TAR panel unit root tests and real convergence: an application to the EU enlargement process*

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Abstract

We propose a new panel data methodology to test real convergence in a non-linear framework. It extends the existing methods by combining three approaches: the threshold model, the panel data unit root tests and the computation of critical values by bootstrap simulation. We apply our methodology on the per-capita outputs of a total of fifteen European countries, including some of the East-European countries that have recently joined the EU.

Keywords: real convergence, panel data unit root tests, bootstrap, threshold model, non-linearity.

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1 Introduction

The construction of an economic union is based on the idea that an economic and political union is a guarantee for more growth and welfare for its members taken as a whole. However, it is not clear that there exists an automatic mechanism that redistributes these benefits among all the members in a fair way. According to Solow (1956) and subsequent related literature, economic integration will automatically promote economic convergence under free factor mobility and international diffusion of technological knowledge. However, some other authors postulate that economic integration increases geographical disparities, because the production factors will be concentrated in the more developed regions as a result of increasing returns to scale and externalities. Significant examples are the agglomeration theory of Krugman (1990) and the endogenous growth models of Romer (1986 and 1990). Therefore, discriminating between these two theoretical alternatives is an important issue for policy makers at the national and international level.

To answer this question, we need efficient statistical and econometric tools. There already exists a vast theoretical and empirical literature concerning statistical and econometric techniques for the analysis of output convergence. This type of convergence has been tested with different sorts of samples, and different econometric procedures. A well-known and somewhat pioneering methodology is the so-called "β-convergence" regression, associated to the works of Barro and Sala-i-Martín (1992). It consists of a cross-sectional regression of the average growth rate of per-capita output over some long enough time-period on a constant, on the level of per-capita output at the beginning of the period and, if necessary, on a set of country-specific additional variables. For convergence, the coefficient of the output level at the beginning of the period should be negative. However, as reviewed by Islam (2003), this cross-section approach has received many criticisms. Among the critics, Evans (1998) and Evans and Karras (1996) show that the β-convergence regression in general provides invalid inference and conclusions and argue in favor of a panel data approach. The basic idea in this approach consists of starting from conventional linear panel-data unit root tests and adapt (and extend) them to the analysis of output convergence. It can indeed be shown that divergence takes place if the per capita GDP series have a unit root, whereas convergence requires the absence of such a unit root.

In this paper, we propose a non-linear extension of Evans-Karras approach. The rational is based on the belief that the convergence process is not uniform, in the following sense. In the first place, it could be that the countries may only converge if certain institutional, political or economic conditions are fulfilled, whereas they may diverge otherwise. Another possibility is that convergence takes place at one rate in certain conditions and at another rate under other conditions. If this is true, it is straightforward to understand that a linear panel data model may lead to misleading results and that a two-regime model is required. In order to provide an appropriate framework to deal with these cases, we use here a panel data Threshold Autoregressive (TAR) specification in which the series of the panel may exhibit unit roots. In the recent literature of TAR models, Tsay (1998) considered a multivariate TAR specification but excluded the possibility of a unit root. On the other hand, Caner and Hansen (2001) analyzed how to test for a unit root in TAR models but they restricted their analysis to univariate time series. Our approach tries to fill this gap in the literature: we develop unit root tests in a multivariate TAR model and show how to apply these results to test real convergence. Therefore, our paper not only may be considered as an extension of the convergence literature, but also of the nonlinear TAR literature.

From a theoretical point of view, we contribute to the literature by proposing an estimation procedure for this new TAR model. This is based on a combination of grid-search procedures and feasible Generalized Least Squares methods. In addition, we propose several new testing procedures. The first family of tests are aimed at checking non-linearity. It is inspired in the Likelihood Ratio principle and solves the problem of unidentified nuisance parameters under the null hypothesis by computing bootstrap probability values. Two sets of bootstrap p-values are proposed. The first ones are obtained under the hypothesis of no unit root, whereas the second one covers the case of the opposite hypothesis. The second family of testing procedures are designed for the convergence analysis, once the TAR model has been confirmed to be better than the linear one. To discriminate between convergence and divergence, we propose specific tests, which are TAR extensions of the recent literature on panel data linear unit roots tests advocated by Chang (2004).

These extensions take into account that the countries might converge under one regime and diverge under the other. We also develop tests to discriminate between absolute and conditional convergence. Again, these tests include the possibility of different convergence patterns under each regime. In both cases, the probability values are obtained by bootstrap in order to tackle the problem caused by the cross-country contemporaneous correlations of the data.

On the empirical side, we apply the proposed techniques to check for real convergence in different sets of European Union (EU) countries by using their GDP per capita series for the sample $1950 - 2004$. The first group of countries refer to nine rich EU countries (Austria, Belgium, France, Italy, the Netherlands, Sweden, and the United Kingdom) for which we find that the degree of convergence depends on two distinct regimes. Next, in order to analyze the effect on the convergence of adding new members to the existing EU, we include in the sample three poorer countries that joined the union during the eighties: Spain, Portugal and Greece. In this case, we show that the convergence appears only in one regime. Finally, we examine the effect on the convergence of the last EU enlargement. To this end, we include in the analysis the three countries for which data were available: Hungary, Poland and Czechoslovakia. In this case, we fail to detect convergence in any of the two regimes.

