AUTOMATIC LOOP SHAPING IN QFT BY USING FRACTIONAL STRUCTURES

Joaquín Cervera [1] **Alfonso Baños** [1]

Faculty of Computer Engineering, Computer and Systems Engineering Department, University of Murcia, Campus de Espinardo, 30100 Murcia, Spain; [jcervera,abanos]@um.es

Abstract: This work focus on the problem of automatic loop shaping in QFT, where traditionally the search of an optimum design, a non convex and nonlinear optimization problem, is simplified by linearizing and/or convexifying the problem. In this work, the authors propose a suboptimal solution using a fixed structure in the compensator. The main idea in relation to previous work consists of the study of the use of fractional compensators, which give singular properties to automatically shape the open loop gain function with a minimum set of parameters.

Keywords: Automatic loop shaping, computer-aided control systems design, robust control, QFT, TID, CRONE, fractional PID.

1. INTRODUCTION

Quantitative Feedback Theory is a robust frequency domain control design methodology which has been successfully applied in practical problems from different domains (Horowitz, 1993). One of the key design steps is loop shaping of the open loop gain function to a set of restrictions (or *boundaries*) given by the design specifications and the (uncertain) model of the plant. Although this step has been traditionally performed by hand, the use of CACSD tools (e. g. the QFT Matlab Toolbox (Borghesani *et al.*, 1995)), has made the manual loop shaping much more simple. However, the problem of automatic loop shaping is of enormous interest in practice, since the manual loop shaping can be hard for the non experienced engineer, and thus it has received a considerable attention, specially in the last two decades.

Optimal loop computation is a non linear and non convex optimization problem for which it is difficult to find a satisfactory solution, since there is no optimization algorithm which computes, in a reasonable time in terms of interactive design purposes, a globally optimum solution for such a problem . It must be noticed, however, that the work by Nataraj and others on this subject, based on interval analysis techniques, is very promising (see for instance (Nataraj and Tharewal, 2004; Nataraj and Kubal, 2006)).

A possible approach is to simplify the problem in some way, in order to obtain a different optimization problem for which there exists a closed solution, or an optimization algorithm which does guarantees a global optimum. A trade-off between necessarily conservative simplification of the problem and computational solvability has to be chosen. This is the approach in, for instance, (Thomson, 1990; Gera and Horowitz, 1980). Some authors have investigated the loop shaping problem in terms of particular structures, with a certain degree of freedom, which can be shaped to

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the particular problem to be solved. This is the case in (Fransson *et al.*, 2002), (Chait *et al.*, 1999) and (Yaniv and Nagurka, 2004). Another possibility is to use evolutionary algorithms (Spears *et al.*, 1993), able to face nonlinear and non convex optimization problems. This is the approach adopted in (Chen *et al.*, 1998; Raimúndez *et al.*, 2001). Evolutionary algorithms' drawbacks are that they are computationally demanding and they do not guarantee, in general, an optimal solution. Interval analysis techniques are also able to face such kind of problems, and do guarantee global optimum; however, computation time has still to be improved for interactive design purposes.

By the use of information about the problem computation effort can be reduced and reasonably close to the optimum solutions can be obtained by evolutionary algorithms. In this sense, a good structure for the compensator, in terms of using a reduced set of parameters, but with a rich frequency domain behavior, is of crucial importance. This is the idea explored in this paper: to use evolutionary algorithms together with a flexible structure, able to get close to the optimal solution, but with a reduced number of parameters. In previous work, the compensator has been fixed to a rational structure, with a finite (but no necessarily small) number of zeros and poles. In this work, the main contribution is to introduce a fractional compensator that, with a minimum number of parameters, gives a flexible structure in the frequency domain regarding automatic loop shaping. In fact, it can be approximated by a rational compensator, but with a considerably large number of parameters. This dramatic reduction in the number of parameters has shown to be of capital importance for the success of evolutionary algorithms in the resolution of the automatic loop shaping problem.

