Automatic Loop Shaping in QFT by Using CRONE Structures

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Outline

The QFT Automatic Loop Shaping Problem

- QFT = Quantitative Feedback Theory: robust frequency domain control design methodology
- Key design step: loop shaping
- Loop shaping = nonlinear nonconvex optimization problem.

Previous Approaches

APPROACH	DRAWBACKS
Simplify the problem (linearize/convexify)	conservative
Use fixed (rational) structure (with few parameters)	not flexible
nonlinear + nonconvex optimization algorithm	- computationally demanding - global optimum not always guaranteed

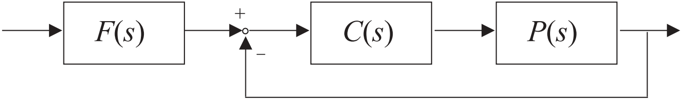
Proposed Approach

- No simplification = not conservative.
- Evolutionary algorithms.
- Fixed structure:
  - Flexible enough to approach optimum
  - With few parameters

$\Rightarrow$ **FRACTIONAL STRUCTURES**, in particular
CRONE STRUCTURES

QFT introduction (1)

- Basic idea: quantitative relation uncertainty ⇔ control effort
- Typical configuration:

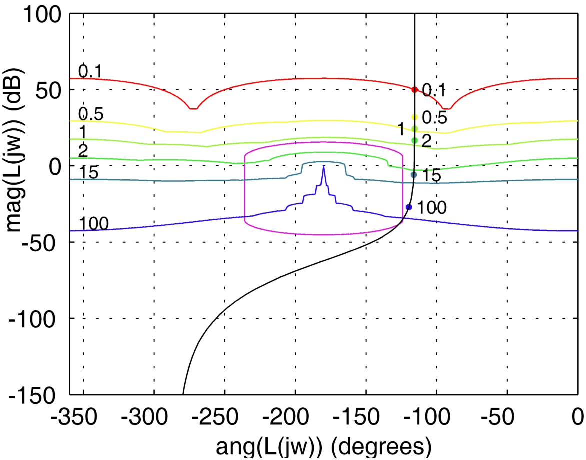


- For $P(s)$, *template* $\mathcal{P}$ and *nominal* P_0.
- Design of $C(s)$: in Nichols chart, with $L_0(s) = P_0(s)C(s)$

QFT introduction (2)

- Ω: discrete set of design frequencies.
- Robust stability/performance specifications $\Rightarrow$
  $\Rightarrow$ *boundaries* $\mathcal{B}_\omega$, $\omega \in \Omega$
- Basic step: *loop shaping* – design of $L_0(j\omega)$ which
  - satisfies boundaries
  - is reasonably close to optimum
- Optimization: minimization of high frequency gain (K_{hf})
- Optimum characteristics:
  - on performance boundaries
  - tightly following UHFB.

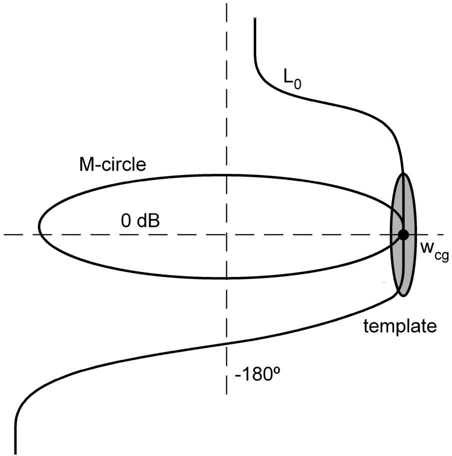
QFT design example



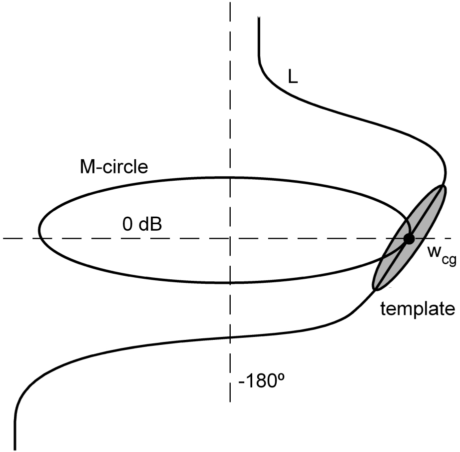
CRONE - Introduction

- CRONE = *Contrôle Robuste d'Ordre Non Entier*
- Based on the use of noninteger differential operator
- CRONE 1 & 2:
  - real non integer differentiation: $\beta(s) = ks^n, \ n, k \in \mathbb{R}$
  - band defined: $\beta(s) = k\left(\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^n$
- CRONE 3:
  - complex differentiation: $D(s) = \left(\frac{s}{\omega_u}\right)^{a+ib}, \quad a, b, \omega_u \in \mathbb{R}$
  - band defined: $D(s) = \left(C_0 \frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a+ib}$

CRONE 1 & 2 - Purpose



CRONE 3 - Purpose



CRONE - Structures

- CRONE 1 & 2:

$$L(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_l}\left(\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{n}\frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_F}}$$

- CRONE 3:

$$L(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_l}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a}$$
$$\cos\left[b\,\mathrm{Log}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)\right]\frac{1}{\left(\frac{s}{\omega_h}+1\right)^{n_F}}$$

