ABOUT THE INTEREST OF LINEARISED KALMAN FILTER FOR LOW-COST GPS-BASED HYBRID POSITIONING SYSTEM FOR LAND VEHICLES

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ABSTRACT

GPS-based positioning systems have been widely introduced in cars and land vehicles, but, except for comfort-oriented applications such as simple navigation, the level of availability and integrity provided by these systems are far from satisfying the requirements of most of the new location-based services that are under development for road transportation. The research is very active on the topic of data fusion between GNSS and proprioceptive sensors to improve the naturally poor GPS integrity. Among the numerous fusion techniques that can be used, the well-known Extended Kalman Filter (EKF) is still the most popular, but simpler and less computation demanding techniques can also be used such as Linearised Kalman Filter (LKF). This paper discusses the design of a hybrid localisation system with low cost sensors (odometer and gyro) and compares the efficiency of the 2 filters, especially around long GPS outages, which represent the most constraining situations. Different trials have been carried out on a real circuit and illustrate the comparison.

KEYWORDS
satellite navigation, data fusion, integrity, kalman filtering

INTRODUCTION

Nowadays, there are many low cost positioning systems for land vehicles based on a single GNSS solution. It is not new that the GNSS devices cannot guarantee high integrity positioning, especially in unfriendly environments. The problem is that the GPS systems work well only in open environments with no overhead obstructions and they are subject to large unavoidable errors when the reception from some of the satellites is blocked (forests, tunnels and urban environments).

Day by day, the number of applications where it is necessary to use a high integrity positioning system is growing. Toll collect [6] is a representative example of this kind of applications. The main features of solutions suitable for mass market applications would be low cost (including easy installation and maintenance) and continuous positioning, even during the outages of the GNSS signals, and integrity.

In order to improve integrity in the localisation system, GPS is usually hybridized with additional proprioceptive sensors. There are many studies related to this topic in order to guarantee the proper position quality using low cost sensors with GPS receivers. The most popular and less expensive sensor is the odometer, available in any vehicle. But the development of sensors based on the MEM technology have helped to increase the research into hybrid GPS/MEM systems too.

Many works on the subject use the Extended Kalman Filter (EKF) as a sensor fusion tool. The EKF performance is reliable in many practical situation, but the non-linear state equations may lead to instability problems. Other filtering methods can be found in bibliography, Sequential Monte-Carlo Methods [5], particle-based solution [3] and others.

Different approaches are being studied related to hybrid GPS/MEM/Odometry systems. In [13] we can find a summary of different GPS/INS system structures and the advantages of each one. The author presents a solution based in a low cost IMU system plus the GPS device for land vehicle positioning. Numerous recent publications discuss about similar hybrid systems [14, 1, 3, 9, 10, 8, 12, 4]. On the one hand, the simplest approach is the fusion of the GPS and odometry (ABS system or odometer of a car) readings [3, 8], usually with a EKF. The disadvantage of this system is the lack of precision in the heading. On the other hand, more precise system using complete GPS/INS system are presented in [1, 14]. Also, a new wave is the addition of a digital map as another source of information [9, 14]. Finally, other possibility would be a GPS/odometer system plus a gyro for heading rate [12, 4, 10]. This system is quite simple and the gyro supplies the lack of precision in the heading measurement. This last option has been adopted in our research. Low-cost MEM gyros can be purchased nowadays and, moreover, it is likely that the outputs coming from the ones included into the ESP systems will soon be available for mass-market developments.

Another important aspect of the positioning system is the vehicle model used. Kinematic 2D models [12] provide the evolution of the vehicle pose in a two-dimensional reference frame (2 coordinates and yaw angle), in a compliant way with the non-holonomic constraints. These models are used to propagate the estate (here the pose) and are generally fed by the proprioceptive sensors inputs (odometer + gyrometer or inertial unit), but they are non-linear. As a consequence, it is necessary to implement an EKF to be able to propagate the state covariance matrix using the Kalman formalism.

Other models usually used, mainly in aircraft GPS/INS systems, are the three-dimensional ones, providing the possibility to use strap-down inertial measurement units that offer valuable information on all the degrees of freedom of the vehicle [7]. But 3D models imply high computational cost, especially for the computation of the jacobian matrixes and they are no very suitable for land systems because they do not implicitly take into account the non-holonomic constraints of the vehicle and the fact it moves on a plane.