This work, following (Cervera and Baños, 2005; Cervera and Baños, 2006; Cervera *et al.*, 2006), is a summary of (Cervera, 2006). It is considered the particular case of minimum phase open loop gain functions, for which the investigated compensators can give a good structure with a reduced set of parameters. Some first steps towards the non-minimum phase case are given in (Cervera, 2006), but the focus is on minimum phase problems. The work is structured as follows: in section 2 a brief introduction to QFT is given; in section 3 the proposed QFT automatic loop shaping procedure is presented; in section 4 this procedure is applied to a typical benchmark problem.

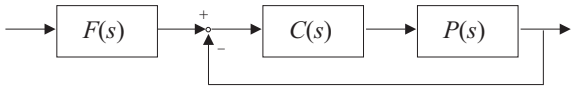

Fig. 1. Two degrees of freedom configuration.

2. INTRODUCTION TO QFT

The basic idea in QFT (Horowitz, 1993) is to define and take into account, along the control design process, the quantitative relation between the amount of uncertainty to deal with and the amount of control effort to use. This way, QFT guarantees (robust) specifications for every plant belonging to the admisible set defined by the a priori specified uncertainty. Typically, the QFT control system configuration (see fig. 1) considers two degrees of freedom: a controller $C(s)$, in the closed loop, which cares for the satisfaction of robust specifications despite uncertainty; and a precompensator, $F(s)$, designed after $C(s)$, to achieve desired frequency response once uncertainty is controlled. For a given plant $P(s)$, its *template* $\mathcal{P}$ is defined as the set of plant possible frequency responses due to uncertainty. A *nominal* plant, $P_0 \in \mathcal{P}$, is chosen.

The design of the controller $C(s)$ is accomplished in the Nichols chart, in terms of the nominal open loop transfer function, $L_0(s) = P_0(s)C(s)$. A discrete set of design frequencies Ω is chosen. Given quantitative specifications on robust stability and robust performance on the closed loop system, *boundaries* $\mathcal{B}_\omega$, $\omega \in \Omega$, are computed. $\mathcal{B}_\omega$ defines the allowed regions for $L_0(j\omega)$ in the Nichols chart, so that $\mathcal{B}_\omega$ being not violated by $L_0(j\omega)$ implies specification satisfaction by $L(j\omega) = P(j\omega)C(j\omega)\forall P(s) \in \mathcal{P}$. The basic step in the design process, *loop shaping*, consists of the design of $L_0(j\omega)$ which satisfies boundaries and is reasonable close to optimum. QFT optimization criterion is the minimization of high frequency gain (Horowitz, 1973)

$$k_{hf} = \lim_{s\to\infty} s^{n_{pe}} L(s) \tag{1}$$

where n_{pe} = excess of poles over zeros. This definition allows to compare (only) loops with the same n_{pe}. The UHFB (universal high frequency boundary) is a special boundary. It is a robust stability boundary which should be satisfied by $L_0(j\omega)\forall\omega \geq$ a certain ω_U. It has been shown ((Horowitz, 1973), (Lurie and Enright, 2000), (Bode, 1945)) that, to get close to the optimum, $L_0(j\omega)$ should be as close as possible to the critical point (0 dB, -180) in the vicinity of the crossover frequency, ω_{cg}, and so should get as close as possible to the right side and bottom of the UHFB. ω_{cg} is defined as the first frequency such that $L(j\omega_{cg}) = 0$ dB.

3. AUTOMATIC LOOP SHAPING

The automatic loop shaping method proposed consists of adapting existing fractional structures to solve the QFT loop shaping problem. Additionally, new fractional structures are proposed. Used structures are introduced in sections 3.1-3.4. For evolutionary optimization, the *Genetic and evolutionary algorithm toolbox for use with Matlab* (GEATbx, (Pohlheim, 2004)) has been used.

