

# A new framework to analyze business cycle synchronization\*

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## Abstract

In this paper, we propose a new framework to analyze pairwise business cycle synchronization across a given set of countries. We show that our approach, that is based on multivariate Markov-switching procedures, leads to more reasonable results than other popular approaches developed in the literature. According to recent findings, we show that the G7 countries seem to exhibit two differentiated “Euro-zone” and “English-speaking” business cycles dynamics.

**Keywords:** Business cycle synchronization, economic integration, Markov-switching.

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# 1 Introduction

The analysis of business cycle synchronization across different countries has become a topic of increasing interest in both academic and policy circles. Terms such as “globalization” or “world integration” can be found everyday in the press, with all kinds of associated implications. What is clear is that developed economies have become more tightly integrated in recent years. In these countries, international trade flows have increased substantially and financial markets have become more homogeneous. Promoted by this international integration, growing attention is being devoted to examine whether the efforts to coordinate their economic policies lead to higher business cycle synchronization. In European context, the European Union is facing a major enlargement to include countries from Eastern Europe. Obviously, the synchronization of their national business cycles is a necessary condition for the success of the stabilizing role of monetary policy, that is left to a supranational authority, and fiscal policy, that is restricted to the achievement of close-to-balanced budget constraints imposed by the stability pacts. The theoretical argument behind this reasoning is that countries with strong linkages in terms of business cycle correlations and concordances, are expected to face smaller costs of joining the union than those countries with relatively less synchronized cycles.

Therefore, when measuring the effect of policy linkages the heart of the argument is the relative importance or even the existence of a common source of shocks that flows across the different economies. If this source exists and explains a big portion of the variance of the growth rates of the individual economies, policy linkages across them should not be costly. Otherwise, the cost of synchronizing economies may be higher than the gains associated with it. However, we think that although measuring comovements is extremely important for checking the consequences of relevant policy decisions, there is no consensus in the literature on how to implement this. Some of the most standard techniques proposed in the literature present drawbacks that we will try to solve in this paper.

On the one hand, one strand of the existing papers, assuming from the beginning that a common cycle exists, estimates it and calculates its importance in explaining country specific movements. Not testing for the existence of this estimated common cycle leaves the reader with the question, under the null hypothesis of nonexistence, what are these papers estimating? Recent examples are Gregory, Head and Raynauld (1997), Lumsdaine and Prasad (2003), and Canova, Ciccarelli and Ortega (2004).

On the other hand, other papers focus on measuring synchronicity attending to the degree of comovement among national measures of output without imposing any common cycle. In this

respect, den Haan (2000) proposes a measure of dynamic correlations in the time domain that is based on VAR forecast errors at different horizons, and Croux, Forni and Reichlin (2001) suggest a measure of comovement in the frequency domain that is based on the estimation of the spectra and cross-spectra of the time series.

In this same spirit of not imposing the common cycle, a final major strand of the literature on business cycle synchronization focuses its attention on pairwise comparison, between the countries' business cycle timing, usually by computing concordance indexes and correlations (see Harding and Pagan, 2003). When identifying the national business cycles, most of these approaches rely on univariate analyses of national series of production. Some of these works use Bry-Boschan based nonparametric algorithms, as the ones proposed by Artis, Kontolemis and Osborn (1997), and Harding and Pagan (2002). One significant example is the well known paper of Harding and Pagan (2003). They take as given the turning points identified in the univariate series of twelve national industrial productions by Artis et al. (1997) to compute the correlation between the corresponding business cycle indicators. Other papers, as Guha and Banerji (1998), and Bodman and Crosby (2002), generate business cycle chronologies following the univariate Markov-switching model proposed by Hamilton (1989). In this paper we show that these approaches, which rely on business cycle indicators obtained from individual series, may lead to misleading results since they are biased to show relatively low values of business cycle synchronization precisely for countries that exhibit synchronized cycles.

In this respect, this paper attempts to provide additional light on the analysis of business cycle synchronization. From a theoretical point of view, based on the analysis of Bengoechea, Camacho, and Perez-Quiros (2005), we propose a novel method to analyze business cycle synchronization across a group of countries that is based on the comparison of their Markov-switching unobserved variables that refer to their business cycle dynamics. As pointed out by Phillips (1991), regarding the analysis of pairwise business cycle synchronization, two extreme cases are presented in the literature; the case of complete independence (two independent Markov processes are hidden in the bivariate specification) and the case of perfect synchronization (only one Markov process for both variables). We think that in most of the real cases, the economies are somewhere in between, so we model the data generating process as a linear combination of these two extreme situations, where the parameters of the linear combination are estimated from the data. Using these parameter estimates, we measure the distance between each pair of countries as the distance from the full dependence case. In this respect, an interesting point of comparison is the work of Smith and Summers (2005), who use the multivariate Markov-switching model developed by Paap and van Dijk (2003) to analyze business cycle synchronization.

From an empirical point of view, it is worth noting that our proposal may lead to a matrix of distances between business cycles that may be used to develop an exhaustive analysis of their business cycle synchronization as, for example the one developed in a recent study by Camacho, Perez-Quiros and Saiz (2005). For this attempt, we employ the method proposed in this paper to evaluate the degree of business cycle synchronization among the G7 countries, finding results in the same spirit as the recent study of Stock and Watson (2003). We observe that the G7 business cycles, instead of exhibiting synchronized dynamics, are more likely to follow one pattern in the G7's "Euro-zone countries" and another in the G7's "English-speaking countries."

The paper is structured as follows. Section 2 proposes an appropriate framework to deal with the analysis of business cycle synchronization. Section 3 describes the data, characterizes the business cycle of our sample of countries, and examines the business cycle synchronization across the G7 economies. Section 4 concludes.

## 2 A framework to analyze business cycle synchronization

### 2.1 Univariate Markov-switching approach

One recognized empirical fact about the industrialized economies' dynamics is that, even though their series of output present upward trends, these trends do not seem to be monotonically increasing curves, but rather exhibit sequences of upturns and downturns that configure the traditional business cycles phases. During the periods that are usually known as recessions, the value of the output growth rates are usually lower (and sometimes negative) than during the periods of expansion. A natural approach to model this particular nonlinear dynamic behavior is the regime switching model proposed by Hamilton (1989). Following his seminal proposal, we assume that the switching mechanism of the  $k$ -th country's output growth at time  $t$ ,  $y_{k,t}$ , is controlled by an unobservable state variable,  $s_{k,t}$ , that is allowed to follow a first-order Markov chain. Thus, a simple switching model may be specified as

$$y_{k,t} - \mu_{s_{k,t}} = \phi_{k,1} \left( y_{k,t-1} - \mu_{k,s_{k,t-1}} \right) + \dots + \phi_{k,p} \left( y_{k,t-p} - \mu_{k,s_{k,t-p}} \right) + \varepsilon_{k,t} \quad (1)$$