The paper is structured as follows. Section 2 reviews the linear framework for the convergence analysis with panel data. Section 3 describes our methodology in a non-linear framework. Section 4 describes the application of this methodology to the analysis of the convergence processes which might have taken place within the EU. Section 6 concludes.

2 Review of the linear framework

2.1 The basic Evans-Karras procedure

Evans and Karras (1996) use the following specification in order to test real convergence with panel data:

$$\Delta g_{n,t} = \delta_n + \rho_n g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i} \Delta g_{n,t-i} + \varepsilon_{n,t}, \tag{1}$$

with $n = 1, \cdots, N$, and $t = 1, \cdots, T$. In this model, the subscript n refers to unit n (typically country n), whereas the subscript t refers to the time period. The variable $g_{n,t}$ is defined as

$$g_{n,t} = y_{n,t} - \bar{y}_t, \tag{2}$$

where $y_{n,t} = \log(Y_{n,t})$, $Y_{n,t}$ is the per-capita income of country n in real terms and $\bar{y}_t = \frac{1}{N}\sum_{n=1}^{N} y_{n,t}$ is the cross-country average log of per capita income at time t.

They show that $\rho_n = 0$ implies that the N countries diverge, whereas $0 < -\rho_n < 1$ for all n is a convergence condition.[1] The convergence is absolute if $\delta_n = 0$ for all n whereas it is conditional if not.

The testing procedure proposed by Evans and Karras (1996) is as follows. First, apply Ordinary Least Squares (OLS) to (1) in order to obtain an estimate of the standard deviation of ε_n, say s_n, and use it to transform the data to $w_{n,t} = g_{n,t}/s_n$. Second, obtain the OLS estimate of ρ and its t-ratio applying OLS to the equation

$$\Delta w_{n,t} = \delta_n + \rho w_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i} \Delta w_{n,t-i} + \varepsilon_{n,t}. \tag{3}$$

Third, if this t-ratio is sufficiently negative, reject the null in the test

$$\rho_n = 0 \ \forall n \text{ against } \rho_n < 0 \ \forall n. \tag{4}$$

Under the alternative, the economies converge. Otherwise, they diverge. Note that this amounts to testing that the series of the panel exhibit a unit root. Finally, if divergence is rejected in the third step, test the null that

$$\delta_n = 0 \ \forall n \text{ against } \delta_n \neq 0, \tag{5}$$

for some n in equation (1); for that purpose, estimate this equation for $n = 1, \ldots N$; compute $\Phi = \frac{1}{N-1} \sum\limits_{n=1}^{N} \left(t^2_{\hat{\delta}_n} \right)$, and reject the null if Φ is too large, in which case convergence would be conditional. Otherwise, convergence is absolute.

Evans and Karras (1996) demonstrate that, under the assumption that the errors in (1) are contemporaneously uncorrelated, the tests for hypothesis (4) and (5) have standard

[1] It can be shown (Beyaert, 2005) that the divergence of one single country in the panel implies that the $g_{n,t}$ will be I(1) for all n.

asymptotic distributions, when N and T tend to ∞. The authors suggest improvements on these asymptotic values by obtaining critical values via simulations from Normal independent distributions.

In our opinion, Evans-Karras approach presents two types of limitations. On the one hand, when the number of economies is moderate to low, the crucial Evans-Karras assumption of cross-sectional independence is difficult to sustain. On the other hand, the linear formulation considered in (1) may be unrealistic when some countries of the panel have experienced important institutional and economic modifications over the sample period. Obviously, these two limitations are not mutually exclusive.

Therefore, we consider in this paper two simultaneous extensions of this approach. The first one consists of relaxing the assumption of cross-sectional independence. This extension takes into account the results of Chang (2004), who recommends the use of bootstrap critical values in panel-data unit roots tests under cross-sectional dependence. It has already been applied in Beyaert (2005). Since it is used in the empirical application and it is helpful to better understand the second extension, it is briefly described in Section 2.2. The second extension adds to the first one the possibility that the dynamics of the convergence process is not uniform over time, but rather varies according to the economic or institutional circumstances of the countries. It is based on a panel-data TAR model, it is described in Section 3 and it constitutes the main contribution of this paper.

2.2 A bootstrap version of Evans-Karras procedure

In model (1), p is supposed to be high enough so that $\varepsilon_{n,t}$ is a white noise process for each n. So serial correlation in the errors is excluded. However cross-country contemporaneous correlation is not. Economically speaking, although shocks are serially uncorrelated, it is likely that convergent countries are affected by the same types of shocks. Therefore, if we define $\varepsilon_n = [\varepsilon_{n,1}, \cdots, \varepsilon_{n,T}]'$ and $\varepsilon = [\varepsilon_1', \varepsilon_2', \cdots, \varepsilon_N']'$, the variance-covariance matrix of ε is not diagonal and is likely to satisfy

$$V = \Omega \otimes I_T, \tag{6}$$

where

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_{NN} \end{bmatrix}, \tag{7}$$

with $\sigma_{nm} = cov(\varepsilon_{n,t}, \varepsilon_{m,t})$ for all n, m.