3.1 TID

TID controller (Lurie, 1994) is a modified version of PID controller, where the proportional term is replaced by a *tilted* (fractional) term, with transfer function s^{-e_T}, which permits a better approach to theoretical optimum. The resulting controller, including a low pass filter, is given by

$$C_{TID}(s) = k\left(\frac{T}{s^{e_T}} + \frac{1}{s} + \frac{qDs}{q+s}\right)\frac{1}{\left(1+\frac{s}{w_h}\right)^{n_h}} \tag{2}$$

3.2 PI$^\lambda$D$^\mu$

PI$^\lambda$D$^\mu$ controller (Podlubny, 1994; Podlubny, 1999) is a PID generalization in which both integrator and derivative terms have real order, λ and μ respectively. In this work, the multiplicative version of PI$^\lambda$D$^\mu$ given in (Monje *et al.*, 2006) will be used, with transfer function

$$C_{\mathrm{PI^\lambda D^\mu}}(s) = k_c x^\mu \left(\frac{\lambda_1 s+1}{s}\right)^\lambda \left(\frac{\lambda_2 s+1}{x\lambda_2 s+1}\right)^\mu \tag{3}$$

CRONE 2 is a particular case of (3), where some parameters are linked.

3.3 CRONE BASED CONTROLLERS

The CRONE approach (Oustaloup *et al.*, 1998; Oustaloup, 1991) defines three generations of fractional controllers based on the use of frequency-band noninteger differentiators. First and second generations (CRONE 1 and CRONE 2) use real non integer differentiation, whereas third generation (CRONE 3) use complex non integer differentiation. Three CRONE structures based compensators are studied. The first one uses the transfer function structure common to CRONE 1 and 2. The basic component of this structure is an order n differentiator in the form of the implementable band-defined transfer function. An order n_I band limited integrator and an order n_F low-pass filter are added to manage accuracy, robustness and control effort problems, being the open loop structure finally defined as

$$L_{CR2}(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_I}\left(\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^n \frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_F}} \tag{4}$$

The application of the CRONE 2 structure to the QFT problem is quite straightforward.

The second compensator uses CRONE 3 structure, consisting of the substitution of the (real) order n integro-differentiator in CRONE 2 for the real part $D_r(s)$ of a (complex) order $n = a + ib$ integro-differentiator in the form of the implementable band-defined transfer function

$$D(s) = D_r(s) + iD_i(s) = \left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a+ib} \tag{5}$$

with $C_0 = \omega_h/\omega_u = \omega_u/\omega_l$ and $\omega_u = \sqrt{\omega_l\omega_h}$. The open loop structure $L(s)$ in CRONE 3 is finally defined as

$$L_{CR3}(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_I}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^a \cos\left[b\,\mathrm{Log}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)\right]\frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_F}} \tag{6}$$

The third compensator is decoupled CRONE 3 (Cervera and Baños, 2006; Cervera and Baños, 2007), a modified version of CRONE 3 structure (6), where some parameters are decoupled in order to obtain higher flexibility. Frequencies ω_h and ω_l are decoupled by defining new frequencies ω'_h, ω'_l and ω_{h4}. C_0 is decoupled by redefining C_0 in cos() as $C'_0 = cC_0$, where c is a new free parameter, $C_0 = \omega'_h/\omega_u = \omega_u/\omega'_l$, and $\omega_u = \sqrt{\omega'_l\omega'_h}$. The new structure to be shaped is

$$L_{decCR3}(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_I}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^a \cos\left[b\,\mathrm{Log}\left(cC_0\frac{1+\frac{s}{\omega'_h}}{1+\frac{s}{\omega'_l}}\right)\right]\frac{1}{\left(\frac{s}{\omega_{h4}}+1\right)^{n_F}} \tag{7}$$

For both CRONE 3 and decoupled CRONE 3, a detailed study of the parameters involved, its interrelations and their allowed ranks in order to obtain desired behavior, is developed in (Cervera and Baños, 2007).