Compared to EKF, the so-called "error models" or LKF use an alternative way of linearising and they are often said to require less computational resource [13, 1, 10]. The general idea is to consider the error of the pose respect to the actual (or nominal) trajectory, as the state vector to estimate, instead of the pose itself. The 3D error model, based on Pinson error model [11], computes the dynamic error propagation model of the INS, and it is independent of the vehicle kinematics. A more simplified 2D error model can be obtained through the kinematic model of the vehicle, and so it is possible to obtain a low-computational filter. Actually, this "error model" method is nothing but the well-known "Linearised Kalman Filter", with consists in linearising the state equation on the nominal trajectory while the EKF does it on the estimated trajectory.

This paper presents a low-cost positioning system based on DGPS, the standard odometer of the vehicle and two gyros based on different technologies (MEMS and FOG). A 2D simplified kinematic model is used to propagate the pose and the two methods for linearising the state equation are presented and compared on real tests.

DESCRIPTION OF THE METHODS

In our computation we use the classical 2D Kinematic model of a vehicle in a plane (position of a point of the vehicle plus yaw angle), which in discrete time can be computed with the following equation:

$$\begin{bmatrix} x(k) \\ y(k) \\ \psi(k) \end{bmatrix} = \begin{bmatrix} x(k-1) \\ y(k-1) \\ \psi(k-1) \end{bmatrix} + T \cdot \begin{bmatrix} \cos(\psi(k-1) + Tw(k-1)/2) & 0 \\ \sin(\psi(k-1) + Tw(k-1)/2) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v\,(k-1) \\ w\,(k-1) \end{bmatrix} \quad (1)$$

where (x, y) is the position of the vehicle at the rear axle center, ψ the yaw angle, and T the sampling period. The velocity v and angular rate w are measured with an odometer and a gyro sensor. The odometer is situated at the rear wheels axle, and the gyro in the origin of the local frame. The x axis corresponds to the North and y to the East.

The vehicle model can be expressed in a compact way as:

$$\mathbf{x}(k) = f[\mathbf{x}(k-1), \mathbf{u}(k-1)] + \mu(k-1) \quad (2)$$

where μ is assumed to be a Gaussian white noise. The pose and input vectors are:

$$\mathbf{x}(k) = (x(k) \;\; y(k) \;\; \psi(k))^T \quad (3)$$

$$\mathbf{u}(k) = (v(k) \;\; w(k))^T \quad (4)$$

EXTENDED KALMAN FILTER EQUATIONS

Taking into account the previous kinematic model, the evolution of the centre of phase of the GPS antenna can be expressed:

$$x(k) = x(k-1) + ds(k-1)cos(\psi(k-1) + Tw(k-1)/2) - w(k)T(D_x sin\psi(k-1) + D_y cos\psi(k-1))$$
$$y(k) = y(k-1) + ds(k-1)sin(\psi(k-1) + Tw(k-1)/2) + w(k)T(D_x cos\psi(k-1) - D_y sin\psi(k-1))$$
$$\psi(k) = \psi(k-1) + w(k)T \qquad (5)$$

with $ds(k) = v(k)T$ and (D_x, D_y) are the coordinates of the GPS antenna in body frame (figure 1).

The evolution of the covariance matrix of the state with the EKF is:

$$P(k|k-1) = F_x(k-1)P(k-1|k-1)F_x(k-1)^T + F_u(k-1)Q_u F_u(k-1) + Q \qquad (6)$$

where F_x and F_u are the jacobian matrixes with respect to the state and the input. Matrix Q_u represents the covariance matrix of the noise of the odometry, and Q the covariance matrix of the mismatch of the kinematic model in the prediction step.

The measurement vector of the EKF is only made up for GPS readings:

$$\mathbf{z}(k) = [\ x^{GPS}(k) \quad y^{GPS}(k) \quad v_x^{GPS}(k) \quad v_y^{GPS}(k)\ ]^T \qquad (7)$$

and the innovation is computed with the equation:

$$\nu(k) = \mathbf{z}(k) - h(\hat{\mathbf{x}}(k|k-1), \mathbf{u}(k)) \qquad (8)$$

where h is the measurement function. Again it can be seen that the if we use the GPS velocities in the measurement vector we have a non-linear relationship and it is necessary to use the jacobian matrix H. Using only the GPS position the measurement equation will be linear.