This has to be taken into account both in the estimation process and in the testing strategy. As far as estimation is concerned, a Feasible Generalized Least Squares (FGLS) estimation procedure should be applied. Let $G_n = [g_{n,1}, \cdots, g_{n,T}]'$ for $n = 1, \cdots, N$ and $G = [G_1', \cdots, G_N']'$; similarly, $\Delta G = [\Delta G_1', \cdots, \Delta G_N']'$ where ΔG_n is the first difference of G_n. Define also a column-vector of parameter β in which the parameters of each country are stacked by type in a column:

$$\beta = (\delta_1, \cdots, \delta_N; \rho_1, \cdots, \rho_N; \varphi_{1,1}, \cdots, \varphi_{N,1}; \cdots; \varphi_{1,p}, \cdots, \varphi_{N,p})'. \tag{8}$$

Then, we can express (1) compactly in Seemingly Unrelated Regression Estimation (SURE) form as,

$$\Delta G = X\beta + \varepsilon, \tag{9}$$

where $X = (\breve{\imath}, \breve{G}_{-1}, \Delta\breve{G}_{-1}, \cdots, \Delta\breve{G}_{-p})$. The components of X are as follows:

$$\breve{\imath} = \begin{bmatrix} \bar{1}_T & & 0 \\ & \ddots & \\ 0 & & \bar{1}_T \end{bmatrix},$$

with $\bar{1}_T = [1, \cdots, 1]'_{T\times 1}$,

$$\breve{G}_{-1} = \begin{bmatrix} G_{1,-1} & & 0 \\ & \ddots & \\ 0 & & G_{N,-1} \end{bmatrix},$$

where $G_{n,-1}$ is G_n lagged one period; similarly

$$\Delta\breve{G}_{-i} = \begin{bmatrix} \Delta G_{1,-i} & & 0 \\ & \ddots & \\ 0 & & \Delta G_{N,-i} \end{bmatrix},$$

for $i = 1, \cdots, p$, where $\Delta G_{n,-i}$ is ΔG_n lagged i periods. Given the structure of V as specified in (6) and (7), an estimate of the components of Ω is required. For that purpose, model (9) can be estimated by OLS and an estimate $\hat{\Omega} = [s_{nm}]$ can be computed, with $s_{nm} = \frac{1}{T}\sum_{t=1}^{T} e_{nt}e_{mt}$ for $n, m = 1, \cdots, N$, where e_{lt} is the OLS residual of model (9) corresponding to observation t for country l. The FGLS estimator of β is then

$$\hat{\beta}_{FGLS} = \left[X'\hat{V}^{-1}X\right]^{-1} X'\hat{V}^{-1}\Delta G, \tag{10}$$

where $\hat{V} = \hat{\Omega} \otimes I_T$.

In order to test divergence against convergence as described by (4), model (9) is estimated by FGLS under the restriction that $\rho_n = \rho$ for all n, and the t-statistic associated with the estimation of this restricted coefficient is computed. The probability value is obtained by bootstrap to control for the structure of V. For that purpose, an FGLS estimate of model (1) is obtained under the additional restriction that $\rho = 0$ and the residuals $e^{\circ}_{n,t}$ for $n = 1, \cdots, N$ and $t = 1, \cdots, T$ are then recentered and arranged in the following matrix:[2]

$$E^{\circ} = \begin{bmatrix} e^{\circ}_{1,1} & e^{\circ}_{2,1} & \cdots & e^{\circ}_{N,1} \\ e^{\circ}_{1,2} & e^{\circ}_{2,2} & \cdots & e^{\circ}_{N,2} \\ \vdots & \vdots & \cdots & \vdots \\ e^{\circ}_{1,T} & e^{\circ}_{2,T} & \cdots & e^{\circ}_{N,T} \end{bmatrix}. \tag{11}$$

The rows of this matrix are then resampled with replacement, in order to obtain new time-series $\left\{e^{\circ *}_{n,t}, t = 1, \cdots, T\right\}$ of residuals for each n that preserve the initial contemporaneous correlation among the series (Maddala and Wu, 1999, Chang, 2004). Bootstrap data are then generated back from these resampled residuals using the FGLS coefficient estimates of model (1) under $\rho = 0$. This resampling and data generation process is repeated a very large number of times. In each replication, the value of the test statistic is computed in the same way as on the observed data. The bootstrap probability value is the percentage of bootsrapped t-statistics falling to the left of the observed t-statistic.

The bootstrapped version of test (5) is carried out in a similar way. Model (1) is esti-

[2] For each country n, the sample mean over time is substracted from the residuals to obtain zero-mean residuals.

mated in SURE form by unrestricted FGLS. The following test-statistic is then computed:

$$\Phi = \frac{1}{N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_{FGLS,n})\right]^2\right\}, \tag{12}$$

where $\hat{\delta}_{FGLS,n}$ is the FGLS estimate of δ_n in (9). Then, (9) is estimated under the restriction that $\delta_n = 0$ for all n, and the residuals are recentered and resampled by row. The bootstrap data are generated from these bootstrap residuals under this restriction and the corresponding bootstrap Φ statistics are computed. The bootstrap p-value for (12) is then obtained from the relative position of the observed Φ statistic in the empirical distribution of the bootstrapped Φ statistics.