$$= \phi_k(L) \left( y_{k,t-1} - \mu_{k,s_{k,t-1}} \right) + \varepsilon_{k,t}, \quad (2)$$

where  $\varepsilon_{k,t}$  is a Gaussian process with mean zero and variance  $\sigma_k^2$  which is identically and independently distributed over time, and where  $\phi_k(L)$  is a  $p$ -th order lag polynomial. Since  $s_{k,t}$  is assumed to evolve according to an irreducible 2-state Markov process, its transition probabilities are defined by

$$p(s_{k,t} = j | s_{k,t-1} = i, s_{k,t-2} = h, \dots, \chi_{k,t-1}) = p(s_{k,t} = j | s_{k,t-1} = i) = p_{k,ij}, \quad (3)$$

where  $i, j = 1, 2$ , and  $\chi_{k,t} = (y_{k,t}, y_{k,t-1}, \dots)'$ . It is convenient to collect the transition probabilities in a  $(2 \times 2)$  transition matrix

$$P_k = \begin{pmatrix} p_{k,11} & p_{k,21} \\ p_{k,12} & p_{k,22} \end{pmatrix}, \quad (4)$$

whose columns sum to unity. Within this framework, we can label  $s_{k,t} = 1$  and  $s_{k,t} = 2$  as the expansion and recession states in country  $k$  at time  $t$ , respectively. Hence, the average output growth rate of this country is given by  $\mu_{k,1}$  in expansions and by  $\mu_{k,2}$  in recessions, where  $\mu_{k,2} < \mu_{k,1}$ . This is the method used by Guha and Banerji (1998) to obtain the business cycle chronologies for each of the individual countries that they include in the sample.

## 2.2 Multivariate Markov-switching approach

The previous baseline specification may be readily extended to account for pairwise business cycle comparisons. Let us assume that we are interested in measuring the degree of business cycle synchronization between two countries  $a$  and  $b$ . In this case, their output growths are driven by two (possibly dependent) Markov-switching processes,  $s_{a,t}$  and  $s_{b,t}$ , with the same statistical properties as the previous latent variable. The multivariate state dependent model is given by

$$\begin{aligned} y_{a,t} - \mu_{s_{a,t}} &= \phi_{aa}(L) (y_{a,t-1} - \mu_{a,s_{a,t-1}}) + \phi_{ab}(L) (y_{b,t-1} - \mu_{b,s_{b,t-1}}) + \varepsilon_{a,t}, \\ y_{b,t} - \mu_{s_{b,t}} &= \phi_{ba}(L) (y_{a,t-1} - \mu_{a,s_{a,t-1}}) + \phi_{bb}(L) (y_{b,t-1} - \mu_{b,s_{b,t-1}}) + \varepsilon_{b,t}, \end{aligned} \quad (5)$$

where  $(\varepsilon_{a,t}, \varepsilon_{b,t})'$  is an identically and independently distributed bivariate Gaussian process with zero mean and covariance matrix  $\Omega_{ab}$ , and  $\phi_{ij}(L)$  are  $p$ -th order lag polynomials, with  $i, j = a, b$ .<sup>1</sup> To complete the dynamic specification of the process, one can define a new state variable  $s_{ab,t}$  that characterizes the regime for date  $t$  in a way consistent with the previous univariate specification, and whose basic states are

$$s_{ab,t} = \begin{cases} 1 & \text{if } s_{a,t} = 1 \text{ and } s_{b,t} = 1 \\ 2 & \text{if } s_{a,t} = 2 \text{ and } s_{b,t} = 1 \\ 3 & \text{if } s_{a,t} = 1 \text{ and } s_{b,t} = 2 \\ 4 & \text{if } s_{a,t} = 2 \text{ and } s_{b,t} = 2 \end{cases}. \quad (6)$$

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<sup>1</sup>Notice that, for simplicity, we are not considering the possibility of cointegration.

The four-state matrix of transition probabilities states may be defined in the multivariate specification as

$$P_{ab} = \begin{pmatrix} p_{ab,11} & p_{ab,21} & p_{ab,31} & p_{ab,41} \\ p_{ab,12} & p_{ab,22} & p_{ab,32} & p_{ab,42} \\ p_{ab,13} & p_{ab,23} & p_{ab,33} & p_{ab,43} \\ p_{ab,14} & p_{ab,24} & p_{ab,34} & p_{ab,44} \end{pmatrix}, \quad (7)$$

whose columns sum to unity.

As pointed out by Phillips (1991), this specification allows for two extreme kinds of interdependence between the individual countries business cycles. The first case characterizes countries for which their individual business cycle fluctuations are completely independent. The opposite case of perfect synchronization refers to the case in which both countries share the state of the business cycle, that is, their business cycles are generated by the same state variable, so  $s_{a,t} = s_{b,t}$ . In empirical applications, the national business cycles usually exhibit a different degree of synchronization that is located between these two extreme possibilities (in the sense of a weighted average). In particular, following Bengoechea et al. (2005), we consider that actual business cycle synchronization is  $\delta_{ab}$  times the case of independence and  $(1 - \delta_{ab})$  times the case of perfect dependence, where  $0 \leq \delta_{ab} \leq 1$ . The weights  $\delta_{ab}$  may be interpreted as measures of business cycle desynchronization since they evaluate the proximity of their business cycles to the case of complete independence. Hence, it follows that an intuitive measure of business cycle comovement is then  $1 - \delta_{ab}$ .

From now on, for simplicity in the exposition we assume that the lag polynomials in expression (5) are of order zero. Following the line of Hamilton (1994), the filter used to obtain the maximum likelihood estimates goes through the data one observation at a time. It takes the probabilities conditional on observations of  $y$  up to time  $t - 1$ , computes the likelihood function, and updates the probabilities according to the following two steps.

STEP 1: *Computing the likelihoods.* Collect the parameters to be estimated ( $\mu_{s_k,t}$ ,  $\Omega_{ab}$ ,  $\delta_{ab}$  and  $p_{k,ij}$ , with  $k = a, b$ ) in a vector  $\theta$ . Consider the joint density of  $y_t$  at any particular realization of their unobserved state variables, which is the product of the conditional densities and prediction probabilities. Letting  $\chi_{ab,t}$  be  $(y_{a,t}, y_{a,t-1}, \dots, y_{b,t}, y_{b,t-1}, \dots)$ , the univariate and bivariate joint densities are

$$f_a(y_{a,t}, s_{a,t} = j | \chi_{a,t-1}; \theta) = f_a(y_{a,t} | s_{a,t} = j, \chi_{a,t-1}; \theta) P(s_{a,t} = j | \chi_{a,t-1}; \theta), \quad (8)$$

$$f_b(y_{b,t}, s_{b,t} = j | \chi_{b,t-1}; \theta) = f_b(y_{b,t} | s_{b,t} = j, \chi_{b,t-1}; \theta) P(s_{b,t} = j | \chi_{b,t-1}; \theta), \quad (9)$$

$$f_{ab}(y_t, s_{ab,t} = j | \chi_{ab,t-1}; \theta) = f_{ab}(y_t | s_{ab,t} = j, \chi_{ab,t-1}; \theta) P(s_{ab,t} = j | \chi_{ab,t-1}; \theta), \quad (10)$$

respectively. The prediction probabilities  $P(s_{a,t} = j | \chi_{a,t-1}; \theta)$ ,  $P(s_{b,t} = j | \chi_{b,t-1}; \theta)$ , and  $P(s_{ab,t} = j | \chi_{ab,t-1}; \theta)$  are usually collected in the vectors  $\xi_{a,t|t-1}$ ,  $\xi_{b,t|t-1}$ , and  $\xi_{ab,t|t-1}$ , respectively.