Summarizing the estimation step with EKF:

$$W(k) = P(k|k-1)H^T\left(HP(k|k-1)H^T + R\right)^{-1} \qquad (9)$$
$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + W(k)\nu(k) \qquad (10)$$
$$P(k|k) = P(k|k-1) - W(k)\left(HP(k|k-1)H^T + R\right)W^T(k) \qquad (11)$$

R denotes the covariance matrix of the measurement.

THE LINEARISED KALMAN FILTER

In this method we assume that the pose can be expressed as the odometric (or dead-reckoning) pose plus an error that is supposed to remain small:

$$\mathbf{x}(k) = \mathbf{x}(k)^{ODO} + \delta\mathbf{x}(k) \qquad (12)$$

If we apply equation 12 to equation 2:

$$\mathbf{x}(k)^{ODO} + \delta\mathbf{x}(k) = f[\mathbf{x}(k-1)^{ODO} + \delta\mathbf{x}(k-1), \mathbf{u}(k-1)] + \mu(k-1) \qquad (13)$$

Applying first order Taylor linearisation to the previous equation, we obtain:

$$\mathbf{x}(k) \simeq f[\mathbf{x}(k-1)^{ODO}, \mathbf{u}(k-1)] + \left|\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{X}=\mathbf{X}(k-1)^{ODO}} \cdot \delta\mathbf{x}(k-1) \quad (14)$$

and applying again equation 2:

$$\mathbf{x}(k) \simeq \mathbf{x}(k)^{ODO} + \left|\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{X}=\mathbf{X}(k-1)^{ODO}} \cdot \delta\mathbf{x}(k-1) \quad (15)$$

Therefore, we obtain the evolution of the error in the vehicle pose (rear axle) in discrete time:

$$\begin{bmatrix} \delta x(k) \\ \delta y(k) \\ \delta\psi(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -v(k-1)T \cdot sin(\psi(k-1)) \\ 0 & 1 & v(k-1)T \cdot cos(\psi(k-1)) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta x(k-1) \\ \delta y(k-1) \\ \delta\psi(k-1) \end{bmatrix} \quad (16)$$

where the state vector is composed this time of the errors in vehicle pose:

$$\delta\mathbf{x}(k) = (\delta x(k) \quad \delta y(k) \quad \delta\psi(k))^T \quad (17)$$

Now we can see that the evolution of the state is driven by the same jacobian matrix F_x as in the EKF, but this matrix is always assessed at the odometric pose. In this method, the inputs (odometer and gyro) are considered as deterministic parameters in the evolution matrix of the state: this is an approximation that we compensate by tuning directly the Q matrix.

The advantage of this approximation is that the jacobian matrix F_u is not necessary anymore.

In order to keep the odometric error small, LKF proceeds to regular reset of the error by setting the dead-reckoning pose onto the corrected pose. This is equivalent as a restart of the LKF with new initial conditions.

Figure 2 shows the architecture of the method, that is summarized in the following algorithm:

1. `State and Pose Prediction. Compute:`
   - $\mathtt{x}(k)^{ODO} = f[\mathtt{x}(k-1)^{ODO}, \mathtt{u}(k-1)]$
   - $\delta\hat{\mathtt{x}}(k|k-1) = F[\mathtt{x}(k)^{ODO}, \mathtt{u}(k-1)] \cdot \delta\hat{\mathtt{x}}(k-1|k-1)$
   - $P(k|k-1) = F(k)P(k-1|k-1)F^T(k) + Q(k-1)$
2. `If GPS measurement is available, compute Kalman Estimation (otherwise Go to 1):`
   - $\delta\hat{\mathtt{x}}(k|k) = \delta\hat{\mathtt{x}}(k|k-1) + W(k)\nu(k)$
   - $P(k|k) = P(k|k-1) - W(k)\left(HP(k|k-1)H^T + R\right)W(k)^T$

3. `If the Reset Time is up and GPS measurement is available, compute Reset Step (otherwise Go to 1):`