3 Convergence analysis with TAR models

3.1 The non-linear model

Let us now suppose that the convergence process is not uniform, in the following sense. In the first place, it could be that the N countries converge only if certain institutional, political or economic conditions are fulfilled whereas they diverge otherwise. In this case, it may happen that $0 < -\rho_n < 1$ for all n under certain circumstances but that $\rho_n = 0$ if these circumstances are not met. Another possibility would be that convergence takes place at one rate in certain conditions and another rate under other conditions. That is, it may happen that $0 < -\rho_n < 1$ for all n but that its specific value differs according to the prevailing conditions at time t. A model that would be able to represent such a behavior can be specified as follows:

$$\begin{aligned}\Delta g_{n,t} = &\left[\delta_n^I + \rho_n^I g_{n,t-1} + \sum_{i=1}^{p}\varphi_{n,i}^I \Delta g_{n,t-i}\right] I_{\{z_{t-1}<\lambda\}} \\ &+ \left[\delta_n^{II} + \rho_n^{II} g_{n,t-1} + \sum_{i=1}^{p}\varphi_{n,i}^{II} \Delta g_{n,t-i}\right] I_{\{z_{t-1}\geq\lambda\}} + \varepsilon_{n,t},\end{aligned} \tag{13}$$

with $n = 1, \cdots, N$, and $t = 1, \cdots, T$. In this model, $I\{x\}$ is an indicator which takes value 1 when x is true, and zero otherwise. It therefore acts as a dummy variable which takes a unit value if the condition $z_{t-1} < \lambda$ is fulfilled. So when $z_{t-1} < \lambda$, the model is $\Delta g_{n,t} = \delta_n^I + \rho_n^I g_{n,t-1} + \sum_{i=1}^{p}\varphi_{n,i}^I \Delta g_{n,t-i} + \varepsilon_t$, whereas it is $\Delta g_{n,t} = \delta_n^I + \rho_n^I g_{n,t-1} +$

$\sum_{i=1}^{p} \varphi_{n,i}^{I} \Delta g_{n,t-i} + \varepsilon_t$ when $z_{t-1} \geq \lambda$. In other words, at any t, the dynamics of the per-capita incomes follows one of two possible regimes. We will call "regime I" the case where $z_{t-1} < \lambda$ and "regime II" the case where $z_{t-1} \geq \lambda$. The parameter λ is therefore a "threshold" parameter and equation (13) belongs to the class of threshold autoregressive (TAR) models first introduced by Tong (1978). Note that model (13) includes the linear model (1) as a particular case, which takes place when z_{t-1} stands on the same side of λ for all t. As is usual in this type of model, the threshold parameter is unknown. However, in order to carry out the estimation process, the restriction that $0 < \pi_1 \leq P(z_{t-1} \leq \lambda) \leq 1 - \pi_1$ is imposed so that no regime takes place in less than a π_1 fraction of the total sample, with π_1 typically around 0.10 or 0.15. If π_1 falls below this limit, the linear process is preferred.

With respect to the basic TAR model of Tong (1978), ours proposes extensions in two directions. The first extension consists of abandoning the single-equation time-series TAR model in favour of a multivariate panel-data model. The second extension refers to the possible non-stationarity of the data, in the form of a unit-root in the individual (country) series when $\rho_n = 0$. This second extension has been considered by Caner and Hansen (2001) although their model is limited to a single series, whereas we consider a panel of N time series.

Note that in model (13) divergence would take place if $\rho_n^I = \rho_n^{II} = 0$ for all n. Alternatively, global convergence would correspond to $0 < -\rho_n^i < 1$ for all n and $i = I, II$. Finally, there would be partial convergence if $0 < -\rho_n^i < 1$ but $\rho_n^j = 0$ for all n and $i \neq j$.

In (13), the so-called transition variable, z_t, can be either endogenous, when its values are directly obtained from the $g_{n,t}$ variables, or exogenous, when it refers to an economic variable different from any $g_{n,t}$. In the endogeneity case, it makes sense to choose $z_t = g_{m,t} - g_{m,t-d}$, for some m and some $0 < d \leq p$ (where m and d are not a priory fixed but rather determined endogenously). In this way, from the statistical point of view, z_t would be stationary, whether the economies converge ($g_{n,t} \sim I(0)$, for all n and all regimes) or not ($g_{n,t} \sim I(1)$, for one or both regimes). From the economic point of view, it amounts to saying that the shift from one regime to another is related to the growth rate of country j in the last d periods. Another possibility would be to choose z_t exogenously, on the basis of economic arguments. For instance, we might think that the intensity of the convergence process among the countries in the panel varies as a function of their degree of openness towards each other. Let op_t be some measure of the intensity of the international trade relations linking the countries of the panel. Then $z_t = op_{t-d}$ (d could be determined endogenously from the data or fixed exogenously). However, in this paper and in what follows we focus on the endogeneity case.

Note also that (13) assumes that all the parameters change when the economies shift from regime I to regime II. However, restricted versions of this specification could of course be considered. For instance, the following specification

$$\Delta g_{n,t} = \delta_n + \left[\rho_n^I g_{n,t-1}\right] I_{\{z_{t-1}<\lambda\}} + \left[\rho_n^{II} g_{n,t-1}\right] I_{\left\{z_{t-1}\geq\lambda\right\}} + \sum_{i=1}^{p} \varphi_{n,i} \Delta g_{n,t-i} + \varepsilon_{n,t}, \quad (14)$$

with $n = 1, \cdots, N$, and $t = 1, \cdots, T$, assumes that only the convergence rate varies with the regime.

In (13) and (14), p is assumed to be high enough so that $\varepsilon_{n,t}$ is a white noise process for each n. So serial correlation in the errors is excluded. However, for the same reasons as in model (1), cross-country contemporaneous correlation cannot be excluded. The relevant assumption on the errors is of the type described in (6) and (7).

