Distance measures, weighted averages, OWA operators and Bonferroni means

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A B S T R A C T

The ordered weighted average (OWA) is an aggregation operator that provides a parameterized family of operators between the minimum and the maximum. This paper presents the OWA weighted average distance operator. The main advantage of this new approach is that it unifies the weighted Hamming distance and the OWA distance in the same formulation and considering the degree of importance that each concept has in the analysis. This operator includes a wide range of particular cases from the minimum to the maximum distance. Some further generalizations are also developed with generalized and quasi-arithmetic means. The use of Bonferroni means under this framework is also studied. The paper ends with an application of the new approach in a group decision making problem with Dempster-Shafer belief structure regarding the selection of strategies.

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1. Introduction

In the literature, there are a wide range of methods for the aggregation of the information [2,10] and decision making [43]. A very useful technique for doing this is the Hamming distance [11] and, more generally, all the distance measures [9,13]. The main advantage of using distance measures is that we can compare the alternatives of the problem with some ideal result [13]. Therefore, by doing this comparison, the alternative with closest results to the ideal is the optimal choice.

Usually, when dealing with distance measures in aggregation problems, we normalize it by using the arithmetic mean or the weighted average (WA) obtaining the normalized Hamming distance and the weighted Hamming distance, respectively. However, sometimes it would be interesting to consider other alternatives such as the parameterization of the results from the maximum distance to the minimum distance. Thus, it would be useful to use the ordered weighted averaging (OWA) operator [34]. The OWA operator is a very useful technique for aggregating the information providing a parameterized family of aggregation operators that includes the maximum, the minimum and the average. It has been studied by many authors [42,43].

The use of the OWA operator in different types of distance measures has been studied by many authors [14]. Xu and Chen [32] and Merigó and Gil-Lafuente [20] suggested the OWA distance (OWAD) operator with the objective of introducing a parameterized family of distance operators between the minimum and the maximum distance. Merigó and Casanovas [16,17] developed further extensions by using induced aggregation operators. They also considered the use of heavy aggregation operators [18]. Zeng and collaborators extended these approaches for uncertain environments that can be assessed with interval numbers [45], fuzzy information [47] and linguistic variables [46]. Xu also studied the use of fuzzy information [31]. Zhou et al. [51] presented a new approach that could consider continuous aggregations and distance measures in the same formulation. Merigó and Yager [24] studied the use of moving distance measures. Some other authors studied the use of Choquet integrals [5]. Moreover, note that it is also possible to use other related techniques [4] under this framework such as norms [40] and probabilities [23].

An important issue when dealing with the OWA operator is to consider the importance weights that may also appear in the aggregation process. In order to integrate this issue with the OWA operator, Torra [26] suggested the weighted OWA (WOWA) operator. Yager [37] developed another approach that he...