   - $\mathtt{x}(k|k) = \mathtt{x}(k|k-1) + \delta\hat{\mathtt{x}}(k|k)$
   - $\delta\hat{\mathtt{x}}(k|k) = 0$

4. `Go to 1`

THE TEST PERFORMED

To test the two algorithms, we have defined and performed a set of experiments. The circuit is 3 kilometers long (figure 5), the experiments are carry out with a Peugeot van equipped with the following sensors:

- The odometer that is situated at the front wheels axle (but we will admit that it is valid as rotation measurement of the rear wheels axle).
- The fiber optic gyrometer KVH 2000 e-core series, 100 Hz, 100 deg/s full range.
- The DGPS using Omnistar differential corrections.

Different velocities are tested (test 1, 2 and 3), and another test (test 4) is made in opposite sense at medium speed. The maximum speed tested was 100 km/h in test 3.

The EKF was tuned on the logged FOG data for the test at medium speed [2]. 50 seconds GPS masks have been applied, separated by 20 seconds. The variances of the model were tuned so that the error between the estimated and reference positions are included inside the 2σ envelop predicted by the filter. The parameters to tune are the variances of the gyro, odometer and GPS measurements, as well as variances in the mismatch of the model (Q matrix).

The LKF has been tuned with the same methodology as in the case of the EKF. The parameters to tune are the variance of the GPS measurement and the mismatch of the model only. Figure 6 shows the variation of the positioning error as a function of the variance of the position (first and second diagonal elements of Q) horizontally in the figure and vertically the variance of the yaw angle (last diagonal element of Q).

The variance of the GPS measurements is fixed: 0.8 m. The tuning that corresponds to the central test case is 0.05 m for x and y positioning components and 0.05 deg for the yaw angle (table 1). The parameters are divided by 10 from the upper left test case to the next one.

The statistical indicators that we used are maximum and RMS errors. These indicators are included in the title of each test case and also in every table.

A few comments arise from figure 6: note that it corresponds to using FOG gyro data.

- In the lower left corner, the yaw angle is too much precise compared to the positioning components. This entails a saturation of the error envelop.

$Q_{xy} = 0.5^2 m^2$ $Q_{yaw} = 0.5^2 deg^2$	$Q_{xy} = 0.05^2 m^2$ $Q_{yaw} = 0.5^2 deg^2$	$Q_{xy} = 0.005^2 m^2$ $Q_{yaw} = 0.5^2 deg^2$
$Q_{xy} = 0.5^2 m^2$ $Q_{yaw} = 0.05^2 deg^2$	$Q_{xy} = 0.05^2 m^2$ $Q_{yaw} = 0.05^2 deg^2$	$Q_{xy} = 0.005^2 m^2$ $Q_{yaw} = 0.05^2 deg^2$
$Q_{xy} = 0.5^2 m^2$ $Q_{yaw} = 0.005^2 deg^2$	$Q_{xy} = 0.05^2 m^2$ $Q_{yaw} = 0.005^2 deg^2$	$Q_{xy} = 0.005^2 m^2$ $Q_{yaw} = 0.005^2 deg^2$

Table 1: Tuning test cases.

- The lower line shows too small error envelops. Similarly, the upper line shows too large envelops.
- The upper left test case of figure 6 corresponds to the more pessimistic envelop.
- The balance between yaw and the position is well done along the diagonal.

We finally chose the tuning between the two upper left diagonal test cases (0.1 m for x and y positioning components and 0.1 degrees for the yaw angle). With this tuning, the performance are good and the error envelop is reasonably pessimistic. This means that we reduce the confidence of our sensors and model compared to GPS.

In order to test the capability of the methods with a low-cost gyro, we simulated rotation rate data by adding noise to KVH measurement. A static experiment has been done in our laboratory for that purpose. We logged during three minutes data output by the previous KVH FOG and also vertical rotation rate output by a Crossbow MEMS VG-400. Both were mounted on the same platform in a vehicle and stabilised in temperature during this experiment. The beginning and the end of the experiment correspond to perturbations of the vehicle stability due to the movement of the operators (figure 3). We notice that the level of noise of the FOG is particulary low (0.007 deg/sec standard deviation) when absolutely no perturbation occurs (i.e. between 15 and 135 sec). In the standard conditions of use of a vehicle, this level of noise would not be so low: this is visible in the beginning and the end of figure 3 and also in figure 4. In the mean time, the level of noise of the MEMS gyro is much larger (0.19 deg/sec standard deviation): this last level of noise was that we used for the simulation. Both gyros are affected by the rotation rate of the earth (0.003 deg/sec at our latitude): the difference between the two mean values is 0.07 deg/sec, which fixed the magnitude of a possible value for the simulated bias.