3.2 Estimation

Model (13) is estimated by least squares. However, given the dependence of the coefficients on the threshold value of the transition variable (both unknown), and given the structures (6) and (7), it is convenient to use concentration in a GLS approach.

To simplify the exposition, lest us first suppose that λ, m and d are known and collect their known values in vector $\theta_0 = (\lambda_0, m_0, d_0)'$. So, conditional on θ_0, model (13) can be seen as a panel data equation with known dummy variables. Assume that the total number of available time observations for each country is $(T+p+1)$ so that $(p+1)$ initial values prior to $t = 1$ exist.

If $\odot$ denotes an element-by-element multiplication, (13) can be written in SURE form as

$$\Delta G = \left[X \odot \breve{I}_{I,\theta_0} \vdots X \odot \breve{I}_{II,\theta_0} \right] \begin{bmatrix} \beta_{I,\theta_0} \\ \cdots \\ \beta_{II,\theta_0} \end{bmatrix} + \varepsilon, \tag{15}$$

where $\beta_{I,,\theta_0}$ (β_{II,θ_0}) is defined like β in (8) but refers to the coefficients under regime I (II) when $\lambda = \lambda_0$, $m = m_0$ and $d = d_0$. The expression $\breve{I}_{I,\theta_0}$ refers to an $(NT \times 1)$ vector obtained by stacking N times the $(T \times 1)$ dummy-variable vector

$$I_{I,\theta_0} = \left[I_{z_{0,p}<\lambda_0}, I_{z_{0,p+1}<\lambda_0}, \cdots, I_{z_{0,T-1}<\lambda_0} \right]', \tag{16}$$

where $z_{0,t} = g_{m_0,t} - g_{m_0,t-d_0}$. Similarly, $\breve{I}_{II,\theta_0}$ is obtained by stacking N times the vector

$$I_{II,\theta_0} = \left[1 - I_{z_{0,p}<\lambda_0}, 1 - I_{z_{0,p+1}<\lambda_0}, \cdots, 1 - I_{z_{0,T-1}<\lambda_0} \right]'. \tag{17}$$

Model (15) can be written more compactly as

$$\Delta G = \breve{X}_{\theta_0} \beta_{\theta_0} + \varepsilon. \tag{18}$$

Estimating this model by feasible GLS is justified by the characteristics of the variance-covariance matrix of ε specified in (6) and (7). So

$$\hat{\beta}_{\theta_0,FGLS} = \left[\breve{X}'_{\theta_0} \hat{V}_0^{-1} \breve{X}_{\theta_0} \right]^{-1} \breve{X}'_{\theta_0} \hat{V}_0^{-1} \Delta G, \tag{19}$$

where $\hat{V}_0 = \hat{\Omega}_0 \otimes I_T$ and $\hat{\Omega}_0 = [s_{nm,0}]$ with $s_{nm,0} = \frac{1}{T} \sum_{t=1}^{T} e_{nt,0} e_{mt,0}$ for $n, m = 1, \cdots, N$, and with $e_{lt,0}$ being the OLS residual of model (15) corresponding to observation t for country l.

In practice, we do not know the true value of λ, m and d. However, we can infer appropiate values for these parameters from the data. Denote $\hat{\varepsilon}_{\theta_0}$ the FGLS residual vector of model (15) and define the weighted sum of squared residuals $s^2_{\theta_0} = \frac{1}{T} \hat{\varepsilon}'_{\theta_0} \hat{V}_0 \hat{\varepsilon}_{\theta_0}$. Since this sum is a function of θ_0, a grid-search procedure can be applied to obtain

$$\hat{\theta} \equiv \left[\hat{\lambda}, \hat{m}, \hat{d} \right] = \arg\min_{\theta_0} (s^2_{\theta_0})$$

and the Least-Squares estimates of the other parameters can be obtained by plugging in the point estimate $\hat{\theta}$ in model (15) and obtain the corresponding $\hat{\beta}_{\hat{\theta},FGLS}$.

To implement the grid-search procedure, for each $m \in [1, 2, \cdots, N]$ and each $d \in [1, \cdots, p]$, λ is given the value $(g_{m,\tau} - g_{m,\tau-d})$ for each $\tau \in (1, 2, \cdots, T)$. The fraction of the sample falling in the implied regime I is then computed. If this fraction lies in the interval $[\pi_{1,} 1 - \pi_1]$, the corresponding FGLS estimator of β_θ and the weighted sum of residuals are computed. If not, this combination of m, d and λ is discarded and the procedure goes to the next point of the grid. Once all the points have been checked, the estimation process ends with the obtention of $\hat{\theta}$ and the corresponding $\hat{\beta}_{\hat{\theta},FGLS}$ We will refer later to this estimation procedure as the "grid-FGLS" method.

Once model (13) is estimated, its superiority has to be checked with respect to the linear Evans-Karras model (1). If confirmed, the next task consists of testing whether there is convergence or not, by applying some type of unit-root test on the ρ coefficients of (13). Finally, if there is evidence of convergence, the last step should test absolute against conditional convergence through a test on the δ coefficients of (13). We propose a testing procedure for each case in what follows.