Conditioned on past observations, the likelihoods  $f_a(y_{a,t} | \chi_{a,t-1}; \theta)$ ,  $f_b(y_{b,t} | \chi_{b,t-1}; \theta)$ , and  $f_{ab}(y_t | \chi_{ab,t-1}; \theta)$  are the sum of (8) to (10) over the possible states of the respective Markov processes. For example, in the bivariate case, the likelihood of  $y_t$  is

$$f_{ab}(y_t | \chi_{ab,t-1}; \theta) = \sum_{j=1}^4 f_{ab}(y_t, s_{ab,t} = j | \chi_{ab,t-1}; \theta). \quad (11)$$

STEP 2: *Updating the prediction probabilities.* If the joint distributions in (8) to (10) are divided by their respective densities, the results are the conditional distributions of the state variables:

$$P(s_{a,t} = j | \chi_{a,t}; \theta) = f_a(y_{a,t}, s_{a,t} = j | \chi_{a,t-1}; \theta) / f_a(y_{a,t} | \chi_{a,t-1}; \theta), \quad (12)$$

$$P(s_{b,t} = j | \chi_{b,t}; \theta) = f_b(y_{b,t}, s_{b,t} = j | \chi_{b,t-1}; \theta) / f_b(y_{b,t} | \chi_{b,t-1}; \theta), \quad (13)$$

$$P(s_{ab,t} = j | \chi_{ab,t}; \theta) = f_{ab}(y_t, s_{ab,t} = j | \chi_{ab,t-1}; \theta) / f_{ab}(y_t | \chi_{ab,t-1}; \theta), \quad (14)$$

that are collected in the vectors  $\xi_{a,t|t}$ ,  $\xi_{b,t|t}$ , and  $\xi_{ab,t|t}$ , respectively. Now, one can form forecasts of how likely the processes are in regime  $j$  in period  $t+1$  given observations up to date  $t$ . These forecasts, denoted by  $P(s_{k,t+1} = j | \chi_{k,t}; \theta)$ , with  $k = a, b$ , can be computed by using the matrices of transition probabilities for each country

$$\xi_{a,t+1|t} = P_a \xi_{a,t|t}, \quad (15)$$

$$\xi_{b,t+1|t} = P_b \xi_{b,t|t}. \quad (16)$$

From these individual forecasts, it is straightforward to deal with the bivariate forecasted probabilities in case of independent cycles

$$\xi_{ab,t+1|t}^I = \begin{pmatrix} P(s_{a,t+1} = 1 | \chi_{a,t}; \theta) P(s_{b,t+1} = 1 | \chi_{b,t}; \theta) \\ P(s_{a,t+1} = 2 | \chi_{a,t}; \theta) P(s_{b,t+1} = 1 | \chi_{b,t}; \theta) \\ P(s_{a,t+1} = 1 | \chi_{a,t}; \theta) P(s_{b,t+1} = 2 | \chi_{b,t}; \theta) \\ P(s_{a,t+1} = 2 | \chi_{a,t}; \theta) P(s_{b,t+1} = 2 | \chi_{b,t}; \theta) \end{pmatrix}, \quad (17)$$

and in case of perfect synchronization

$$\xi_{ab,t+1|t}^D = \begin{pmatrix} P(s_{a,t+1} = 1 | \chi_{a,t}; \theta) \\ 0 \\ 0 \\ P(s_{a,t+1} = 2 | \chi_{a,t}; \theta) \end{pmatrix}. \quad (18)$$

Finally, the probabilities to be used in computing the likelihood for the next period are

$$\xi_{ab,t+1|t} = \delta_{ab} \xi_{ab,t+1|t}^I + (1 - \delta_{ab}) \xi_{ab,t+1|t}^D. \quad (19)$$

The log likelihood function can also be calculated by adding each of the log likelihoods over the  $T$  observations

$$L(\theta) = \sum_{t=1}^T f_{ab}(y_t | \chi_{k,t-1}; \theta). \quad (20)$$

In the numerical optimization procedures used in this paper, we consider additional restrictions such as  $0 \leq p_{ij} \leq 1$ , and  $0 \leq \delta_{ab} \leq 1$ .

### 3 Empirical results

#### 3.1 Preliminary analysis of data

In this section, we consider an application to real data that illustrates the aforementioned procedures. For this attempt, we use the quarterly Gross Domestic Products (GDP) for the G7 countries, Canada, France, Germany, Italy, Japan, UK, and US., covering 1980.2 – 2004.2.

Figure 1 depicts the particular dynamics of the logarithms of these output series. Clearly, these variables present an upward trend but, this trend does not seem to be a smooth curve but rather a sequence of upturns and downturns. However, apart from the US, for which the NBER Business Cycle Dating Committee has been dating expansions and recessions that have been generally recognized as the official US business cycle dates, there is no widely accepted reference chronology of the classical business cycle for other countries. To overcome this problem, we date the turning points by using the dating algorithm of Harding and Pagan (2002) that isolates the local minima and maxima in a quarterly series, subject to reasonable constraints on both the length and amplitude of expansions and contractions.<sup>2</sup> In Figure 1, we highlight in shaded areas the business cycle recessions obtained with the dating algorithm.<sup>3</sup> These areas clearly correspond to slowdowns in the series of output.

Figure 2 shows that the growth rates in the output series are relatively low in the recessionary periods. In this respect, Table 1 reveals that the overall average growth rates of these series are positive (second column), but they are higher in expansions (third column) and become negative during recessions (fourth column). In addition, the fifth row of this table confirms that the mean growth rates of these series of output are statistically lower in periods of recessions than in periods of expansions (the  $p$ -values of the null of no different means are always less than 0.001).

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<sup>2</sup>Broadly speaking, this procedure is a quarterly version of the well known Bry-Boschan method. These authors, in an attempt to stay close to the NBER when choosing the turning points, develop an algorithm to date the peaks and troughs of monthly time series.

<sup>3</sup>Note that the NBER recessions are fairly well identified by the Harding-Pagan dating algorithm.