The same trials of the algorithms are carried out with MEMS data (Tables 2 and 3).

We observe that the performances of both methods are very similar for the tests performed here. The question arises about whether this is due to the simplification (i.e. the consideration of the inputs as deterministic parameters) or due to the difference between the linearisation techniques. To help conclude, we ran the EKF with the same simplification and we obtained exactly the same results as the LKF, providing that this is often reset (i.e. almost at the GPS period). So, it means that it has been relevant for these tests to simplify the tuning process.

Last but not least, we have to pay attention to the main parameter of the LKF: its reset periodicity.

Parameters	**Test 1**	**Test 2**	**Test 3**	**Test 4**
number of laps performed	2	4	8	2
average speed (km/h)	20	40	60	40
time (min)	18	18	18	6
$E_{RMS}(EKF)$ *m*	2.36	3.44	5.63	4.51
$E_{max}(EKF)$ *m*	9.92	13.74	25.88	16.16
$E_{RMS}(LKF)$ *m*	2.01	3.25	5.73	3.38
$E_{max}(LKF)$ *m*	8.85	12.92	28.21	10.35

Table 2: Summary of the EKF and LKF trials with the FOG gyro.

Parameters	**Test 1**	**Test 2**	**Test 3**	**Test 4**
$E_{RMS}(EKF)$ *m*	6.58	9.54	11.98	10.91
$E_{max}(EKF)$ *m*	19.87	31.87	61.69	48.60
$E_{RMS}(LKF)$ *m*	5.27	8.23	11.08	10.89
$E_{max}(LKF)$ *m*	16.20	28.40	61.69	50.74

Table 3: Summary of the EKF and LKF trials with the MEMS gyro.

The duration between two consecutive reset is another parameter to tune. It appears that for our high quality FOG gyro, this time can be very large (a couple of minutes) with no consequence. This would be completely different if the bias was much larger, like with a MEMS gyro. With a bias of an order of magnitude of 0.1 degree per second, the influence became sensible from a duration of 3 minutes (table 4).

Gyro	**Parameters**	**Test 1**	**Test 2**	**Test 3**	**Test 4**
FOG gyro	$E_{RMS}(LKF)$ *m*	2.05	3.27	5.76	3.25
	$E_{max}(LKF)$ *m*	9.18	12.90	28.19	9.51
MEMS gyro	$E_{RMS}(LKF)$ *m*	5.95	10.14	16.18	11.15
	$E_{max}(LKF)$ *m*	20.02	44.88	87.10	50.98

Table 4: Summary of the LKF trials with reset every 3 min for FOG and MEMS gyros.

CONCLUSIONS

The main contribution of this paper is a comparison of the two usual implementations of Kalman filtering for 2D vehicle navigation (EKF and LKF). The first is the Extended Kalman Filter and the second is the Linearised Kalman Filter that respectively linearises around the current pose (EKF) and applies on the error with respect to the dead-reckoning (LKF).

Both were applied on exactly the same data set. We show the results of both methods based on tests on a real circuit, with simulated GPS outages. The performances of both methods are very similar, particularly if the Linearised Kalman Filter is often reset.

An approximation, that consists in gathering both modeling error and inputs errors (gyro and odometer) has been tested and it brings about quite equivalent results as a complete formulation, with the advantage of the lower cost computation (because it is not necessary to compute the jacobian matrixes for inputs at each iteration).

A disadvantage of the LKF appears if one uses a low-cost gyro (noisy and biased signals) and increases the reset interval: actually, if the error grows too large, the LKF linearisation would become more instable than the EKF linearisation.