3.3 The linearity test

The null hypothesis to be tested is that model (1) is correct, versus the alternative of model (13). The problem here is that some parameters (namely, λ, m and d) are not identified under the null since they are defined only under the alternative. As a consequence, conventional test statistics, such as Likelihood Ratio, Wald or LM tests, do not have standard distributions under this null. This problem has been pointed out by Hansen (1996) and is examined for the single-equation multiple-regime TAR model by Hansen (1999). It is also examined by Caner and Hansen (2001) when testing the single-equation two-regime TAR model with a unit root. One of the best solutions proposed in the last two papers consists of obtaining the critical values by bootstrap simulations. We describe an extension of this solution for the panel-data TAR model (13) in what follows.

We want to test

$$H_{o,1} : \delta_n^I = \delta_n^{II}, \rho_n^I = \rho_n^{II}, \varphi_{i,n}^I = \varphi_{i,n}^{II}, \tag{20}$$

$\forall n = 1, \cdots, N$ and $\forall i = 1, \cdots, p$, against the alternative that not all the coefficients are equal in both regimes.

For that purpose, estimate (1) by FGLS and (13) by the grid-FGLS method. For each model, compute the value of the likelihood function at the estimation point and obtain:

$$\pounds_{1,2} = -2\ln(L_1/L_2), \tag{21}$$

where L_1 is the likelihood value of the one-regime linear model (1) and L_2 is the likelihood value of the two-regime TAR model (13).[3] The null of linearity would be rejected if $\pounds_{1,2}$ is too large. In order to know how large $\pounds_{1,2}$ has to be, we obtain the critical value by mimicking Caner and Hansen (2001) single-equation procedure, while we adapt it to take into account the contemporaneous cross-country correlations of the errors as described by (6) and (7). At this point of the analysis we do not know whether the series exhibit or not a unit root; therefore, two sets of bootstrap simulations should be carried out. The first set, called the "unrestricted bootstrap" simulations is based on an unrestricted estimation of the linear model, as specified in (1). The second one, called the "restricted bootstrap", imposes a unit root by restricting $\rho_n = 0$ in (1). That is, the model considered under the null imposes both linearity and a unit root, as described by

$$\Delta g_{n,t} = \delta_n + \sum_{i=1}^{p} \varphi_{n,i} \Delta g_{n,t-i} + \varepsilon_n, \tag{22}$$

with $n = 1, \cdots, N$ and $t = 1, \cdots, T$.

The bootstrap algorithm to obtain the critical value is as follows. In the unrestricted bootstrap simulations, after estimating model (1) by feasible GLS, the corresponding residuals $e^L_{n,t} = \Delta g_{n,t} - \hat{\delta}_n + \hat{\rho}_n g_{n,t-1} + \sum\limits_{i=1}^{p} \hat{\varphi}_{n,i} \Delta g_{n,t-i}$ are obtained for all n and t, recentered and organized in the following $T \times N$ matrix, similar to E° in (11):

$$\tilde{E} = \begin{bmatrix} e^L_{1,1} & e^L_{2,1} & \cdots & e^L_{N,1} \\ e^L_{1,2} & e^L_{2,2} & \cdots & e^L_{N,2} \\ \vdots & \vdots & \cdots & \vdots \\ e^L_{1,T} & e^L_{2,T} & \cdots & e^L_{N,T} \end{bmatrix}. \tag{23}$$

Then, a random sample $\left\{e^*_{n,t}, n = 1, \cdots, N\right\}$ for $t = 1, \cdots, N$ is generated by sampling matrix $\tilde{E}$ by row (with replacement) to preserve the contemporaneous correlation of the

[3] Alternatively, the weighted sum of squared residuals of each model could be computed and a Wald-type test statistic could be built.

errors in the bootstrap populations. The data are then recursively generated from the following equation, where the coefficients take the values of the GLS estimates of (1):[4]

$$\Delta g^*_{n,t} = \hat{\delta}_n + \hat{\rho}_n g^*_{n,t-1} + \sum_{i=1}^{p} \hat{\varphi}_{n,i} \Delta g^*_{n,t-i} + e^*_{n,t}. \tag{24}$$

Let $\pounds^*_{1,2}$ be the value of the test (21) for this generated series. Hence, $\pounds^*_{1,2}$ is computed for each simulated bootstrap series. In this study, we make $1,000$ bootstrap replications. They are then sorted by increasing order and the bootstrap p-value is the fraction of simulated $\pounds^*_{1,2}$ values smaller than $\pounds_{1,2}$.

The restricted bootstrap algorithm is similar, except that the residuals to be resampled are obtained from model (22) instead of model (1):

$$e^{L,r}_{n,t} = \Delta g_{n,t} - \hat{\delta}^{(r)}_n + \sum_{i=1}^{p} \hat{\varphi}^{(r)}_{n,i} \Delta g_{n,t-i},$$

where $\hat{\delta}^{(r)}_n$ and $\hat{\varphi}^{(r)}_{n,i}$ are the GLS estimates of model (22). By the same token, the data are recursively generated from

$$\Delta g^{*(r)}_{n,t} = \hat{\delta}^{(r)}_n + \sum_{i=1}^{p} \hat{\varphi}^{(r)}_{n,i} \Delta g^{*(r)}_{n,t-i} + e^{*(r)}_{n,t}. \tag{25}$$

After both sets of simulations are carried out, inference about linearity is based on the more conservative result, that is, on the higher bootstrap p-value. If the linear model is rejected, the rest of the analysis is based on the TAR model (13).