ALS with CRONE 2 structure

- Original semantic is lost
- Structure: original, slightly rewritten

$$L(s) = k' \left(\frac{\omega_l}{s} + 1\right)^{n_l} \left(\frac{\omega_h + s}{\omega_l + s}\right)^{n} \frac{1}{(s + \omega_h)^{n_F}}$$

- n_l, n_F: fixed/conditioned by n_{p0} and n_{pe}
- k conditioned by specified ω_{cg}

ALS with (modified) CRONE 3

- Original semantic is lost
- Structure: *decoupled* (to obtain more flexibility)

$$L(s) = k\left(\frac{\omega_l}{s}+1\right)^{n_I}\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}}\right)^{a} \cos\left[b\,\mathrm{Log}\left(cC_0\frac{1+\frac{s}{\omega_h'}}{1+\frac{s}{\omega_l'}}\right)\right]\frac{1}{\left(\frac{s}{\omega_{h4}}+1\right)^{n_F}}$$

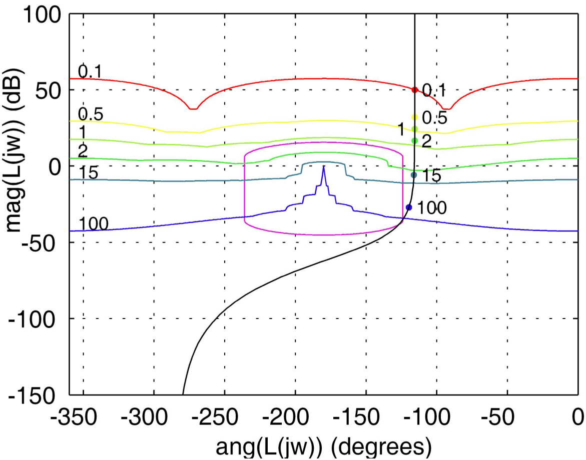
- n_I, n_F: fixed/conditioned by n_{p0} and n_{pe}
- k conditioned by specified ω_{cg}

RHP zeros avoidance

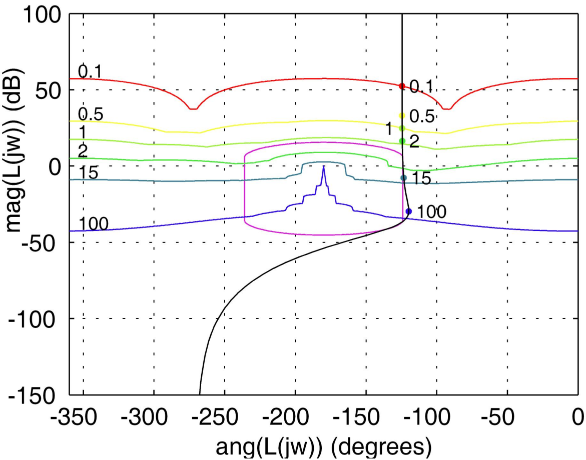
- possible RHP zeros from $Q(s) = \cos\left[b\,\mathrm{Log}\left(cC_0 \frac{1+\frac{s}{\omega_h'}}{1+\frac{s}{\omega_l'}}\right)\right]$
- no RHP zeros if and only if:
  - if $b > 0$,
$$\ln C_0 < \frac{\pi}{2b}(2k_{ceil} - 1) - \ln c$$
$$\ln C_0 < -\frac{\pi}{2b}(2k_{floor} - 1) + \ln c$$
  - if $b < 0$,
$$\ln C_0 < \frac{\pi}{2b}(2k_{floor} - 1) - \ln c$$
$$\ln C_0 < -\frac{\pi}{2b}(2k_{ceil} - 1) + \ln c$$

for $k_{ceil} = \left\lceil \frac{1}{2}\left(\ln c^{\frac{2b}{\pi}} + 1\right)\right\rceil$, $k_{floor} = \left\lfloor \frac{1}{2}\left(\ln c^{\frac{2b}{\pi}} + 1\right)\right\rfloor$

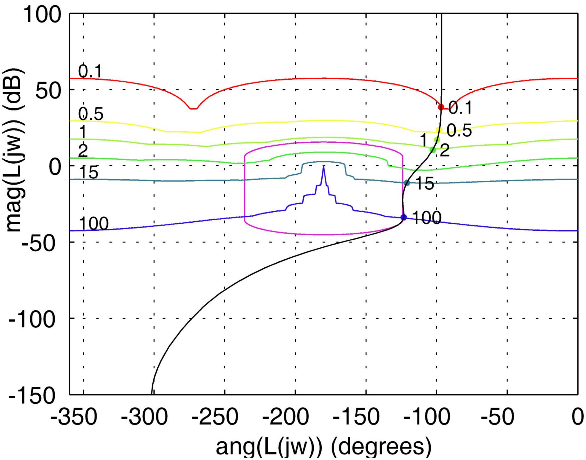
CRONE 2 design



(original) CRONE 3 design



(modified) CRONE 3 design



Comparison

loop structure	K_{hf} (dB)
order 2 in Chen *et al.* 1998	136.60
order 3 in Chen *et al.* 1998	130.00
CRONE 2	129.50
CRONE 3	126.81
decoupled CRONE3	105.32

Summary

- A QFT ALS method, based on fractional structures, has been developed.
- CRONE structures use has been considered, including:
  - decoupling of CRONE 3 structure for higher flexibility
  - RHP zeros characterization in original/modified CRONE 3