Another point of interest to mention in this conclusion is the methodology used for tuning the algorithms, that aims at balancing the influence of the different sources of error (position and heading mismatch of the model), as well as the relevance of the error envelop. This methodology makes a key point in engineering a reliable real time filter for ITS applications that requires a quantified level of integrity.

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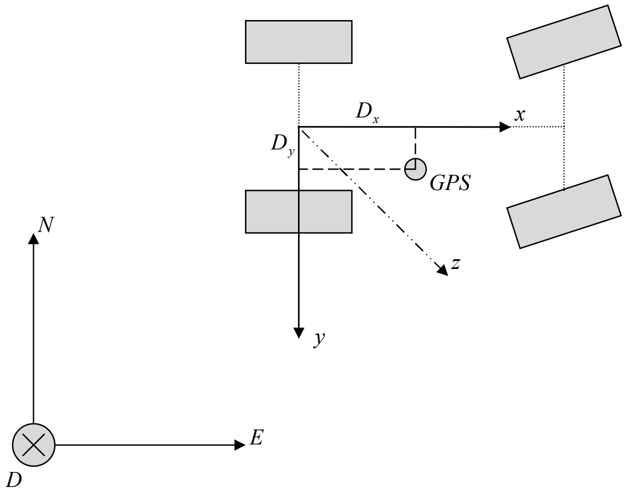

Figure 1: Local and Global Reference Frames.

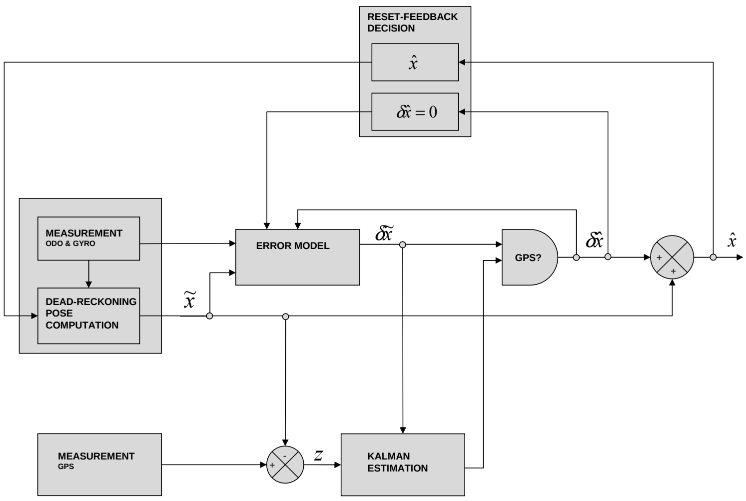

Figure 2: Architecture of the positioning system.

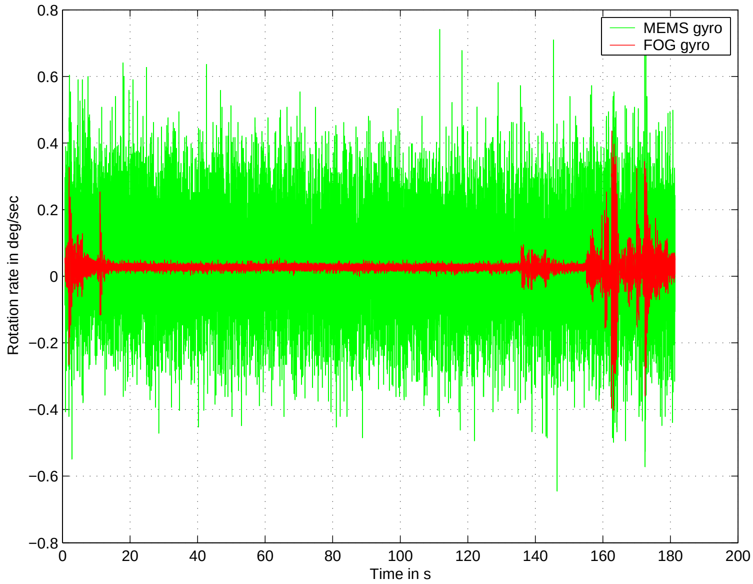

Figure 3: Static test with the MEMS and FOG gyros.

