Bode optimal loop shaping with CRONE compensators

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Outline

Problem Statement

- PROBLEM:
  - for operational bandwith $0 \leq \omega \leq \omega_0$
  - where desired $|L(j\omega)| = M_0 \gg 1$
  - given crossover frequency ω_c
  - compute $L(j\omega)$ which maximizes ω_0
- SOLUTION: decrease $|L(j\omega)|$ as fast as possible...

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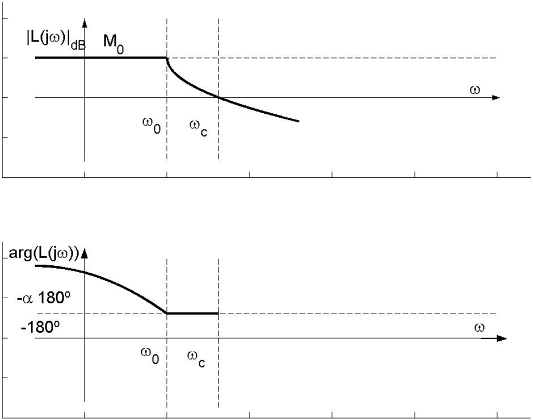

Ideal Bode Characteristic

For ideal optimal structure $\begin{cases} |L(j\omega)| = M_0, & 0 < \omega < \omega_0 \\ \angle L(j\omega) = -\alpha\pi, & \omega > \omega_0 \end{cases}$

solution is equivalent to maximize phase lag,
or minimizing stability margin,
so it is necessary to trade-off between ω_0 and stability margin...

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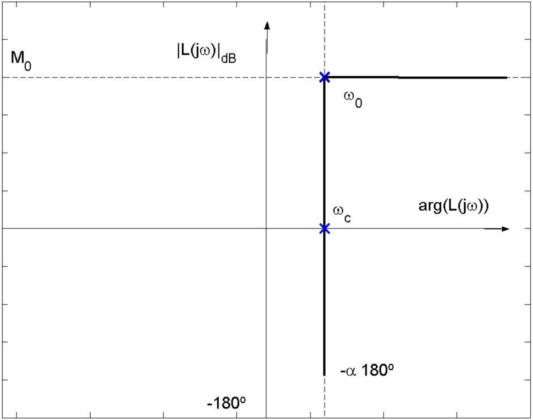

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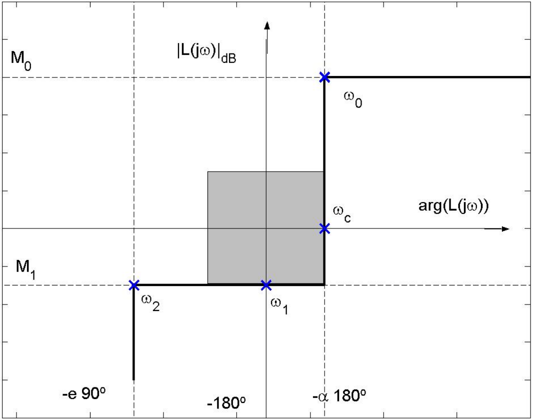
Four Parameters Bode Optimal Loop

- In general, the problem is well defined as a function of parameters:
  - M_0
  - α
  - ω_0
  - ω_c
- Not all of them independent.

Practical considerations about high frequency

- In practice, four parameters Bode Optimal Loop has to be modified:
  - To cope with sensor noise amplification
  - Because it is not realistic to assume good control of $|L(j\omega)|$ for high frequency.