Having detected empirical evidence in favor of the existence of business cycle turning points, we want to analyze the importance of this effect for the dynamics of the GDP series. In a recent paper, Camacho and Perez Quiros (2005) show evidence in the US case in favor of what they call the *jump-and-rest* effect of recessions. These authors show that the US output growth is characterized by a recurrent sequence of shifts between two steady states of high and low mean growth rates that mark the course of the business cycles, rather than by an autocorrelated time series path. They motivate the approach in their paper by regressing output growth on a constant, a dummy variable that takes on value one at the NBER business cycle recessions and the lagged value of output growth, showing that for different nonlinear models that control for the recession periods, the coefficients of the lagged output growth values are never significant. We repeat their approach in this paper but extend the sample to all the G-7 economies and substituting the NBER recession dummy with another dummy variable obtained using the Harding and Pagan (2002) algorithm. The sixth column of Table 1, that exhibits the  $p$ -value of tests for significance of the slope parameters, reveals that with the unique exception of Canada, once the sequence of business cycles is accounted for in the dynamic specification of output growth, the autoregressive coefficients become negligible and statistically insignificant. We show in the last column of this table that the specifications that do not include autoregressive parameters are dynamically complete, in the sense that there is nothing to be gained by adding any lags of output growth, because the residuals of the simple model are white noise. In particular, with the exception of Canada, we obtain  $p$ -values of the Ljung and Box (1978) tests of no autocorrelated residuals that are above any reasonable significance level.

Summing up all of these results, we find that, perhaps with the unique exception of Canada, the output growth variables in the G7 countries display dynamics as simple as series that switch back and forth between two fixed equilibria that correspond to the business cycles phases. The absence of autoregressive parameters minimizes the mathematical complexity and the computational cost of the simulation exercise below.

### 3.2 Comparative analysis of business cycle synchronization

From a theoretical point of view, our proposal is close to those analyses of business cycle synchronization that rely on pairwise comparisons of the underlying unobserved business cycle dynamics. The most popular approaches in the literature are those that base inference about business cycle timing either on nonparametric decision rules, as in Artis et al. (1997), or on Markov-switching models, as in Guha and Banerji (1998). However, in both cases the identification of business cycle patterns is made at the individual level. Accordingly, the univariate methods may work relatively

well when analyzing business cycle synchronization of countries with independent cycles, but they are expected to be ineffective when examining the business cycle comovement of countries with dependent cycles. This is not the case for our multivariate proposal that is expected to be useful in both scenarios.

In this section, we perform a simulation exercise that allows us to analyze the potential gains of our multivariate proposal compared to the alternative univariate approaches. First, we estimate the within recessions mean, within expansions mean, and the variance of output growth, along with the probabilities of being in recession, in expansion, and of switching the business cycle phase in the US economy.<sup>4</sup> Second, using these estimates as reference, we simulate 100 pairs of output growth series for hypothetical countries that share the business cycle and other 100 pairs of output growth series for hypothetical countries whose business cycles are imposed to be independent, where in both cases we use sample sizes that correspond to those observed in our empirical analysis. Third, in each of these two scenarios, we calculate the countries' reference business cycle dummies,  $D_{it}$ , using both the Harding and Pagan (2002) dating procedure and the univariate Markov-switching model discussed in Guha and Banerji (1998). Finally, for these univariate approaches we compute the correlation among their reference business cycle dummies as outlined by Harding and Pagan (2003). In addition, we compute our measure of business cycle distance.

In order to provide a visual analysis of our simulated results, Figure 3 plots the kernel density estimates of the measures of business cycle comovement obtained as in Harding-Pagan (straight line), Guha-Banerji (dashed line) and the approach stated in the previous section (dotted line). When the pairs of time series are generated under the assumption of business cycle independence, the kernel densities of business cycle comovement are tightly centered about zero. In fact, as Table 2 points out, the three measures business cycle comovement exhibit statistically insignificant means ( $p$ -values of 0.55, 0.63, and 0.22, respectively). Hence, these three approaches seem to work well when analyzing comovement of countries with highly desynchronized business cycles. However, when we generate pairs of time series that share the business cycle, our proposal is the only one that leads to measures of comovement close to (and, as shown in Table 2, statistically equal to) one. In our simulation, the first two measures of business cycle synchronization present mean values of 0.57 and 0.51, that are in both cases statistically different from one. This experiment allows us to illustrate that measures that are based on individual identification of the underlying business cycles, even though they work well for analyzing countries with independent cycles, they are biased to show relatively low values of business cycle synchronization precisely for countries that exhibit

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<sup>4</sup>We select the US economy as benchmark since this is the only country for which we have a business cycle reference that is generally accepted, the one proposed by the NBER.

synchronized cycles. This bias is not present in the estimation method that we propose.

### 3.3 Business cycle synchronization across G7 countries

In Table 3 we present the empirical values of our measure of business cycle dissimilarity computed for each pair of countries that belong to the G7, that is  $1 - \delta_{ab}$ , with  $a, b = 1, \dots, 7$ .<sup>5</sup> The result from the previous analysis is a collection of business cycle distances across countries. Country by country, we conclude that the closest country to France and Germany is Italy, and that the closest one to Italy is Germany. In addition, this table shows that the closest country to Canada and UK is US, and that the closest one to US is Canada. Finally, Japan seems to be far away from any of the G7 countries. This finding is closely related to the recent study of Stock and Watson (2003), who detect that the G7 business cycles, instead of exhibiting synchronized dynamics, are more likely to follow one pattern in the G7's "Euro-zone countries" and another in the G7's "English-speaking countries."

Apart from the country by country analysis, it is worthwhile to examine the interdependencies that may arise between these countries business cycles. However, difficulties to reach some intuitive conclusions dramatically grow with the number of countries included in the sample. One intuitive technique to summarize the information of the matrices of distances is *multidimensional scaling*.<sup>6</sup> This approach seeks to find a low dimensional coordinate system to represent  $n$ -dimensional objects and create a map of lower dimension ( $k$ ) which gives approximate distances among objects. The  $k$ -dimensional coordinates of the projection of any two objects,  $r$  and  $s$ , are computed by minimizing a measure of the squared sum of divergences between the true distances ( $d_{r,s}$ ) and the approximate distances ( $\hat{d}_{r,s}$ ) among these objects. That is,

$$\min_{\hat{d}_{r,s}} \frac{\sum_{r,s} (d_{r,s} - \hat{d}_{r,s})^2}{\sum_{r,s} d_{r,s}^2}, \quad (21)$$

with

$$\hat{d}_{r,s} = (\|z_r - z_s\|^2)^{1/2} = \left[ \sum_{i=1}^k (z_{ri} - z_{si})^2 \right]^{1/2}, \quad (22)$$

where  $z_r$  and  $z_s$  are the  $k$ -dimensional projection of the objects  $r$  and  $s$ , and  $z_{ri}$  and  $z_{si}$  are the  $k$  dimensions of each object. In the case of 2-dimensional representations, the resulting picture is much easier to interpret than distances in higher dimensional spaces because it allows plotting the distances in a plane. In the resulting map, countries with higher dissimilarities have representations in the plane which are far away from each other.