3.4 Convergence tests

If the empirical evidence favours model (13), the next step consists of testing convergence against divergence. The null hypothesis is

$$H_{0,2} : \rho^I_n = \rho^{II}_n = 0 \ \forall n \tag{26}$$

[4]For the initial values, we use the $(p+1)$ first data of the sample of observations; this is also the simplification used by Caner and Hansen (2001).

in model (13). If fulfilled, it reflects that the countries diverge both under regime I and regime II. Three types of alternative are of economic interest and can be tested:

$$H_{A,2_a} \quad : \quad \rho_n^I < 0, \rho_n^{II} < 0 \ \forall n, \tag{27a}$$

$$H_{A,2_b} \quad : \quad \rho_n^I < 0, \rho_n^{II} = 0 \ \forall n, \tag{27b}$$

$$H_{A,2_c} \quad : \quad \rho_n^I = 0, \rho_n^{II} < 0 \ \forall n. \tag{27c}$$

The alternative (27a) reflects convergence of the countries both under regime I and II. We will refer to this case as "full convergence". On the opposite, the alternatives (27b) and (27c) imply that convergence takes place only under regime I or only under regime II, respectively. We will refer to such a situation as "partial convergence".

Note that the null and the alternative hypothesis are assuming that the ρ coefficients satisfy the same property for *all* the countries at a time. This is consistent with the definition (2) of the series $g_{n,t}$: since these series are in deviations from their common cross-section mean, as soon as one of the country does not converge to the other (even though the remaining countries do converge to each other), none of the $g_{n,t}$ series can be $I(0)$. In other words, the $g_{n,t}$ series of the panel are all $I(0)$ or all $I(1)$.

In order to discriminate between the three alternatives, we use several test statistics, in the line of Caner and Hansen (2001) for the single-equation case. These authors propose a Wald-type statistics for the test against the global alternative $H_{A,2_a}$ of convergence. Extending their proposition to the panel-data case, the statistic is

$$R_2 = t_I^2 + t_{II}^2, \tag{28}$$

where t_I and t_{II} are t-type statistics associated with the estimation of ρ_n^I and ρ_n^{II}, respectively, in model (13). Namely, if $\hat{\rho}_n^i$ is the grid-GLS estimate of ρ_n^i for each regime i, we have

$$t_i = \frac{\hat{\rho}_n^i}{s_{\rho_n^i}}, \tag{29}$$

for $i = I, II$. Given the definition of R_2, large values of this statistic are favorable to convergence.

For the alternative of partial convergence $H_{A,2_b}$ the statistic to be used would be t_I, while t_{II} would be used to test against the partial convergence hypothesis $H_{A,2_c}$. These are left-sided tests. So, if t_I (t_{II}) is too small, whereas t_{II} (t_I) is not, the data favour the hypothesis of convergence under regime I (II) and divergence under regime II (I).

Here again, bootstrap simulations are used in order to find the appropriate probability values. The first steps of the bootstrap algorithm coincides with the restricted bootstrap of the linearity test. That is, the GLS residuals obtained from adjusting the linear model (22) in which a unit root is imposed are recentered and resampled by row from a matrix similar to $\tilde{E}$. The bootstrapped data are then recursively generated from (25). So the bootstrap generation process imposes not only the null of a unit root but also linearity. The rational for this is twofold. On the one hand, the generation of the data from a linear model is much simpler than from a TAR model when the transition variable is endogenous. On the other hand, Caner and Hansen (2001) obtained much more reliable results, in terms of size of the test, with the linear model than from a TAR generating process, as the results appear to be much less sensitive to nuisance parameters. In fact, they strongly recommend the use of the linearized version of the model in the bootstrap generation.

Once the bootstrap data are obtained, the statistics R_2, t_I and t_{II} are computed and sorted in ascending order to obtain the bootstrap p-values. It must be beard in mind that the test based on R_2 is right-sided, whereas the tests based on t_I and t_{II} are left-sided.

The last step of the convergence analysis consists of discriminating between absolute and conditional convergence. The absolute convergence hypothesis refers to the fact that converging countries share the same steady path. Conditional convergence refers to the existence of parallel, though not coincident, paths. So, in terms of model (13), under the maintained hypothesis that $\rho_n^i < 0$, $\forall n = 1, \cdots, N$, and $i = I, II$, absolute convergence is equivalent to

$$\delta_n^i = 0 \ , \forall n = 1, \cdots, N, \ i = I, II. \tag{30}$$

If the convergence process is partial, in the sense that it takes place only under one of the two regimes, say regime I, then absolute convergence would correspond to

$$\delta_n^I = 0 \ , \forall n = 1, \cdots, N. \tag{31}$$

Note, however, that in this two-regimes model, another case of interest occurs when $\rho_n^i < 0$

for all n and i (global convergence) but $\delta_n^i = 0$ for only one value of i. In this case, convergence is absolute under one regime although conditional under the other one. We therefore need several test statistics to be able to discriminate between these different cases.