3.4 FOCOMP TERMS

Fractional Order Complex (FOComp) terms (Cervera and Baños, 2005), are terms

$$T_i = \left(\frac{\omega_{ni}^2}{s^2+2\delta_i\omega_{ni}s+\omega_{ni}^2}\right)^{e_i} \tag{8}$$

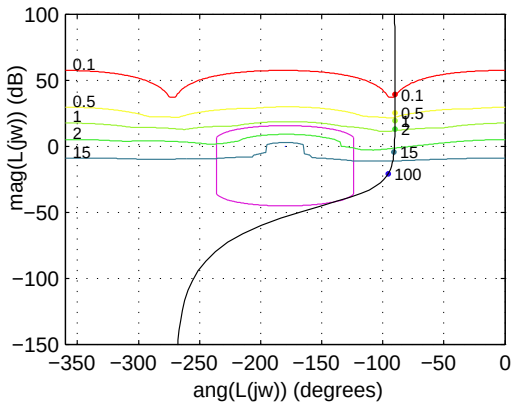

Fig. 2. PID based loop design.

with $e_i \in \mathbb{R}$, $e_i > 0$ corresponding to poles and $e_i > 0$ corresponding to zeros. Different combination of these terms can be used depending on the problem to be solved. For a typical tracking problem with a minimum phase plant, the structure

$$L_0 = KT_1T_2T_3 \tag{9}$$

with $K \in \mathbb{R}^+$, two FOComp poles and one FOComp zero, achieves very good results.

4. DESIGN EXAMPLE

To illustrate the behavior of the proposed design procedure, the QFT Toolbox for Matlab (Borghesani *et al.*, 1995) Benchmark Example number 2 is be used. It has also been used, for instance, in (Chen *et al.*, 1998) and (Raimúndez *et al.*, 2001). In (Chen *et al.*, 1998), two rational loops are designed, a second order one, with $K_{hf} = 136.6$ dB, and a third order one, with $K_{hf} = 130$ dB, with $n_{pe} = 3$ in both cases. A common $n_{pe} = 3$ will be used along this section, so that loops can be compared in terms of K_{hf} . In those structures which cannot get $n_{pe} = 3$ by themselves, a term

$$H(s) = \frac{1}{\left(1 + \frac{s}{\omega_h}\right)^{n_h}} \tag{10}$$

is added to fix $n_{pe} = 3$.

For comparison purposes, a classical PID (TID with $e_T = 0$) based design (fig. 2) is also considered, with $K_{hf} = 152$ dB and parameters in (2): $k = 9.08 * 10^6$, $T = 9.9 * 10^5$, $q = 184.9$, $D = 1.97$, $w_h = 2091.3$, $n_h = 2$.

The result obtained with TID controller is shown in fig. 3, with $K_{hf} = 140$ dB, improving PID. Parameters in (2) are $k = 3.86 * 10^-5$, $T = 4.4 * 10^5$, $e_T = 0.36$, $q = 5685.4$, $D = 0.77$, $w_h = 7625.2$, $n_h = 2$.

PI$^\lambda$D$^\mu$ improves TID result by another 12 dB, with $K_{hf} = 128$ dB, fig. 4. Parameters in (3), with

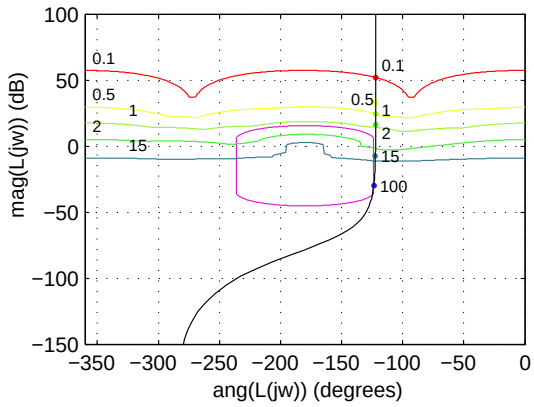

Fig. 3. TID based loop design.