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<sup>5</sup>Just for comparison purposes, we additionally include measures of the business cycle synchronization indexes that are obtained from Harding-Pagan and Guha-Banerji approaches.

<sup>6</sup>We refer the reader to Timm (2002) for details on multidimensional scaling.

Figure 4 represents the multidimensional scaling map of the G7 business cycle distances that takes into account the possible interdependencies among their cycles. This map shows that, according to the results of Stock and Watson (2003), the Euro and the anglosaxon countries form two groups of countries, with synchronized business cycles among these groups, but desynchronized cycles with respect to each group. Again, it seems that Japan exhibit a very particular timing in its business cycle.

## 4 Conclusions

In this paper we provide a new framework to analyze business cycle synchronization. We use the statistical approach proposed by Bengoechea, Camacho and Perez-Quiros (2005), based on multivariate Markov-switching procedures. In a simulation exercise, we show that while the univariate approaches proposed in the literature work relatively well to analyze synchronization of countries with independent cycles, they are no longer appropriate when our sample includes countries with highly synchronized cycles. In this case, univariate approaches are biased to generate excessively low measures of business cycle synchronization. By contrast, we show that our proposal may be used in both scenarios.

In our empirical exercise, we analyze to what extent the G7 countries exhibit synchronized cycles. As the recent findings of Stock and Watson (2003), we find that the Euro and the anglosaxon countries form two groups of countries with synchronized business cycles among each group but desynchronized cycles with respect to each other. However, Japan does not seem to exhibit business cycle synchronization with any of these groups.

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Table 1. Preliminary data analysis

country	means within the phases of the cycle				JAR	LB
	mean	mean-exp	mean-rec	<i>p</i> -val	<i>p</i> -val	<i>p</i> -val
France	0.49	0.57	-0.32	0.00	0.100	0.227
Germany	0.42	0.66	-0.28	0.00	0.618	0.844
Canada	0.68	0.88	-0.72	0.00	0.002	0.002
Italy	0.44	0.62	-0.30	0.00	0.186	0.309
Japan	0.62	0.80	-0.61	0.00	0.461	0.970
UK	0.59	0.73	-0.72	0.00	0.338	0.593
US	0.75	0.88	-0.96	0.00	0.132	0.111

Notes. The business cycle phases have been identified by using the Harding and Pagan (2002) algorithm. The fifth column refers to the *p*-value of standard tests of the null of equal means. The sixth column shows the *p*-values of the non-significance tests of the autoregressive parameters in the regression of output growth on a constant, a dummy variable that equals one during recessions and on lagged output growth. Hence, it tests the Jump-and-rest (JAR) effect of recessions. The last column exhibits the *p*-values of the Ljung-Box (LB) test of no autocorrelated residuals in the regression of output growth on a constant and a business cycle dummy.

Table 2. Comparative analysis in simulated time series

	Independent		Dependent	
	Mean	$p$ -value $H_0 : \text{mean} = 0$	Mean	$p$ -value $H_0 : \text{mean} = 1$
Harding-Pagan	-0.0097	0.5586	0.5776	0.0000
Guha-Banerji	0.0065	0.6336	0.5179	0.0000
Our approach	0.2627	0.2220	0.9226	0.4863

Notes. Business cycle correlation from 100 simulations of pairs of output growth time series for hypothetical countries with independent cycles (first two columns) and completely dependent cycles (last two columns).

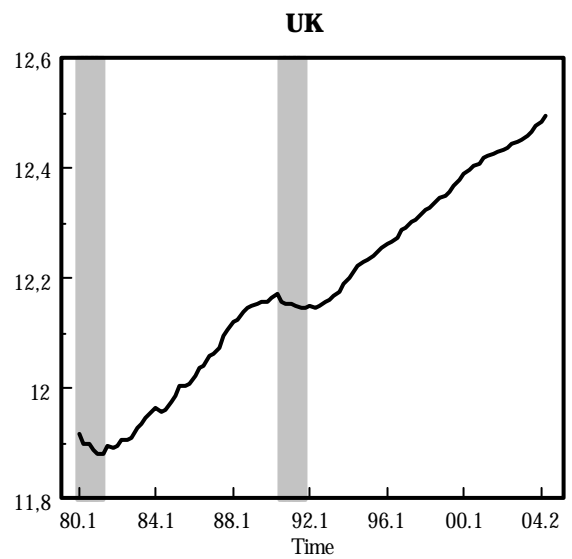
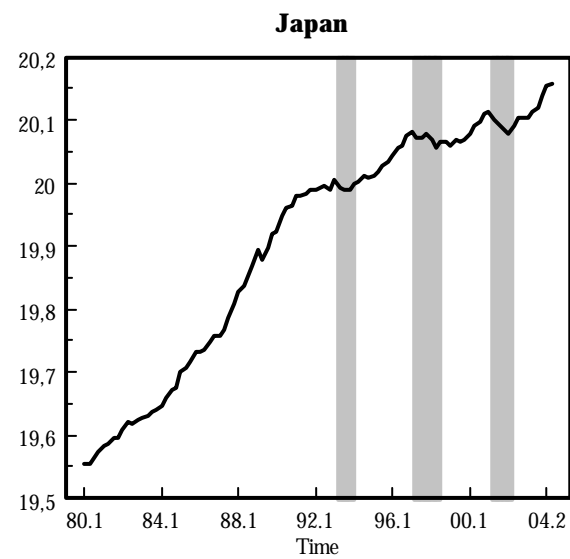
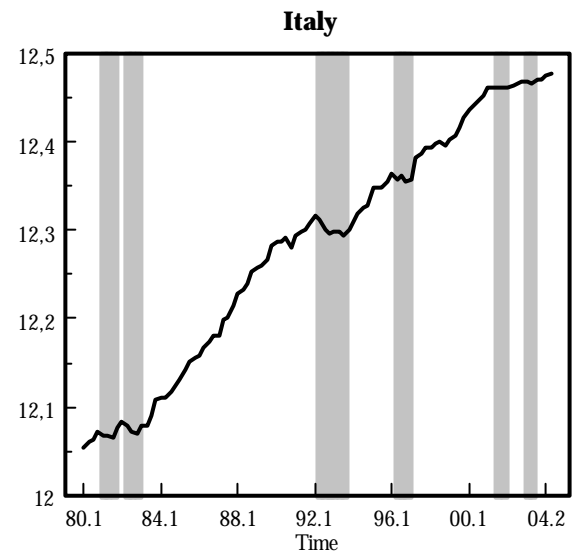
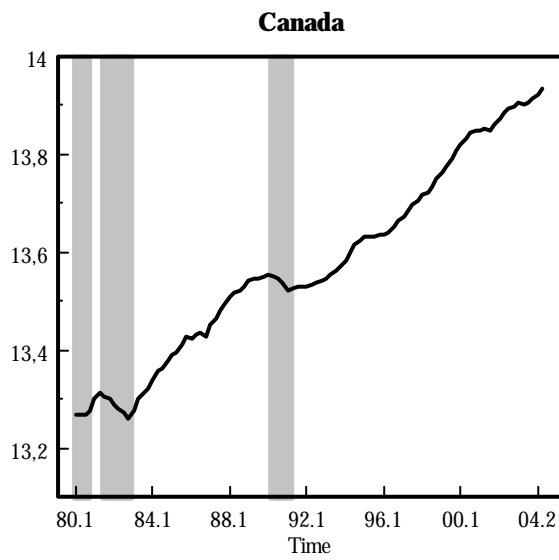
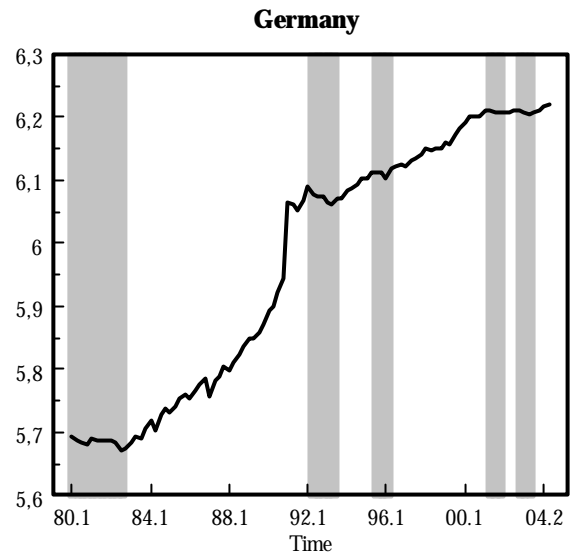
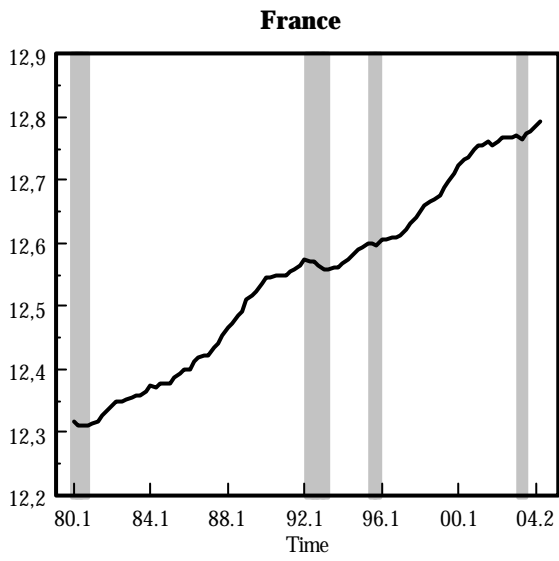