Seven Parameters Bode Optimal Loop



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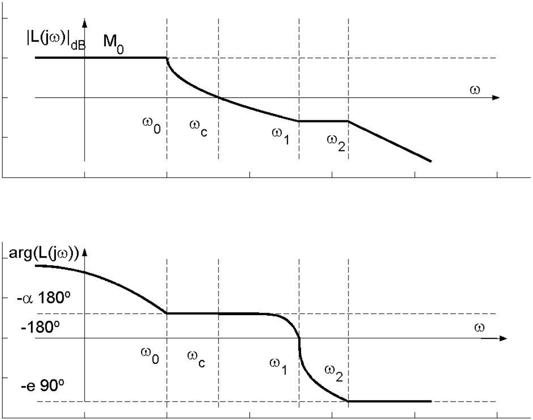
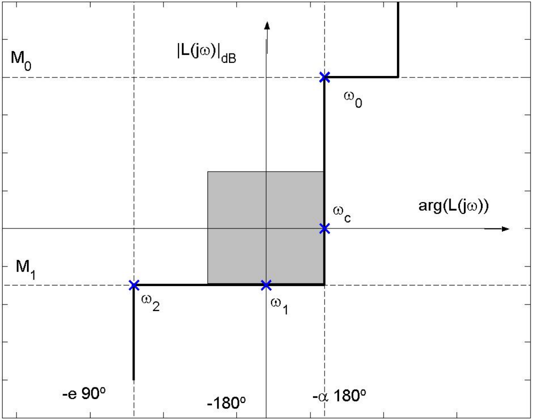


Eight Parameters Bode Optimal Loop

In order to add integrators to the loop, for a good steady state response ...

Eight Parameters Bode Optimal Loop



Eight Parameters Bode Optimal Loop

- Parameters:
  - M_0
  - M_1
  - α
  - ω_0
  - ω_c
  - ω_1
  - n
  - e
- Not all of them are independent.

Our Goal

Establish **relations between** these **eight parameters** and the **parameters of** a proposed **CRONE structure**,
so that a first approach to Bode optimal loop can be obtained in an easy and fast way.

CRONE Features for Bode Optimal Loop Shaping

- Easy to tune
- Few parameters
- For the 2/3 CRONE generation band defined compensator

$$D_r = \left(C_0 \frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}} \right)^a \cos \left[-b \,\mathrm{Log} \left(C_0 \frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}} \right) \right],$$

  - Phase and gain slope only depend on a (real differentiation order)
  - Gain and phase slope only depend on b (complex differentiation order)
- Idea: for a Bode optimal loop shape, with constant phase at (ω_l, ω_h) and constant gain at (ω_l', ω_h'), use real differentiator at (ω_l, ω_h) (b=0) and complex differentiator (a=0) at (ω_l', ω_h').

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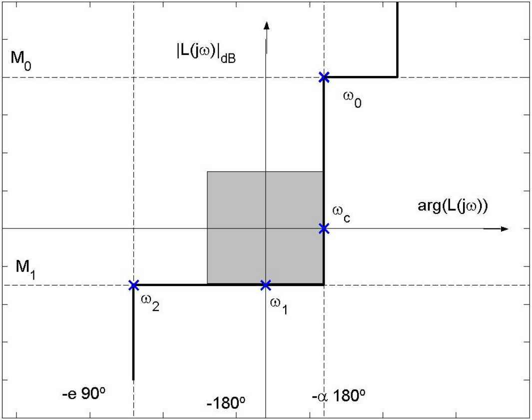

CRONE Features for Bode Optimal Loop Shaping

- Additionally, two terms to shape low and high frequencies
- Final structure:

$$L(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_l}\left(C_0\frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}}\right)^{a}$$

$$\cos\left[-b\,\mathrm{Log}\left(C_0'\frac{1+\frac{s}{\omega_l'}}{1+\frac{s}{\omega_h'}}\right)\right]\frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_h}}$$

Real Differentiator Term

- $L_2(s) = \left(C_0 \frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}} \right)^a$
- Design relations:
  - $a\left(\frac{\pi}{2} - 2\theta_l(\omega_u)\right) = (1-\alpha)\pi$
  - $\left(\frac{\omega_h}{2\omega_l}\right)^{-a} \approx M_0 M_1$