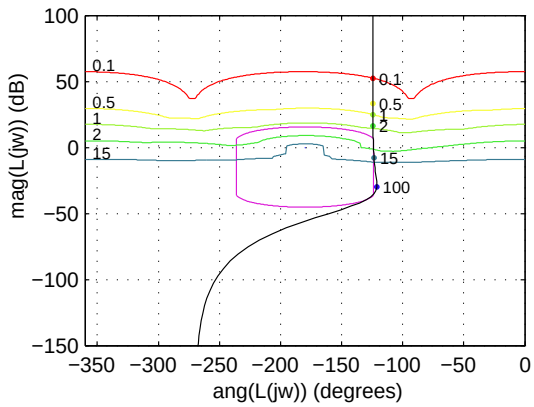

Fig. 4. PI$^\lambda$D$^\mu$ based loop design.

term (10): $k_c = 2.79$, $x = 10^10$, $\mu = 0$, $\lambda = 0.38$, $\lambda_1 = 0.0083$, $\lambda_2 = 10^10$, $\omega_h = 951.87$, $n_h = 2$.

CRONE 2 structure yields the loop shown in fig. 5, with corresponding $K_{hf} = 129.5$ dB and parameters in (4): $k = 2.98 * 10^6$, $\omega_l = 2588$, $n_I = 1.28$, $\omega_h = 1.65 * 10^7$, $n = 3.16$, $n_F = 3$. This result is slightly worse than PI$^\lambda$D$^\mu$'s, which could be expected, since CRONE 2 is the same structure as PI$^\lambda$D$^\mu$, but with some parameters linked, which implies less flexibility.

In fig. 6 it is shown a design using CRONE 3 structure, with $K_{hf} = 126.8$ dB and parameters in (6): $k = 0.0084$, $\omega_l = 153$, $n_I = 1.4$, $C_0 = 2.3$, $\omega_h = 825$, $a = 0.85$, $b = 0.34$, $n_F = 3$. The result is slightly better than using CRONE 2.

In fig. 7 it is shown a design using decoupled CRONE 3 structure, with $K_{hf} = 105.3$ dB and parameters in (7): $k = 1.46$, $\omega_l = 7$, $n_I = 1.07$, $C_0 = 11.2$, $\omega_h = 256.9$, $\omega_l = 7.1$, $a = 1.5$, $b = 0.45$, $c = 0.7$, $\omega'_h = 2 * 10^4$, $\omega'_l = 166$, $\omega_{h4} = 446$, $n_F = 3$. This result significantly improves the original CRONE 3 design (in more than 20 dB).

Finally, in fig. 9 it is shown the result obtained with the FOComp terms structure (9),

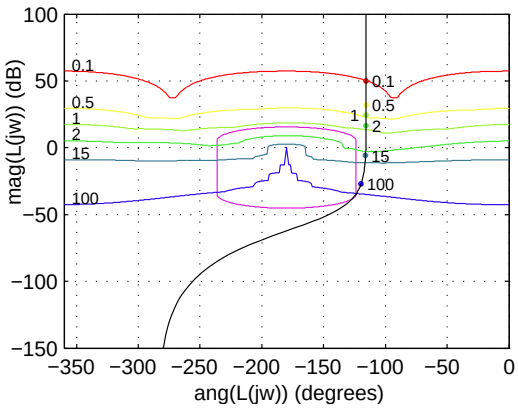

Fig. 5. CRONE 2 based loop design.

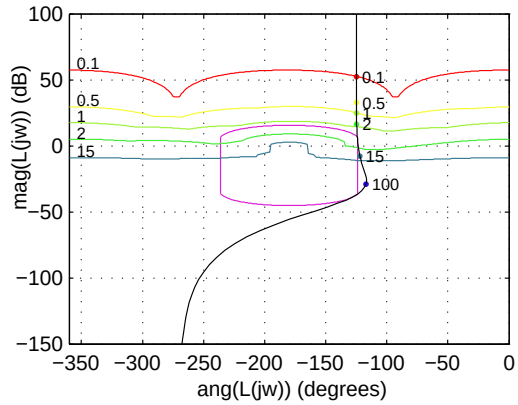

Fig. 6. CRONE 3 based loop design.