Table 3. Measures of distances

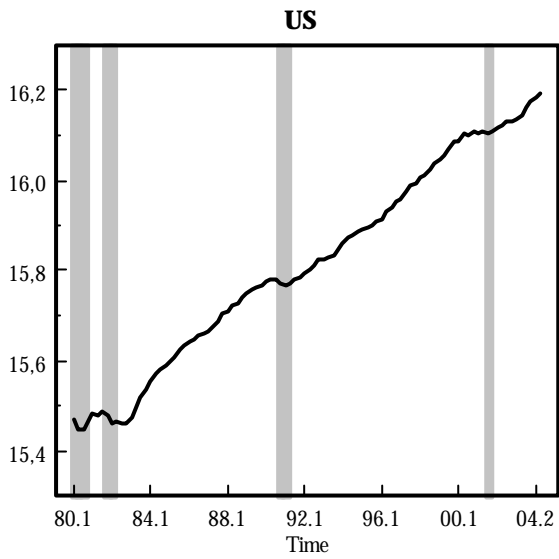
	France	Germany	Canada	Italy	Japan	UK	US
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Harding-Pagan proposal							
France	0.00	0.42	0.83	0.74	1.13	0.78	0.73
Germany	0.42	0.00	0.68	0.46	0.93	0.81	0.66
Canada	0.83	0.68	0.00	0.89	1.15	0.54	0.35
Italy	0.74	0.46	0.89	0.00	0.87	1.08	1.00
Japan	1.13	0.93	1.15	0.87	0.00	1.13	1.00
UK	0.78	0.81	0.54	1.08	1.13	0.00	0.48
US	0.73	0.66	0.35	1.00	1.00	0.48	0.00
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Guha-Banerji proposal							
France	0.00	0.75	1.07	0.75	0.92	0.75	1.01
Germany	0.75	0.00	1.18	0.92	0.54	1.25	1.16
Canada	1.07	1.18	0.00	0.79	1.10	0.79	0.39
Italy	0.75	0.92	0.79	0.00	0.88	1.07	0.95
Japan	0.92	0.54	1.10	0.88	0.00	1.30	1.06
UK	0.75	1.25	0.79	1.07	1.30	0.00	0.68
US	1.01	1.16	0.39	0.95	1.06	0.68	0.00
<hr/>							
Our proposal							
France	0.00	0.55	0.70	0.21	0.91	0.64	0.83
Germany	0.55	0.00	0.71	0.06	0.17	0.85	0.62
Canada	0.70	0.71	0.00	0.64	0.91	0.77	0.03
Italy	0.21	0.06	0.64	0.00	0.47	0.77	0.51
Japan	0.91	0.17	0.91	0.47	0.00	0.92	0.91
UK	0.64	0.85	0.77	0.77	0.92	0.00	0.60
US	0.83	0.62	0.03	0.51	0.91	0.60	0.00

Notes. In Harding-Pagan and Guha-Banerji approaches, the distances are computed as one minus correlation (hence, some entries may be greater than one). In our proposal, these entries refer to the maximum likelihood estimates of parameter  $\delta_{ab}$  (see (9)). This measures the proximity of the business cycles of countries  $a$  and  $b$  to the case of completely independent cycles.