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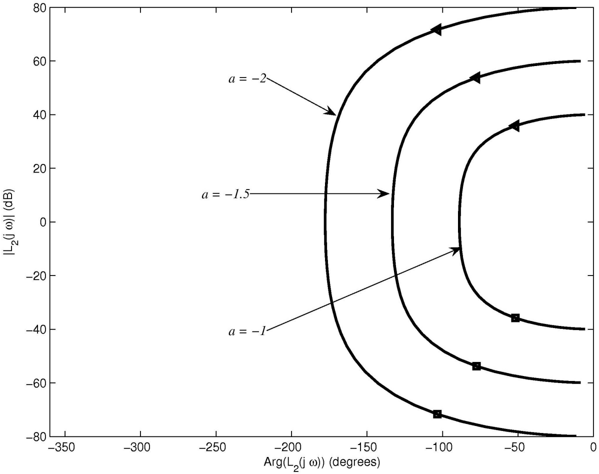

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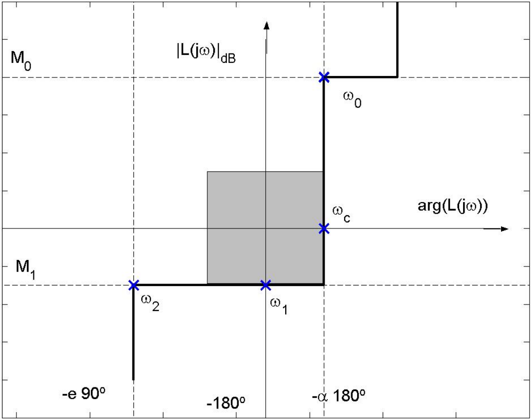

Low and High Frequency Terms

- $L_{CRONE2}(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_l}\left(C_0\frac{1+\frac{s}{\omega_l}}{1+\frac{s}{\omega_h}}\right)^a\frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_h}}$
- Design relations:
  - $n_l \geq n$
  - $n_h \geq e_p \geq n$
  - $|L_{CRONE2}(j\omega_c)| = 1$
  - $|L_{CRONE2}(j\omega_u)| = \frac{M_{0,dB}+M_{1,dB}}{2}$

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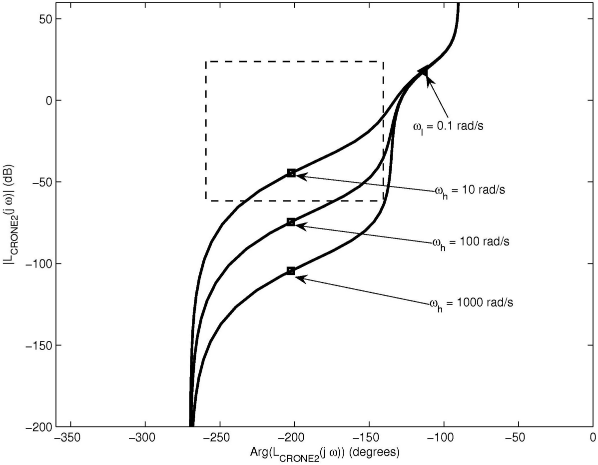

Complex Differentiator Term

- $L_3(s) = \cos\left[-b\,\mathrm{Log}\left(C_0' \frac{1+\frac{s}{\omega_l'}}{1+\frac{s}{\omega_h'}}\right)\right]$
- Complements $L_2(s)$, to increase phase lag at $[\omega_l', \omega_h']$
- To avoid non minimum phase:

$$b \log\left(\frac{\omega_h'}{\omega_l'}\right) < \pi$$

or, equivalently

$$b < b_{max} = \frac{\pi}{\log(\omega_h'/\omega_l')}$$

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Maximizing Loop Phase Lag

- Maximized by $b = b_{max}$, but...
- Design relations:
  - $\omega'_u = \omega_h \approx \omega_1$
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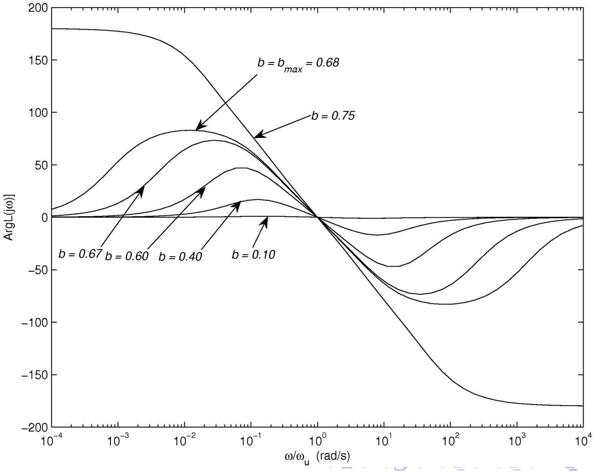