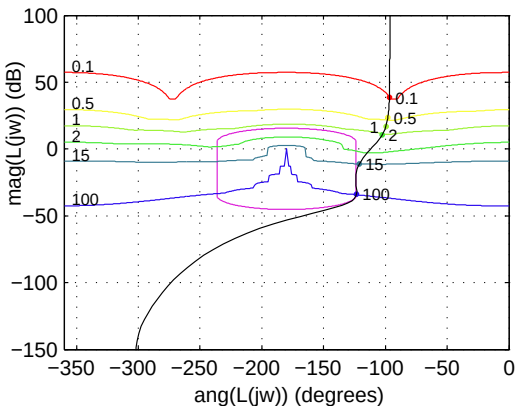

Fig. 7. Decoupled CRONE 3 based loop design.

with $K_{hf} = 94.2$ dB, which improves decoupled CRONE 3 result by more than 10 dB. Parameters for (9): $K = 8.3$, $\omega_{n1} = 2.02$, $e_1 = 0.16$, $\delta_1 = 2.32$, $\omega_{n2} = 743.6$, $e_2 = 1.4$, $\delta_2 = 0.28$, $\omega_{n3} = 7145$, $e_3 = -0.57$, $\delta_3 = 2.55$.

Fig. 9 shows a comparison of the noise amplification at the plant input, $T_N(s) = -\frac{C}{1+L}$, achieved by each design. Table 1 summarizes the results obtained in terms of K_{hf}. As it can be easily

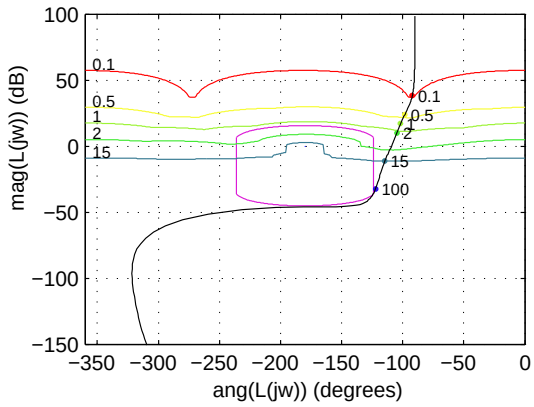

Fig. 8. FOComp terms based loop design.

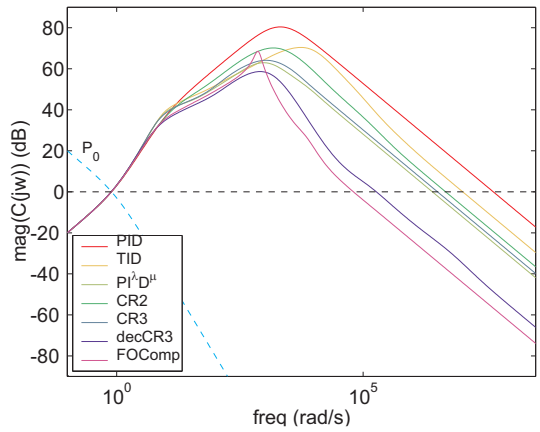

Fig. 9. $T_N(s)$ comparison. From left to right, in terms of ω_{cg}, FOComp terms, decoupled CRONE 3, PI$^\lambda$D$^\mu$, CRONE 3, CRONE 2, TID and PID.

checked, there is a direct correlation between K_{hf} and $T_N(s)$.

CONTROLLER	K_{hf} (dB)
PID	152
TID	140
PI$^\lambda$D$^\mu$	128
CRONE 2	129.5
CRONE 3	126.8
decoupled CRONE 3	105.3
FOComp	94.2

Table 1. K_{hf} comparison.

5. CONCLUSIONS

An automatic loop shaping procedure, based on evolutionary algorithms optimization on the parameters of a fixed structure, has been proposed. The key idea behind this proposal is the introduction of a structure with few parameters (a

must in order to get good results from evolutionary optimization) but, at the same time, flexible enough, thanks to its fractional nature, to get results which are close to the optimum. Fractional structures have been proposed as ideal candidate. Existing fractional structures has been studied and new fractional structures have been proposed. These structures have achieved very good results in terms of QFT classical optimization criterion.

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