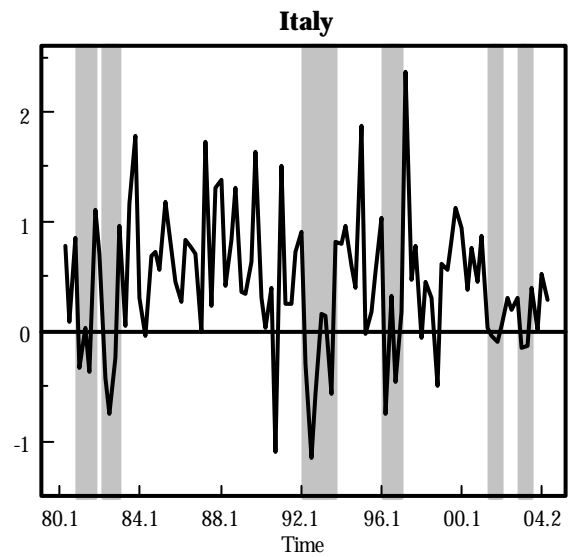
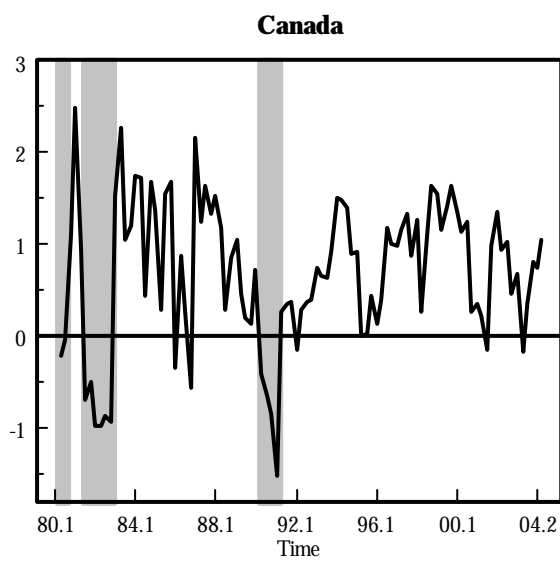
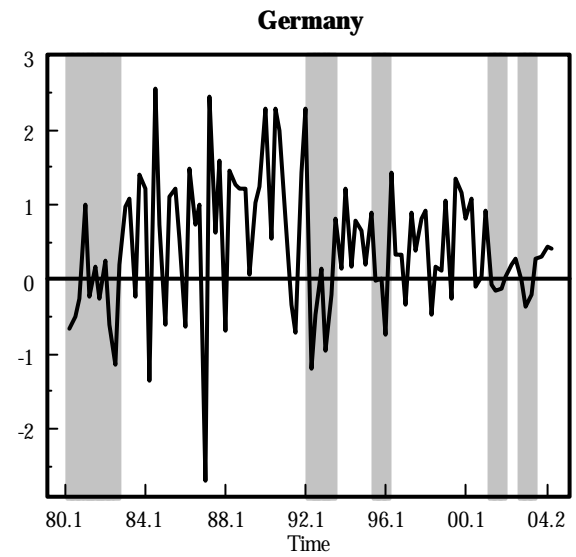
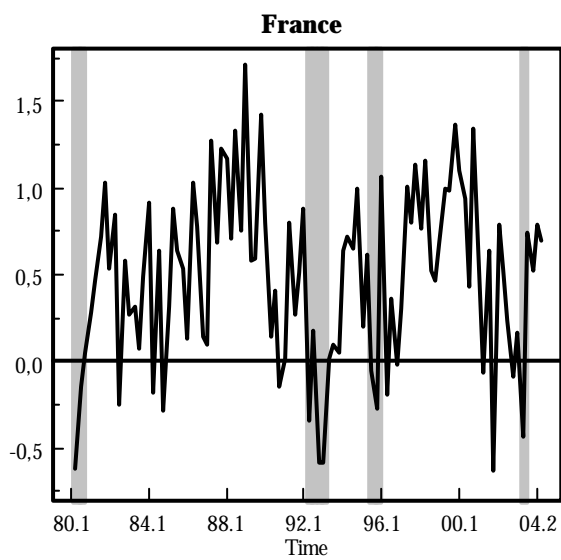
**Figure 1: Logs of GDP 1980.1-2004.2**