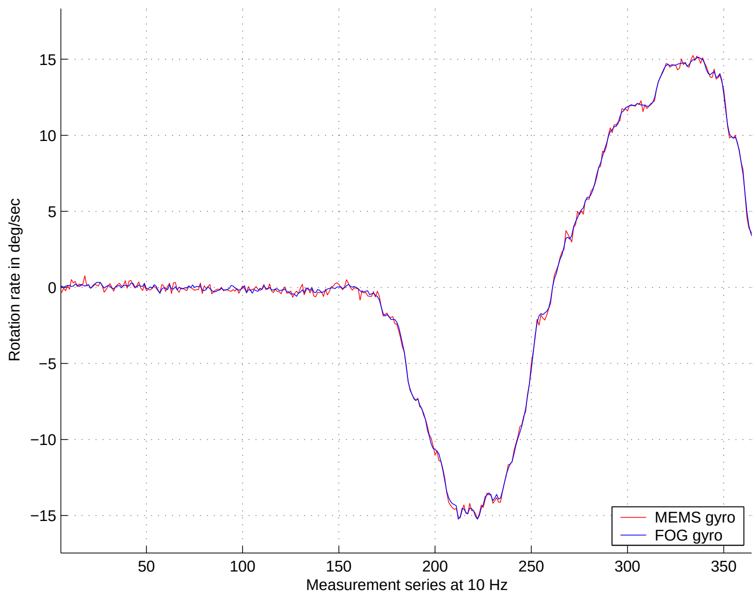

Figure 4: Gyro signals at the beginning of the Test 4.

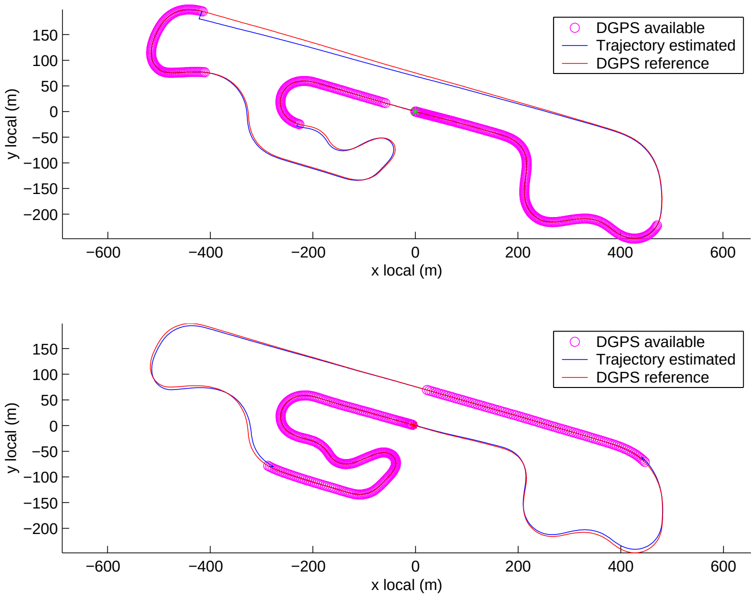

Figure 5: Circuit with the EKF method (FOG gyro).

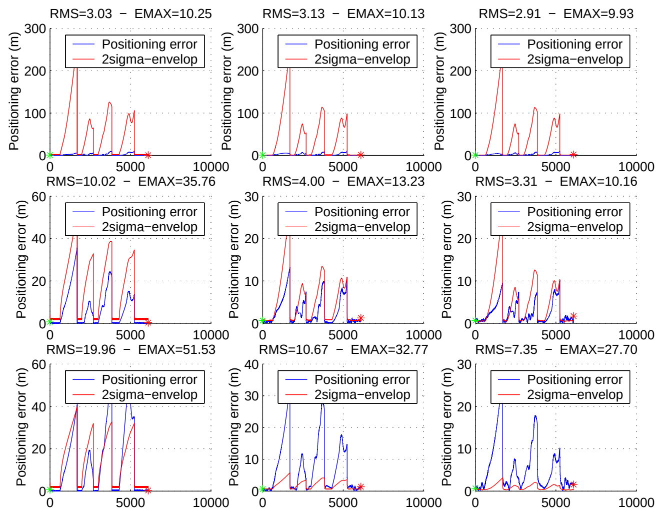

Figure 6: Tuning of the LKF (MEMS gyro).