ for a finite p-group π. Surprisingly, the Bogomolov multiplier of the group π comes into play, and explains why this conjecture is not true in general. We will also explain a nice application to rationality questions in linear algebraic groups.

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