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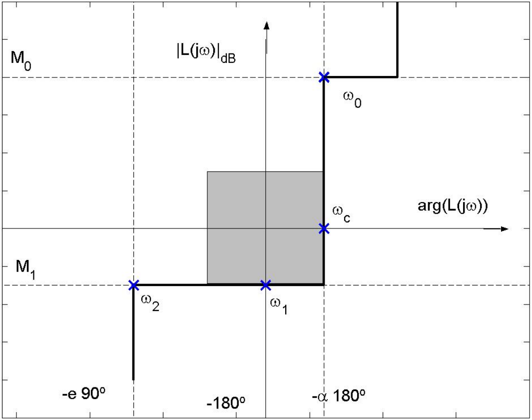

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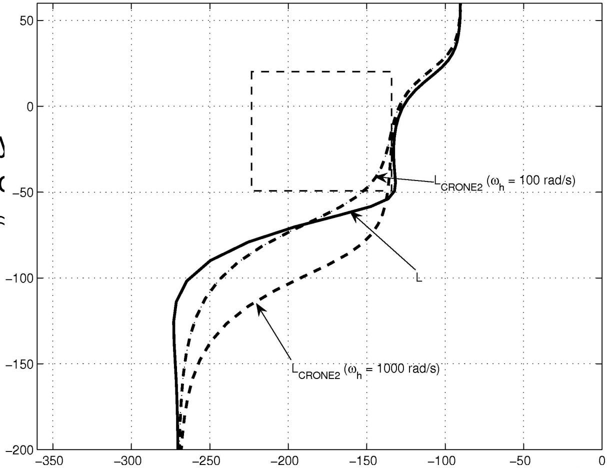

Desig Example

- 8 Parameters Bode Optimal Specifications:
  - $M_{0,dB}$=$M_{1,dB}$=30 dB
  - $\omega_0 = 0.4$ rad/s, $\omega_c = 0.4$ rad/s
  - $\alpha = 0.22$ (40º phase margin)
  - $e = 3$, $n = 2$
  - ω_1= 40 rad/s
- Loop obtained:

$$L_{ex}(s) = 0.87\left(\frac{0.34}{s}+1\right)^2\left(C_0\frac{1+\frac{s}{0.34}}{1+\frac{s}{93.5}}\right)^{-1.45} \cos\left[-1.8374\,\mathrm{Log}\left(C_0'\frac{1+\frac{s}{97.5}}{1+\frac{s}{250}}\right)\right]\frac{1}{\left(\frac{s}{93.5}+1\right)^3}$$

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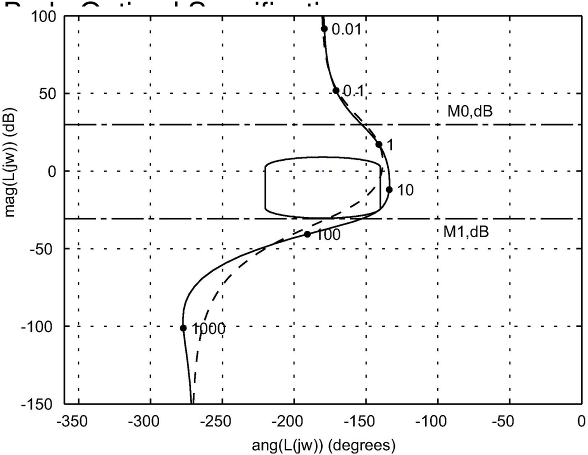

Conclusions

- A special CRONE compensator has been proposed to efficiently approximate Bode optimal loop.
- Bode optimal loop has been defined based on a number of parameters, and simple design rules have been obtained for tuning the proposed compensator.
- These rules yield a first solution of a rather hard problem.
- A finest tuning may require the use of some automatic loop shaping technique.

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