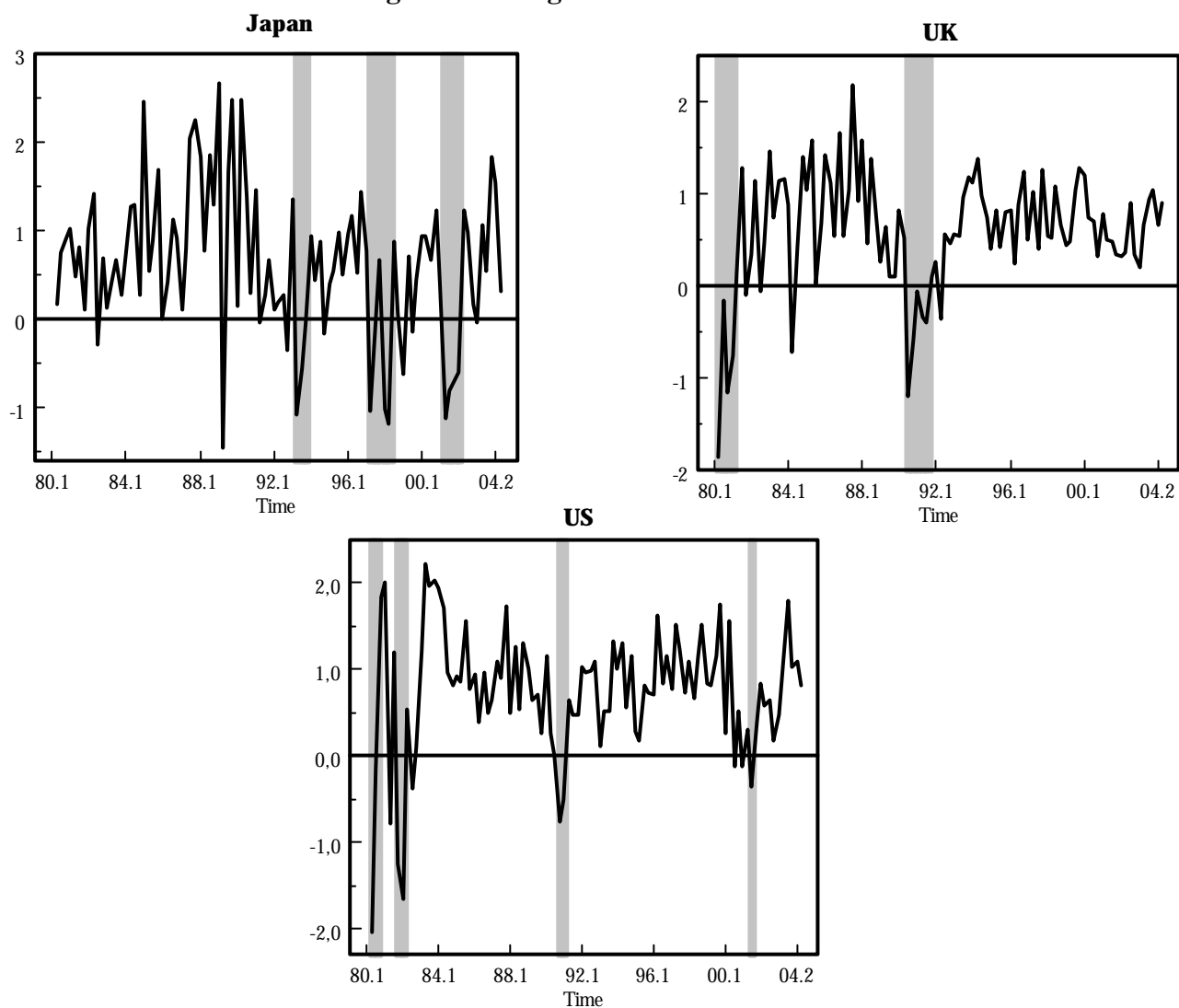
**Figure 1: Logs of GDP 1980.1-2004.2**



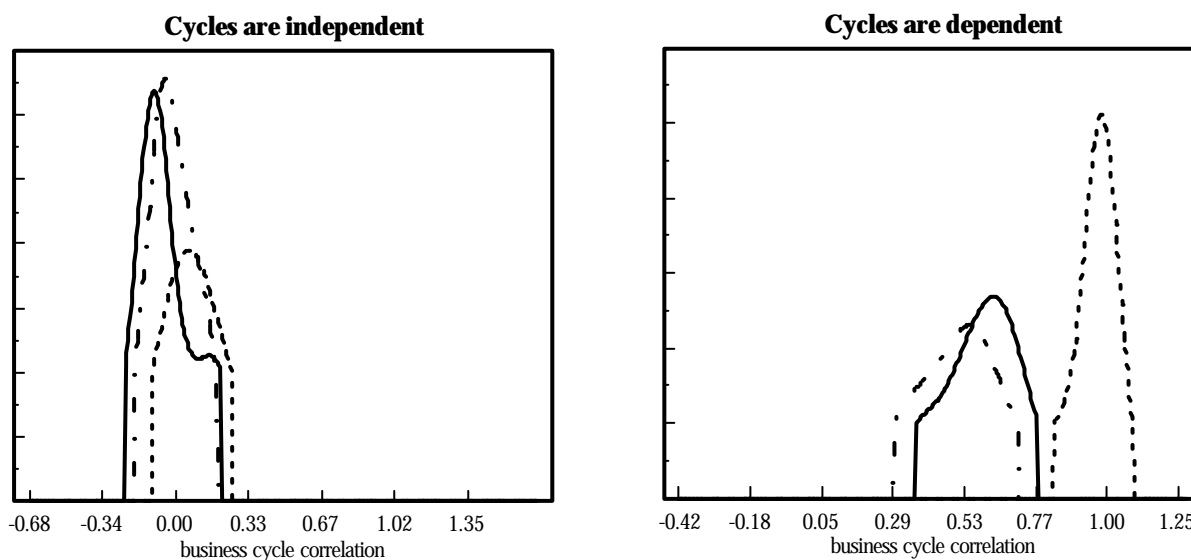
**Figure 2: GDP growth rates 1980.2-2004.2**



**Figure 2: GDP growth rates 1980.2-2004.2**

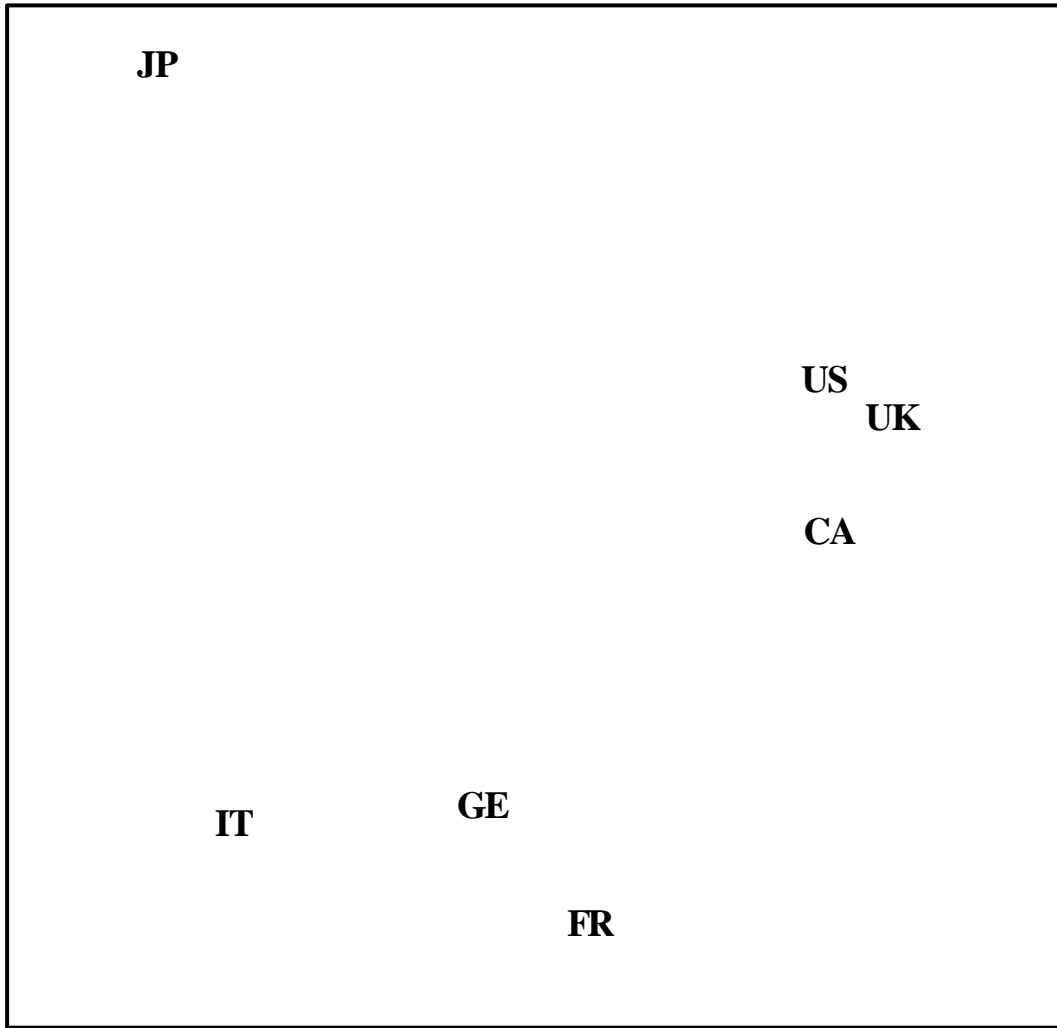


**Figure 3: Kernel densities of simulated correlations**



Notes. Kernel density estimates of simulated business cycles correlations under independent and dependent cycles. Straight, dashed, and dotted lines refer to correlations measured as in Harding-Pagan, Guha-Banerji, and our proposal, respectively

**Figure 4: Multidimensional scaling map**



Notes. Multidimensional scaling map of business cycle distances