The tests that we propose are based on the grid-GLS estimation of model (13). They direct extensions to the TAR model of the statistics proposed by Evans and Karras (1996) for the linear case. In particular, they are derived from the t-statistics $t(\hat{\delta}_n^i) = \frac{\hat{\delta}_n^i}{s_{\hat{\delta}_n^i}}$, with $i = I, II$, and $n = 1, \cdots, N$, associated with the estimated value of the constant terms. They are the following:

$$\Phi_a = \frac{1}{2N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_n^I)\right]^2 + \sum_{n=1}^{N}\left[t(\hat{\delta}_n^{II})\right]^2\right\}, \tag{32a}$$

$$\Phi_b = \frac{1}{N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_n^I)\right]^2\right\}, \tag{32b}$$

$$\Phi_c = \frac{1}{N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_n^{II})\right]^2\right\}. \tag{32c}$$

Given the endogeneity of the transition variable, here too the bootstrap p-values are obtained from adjusting a linear model to the observed data. So we impose a null constant term in model (1) and estimate

$$\Delta g_{n,t} = \rho_n g_{n,t-1} + \sum_{i=1}^{p}\varphi_{n,i}\Delta g_{n,t-i} + \varepsilon_t, \tag{33}$$

with $n = 1, \cdots, N$ and $t = 1, \cdots, T$ by feasible GLS. The matrix of recentered residuals is then resampled by row and the bootstrapped data are generated from the estimates of (33). Model (13) is then adjusted on these data and the three tests Φ_a, Φ_b and Φ_c are computed. The bootstrap right-sided p-values are extracted from their empirical distributions. The three statistics are then used in the following way:

- if $H_{0,2}$ has been rejected in favor of $H_{A,2_a}$:
  - Φ_a is too large $\Rightarrow$ conditional convergence takes place under both regimes.
  - Φ_b is too large, although Φ_c is not $\Rightarrow$ conditional convergence takes place under regime I and absolute convergence takes place under regime II.

- Symmetrically, Φ_c is too large, although $\Phi_b$is not $\Rightarrow$ conditional convergence takes place under regime II and absolute convergence takes place under regime I.

- if $H_{0,2}$ has been rejected in favor of $H_{A,2_b}$:
  - Φ_b is too large $\Rightarrow$ conditional convergence takes place under regime I.
  - Φ_b is not large enough $\Rightarrow$ absolute convergence takes place under regime I.

- if $H_{0,2}$ has been rejected in favor of $H_{A,2_c}$:
  - Φ_c is too large $\Rightarrow$ conditional convergence takes place under regime II.
  - Φ_c is not large enough $\Rightarrow$ absolute convergence takes place under regime II.

4 Empirical application

The methodology described in the previous section has been applied on the data of several European countries, with the main objective of examining the situation of the recently acceded countries in comparison with the older members.

4.1 The data

The panel of countries on which we have applied our procedure refers to annual data of the logarithm of the GDP per capita for fifteen countries which can be gathered in three groups. The first group includes nine so-to-speak "rich" West-European countries: Austria, Belgium, Denmark, Finland, France, Italy, the Netherlands, Sweden and the United Kingdom. The second group refers to three "poorer" West -European countries: Spain, Greece and Portugal. Finally, the third group corresponds to three East-European countries: Hungary, Poland and Czechoslovakia. The data are expressed in 1990 constant and international GK Dollars and cover the period 1950-2004 but for Czechoslovakia for which the data stop in 2003. They have been derived from the database of the University of Groningen.[5]

The selection of these countries deserves two comments. On the one hand, Germany has been excluded from the first group in order to avoid the distorting statistical effect of the 1989 reunification process. Since the former West-Germany was one of the richer European countries and the reunification drastically lowers the per capita output of the new geographical area known as Germany, the inclusion of Germany in the panel would artificially favor the convergence hypothesis. On the other hand, as far as the East-European countries are concerned, our interest is in testing the convergence situation of the so-called accession countries with respect to the older members of the European. Ideally, we would like to include the whole set of countries recently acceded to the EU. However, the data on GDP per capita are available on a long enough period only for Hungary and Poland whereas for the remaining countries, the series start in 1989. So, we have just taken these two countries, together with the data for the former Czechoslovakia as a proxy of the new Czech Republic in order to work with information on a third acceding country.[6]

4.2 Empirical results

As mentioned before, when working with the deviations of per capita output from a common cross-country mean, the divergence of one single country implies that the whole set of $g_{n,t}$ series are $I(1)$. It is therefore important to carefully select the set of countries on which we apply the convergence tests. In particular, it is wise to start with a subset of countries for which convergence is highly probable, and if confirmed, to progressively add more countries to the set and repeat the convergence tests on these augmented set. In this way, it is possible to identify which countries converge and which do not.

As a consequence, we start by testing convergence in the first group of richer countries, which are the most likely to converge. The data of the per capita GDP for these countries are represented in Figure 1. This visual information points towards a high degree of

[5] Groningen Growth and Development Centre and The Conference Board, Total Economy Database, January 2005, http://www.ggdc.net.

[6] Since its creation, the share of the real GDP of the Czech Republic in the real GDP of former Czechoslovakia evolved between 66.3% and 71, 2%, with an average value of 68, 5%.

convergence between these countries. The statitical results are gathered in Table 1.a, for the linear model (1), and in Table 1.b for the TAR model (13). As expected, the linear model reflects that these countries have been converging during the last five decades (p-value for the null of divergence equal to 0.000), and that this convergence has been absolute (p-value of 0.902 for the null of absolute convergence).

Turning to the TAR model, several results deserve comments. First, the test of linearity favours the TAR specification very strongly. Both the unrestricted and restricted bootstrap p-values are below the standard 5% critical value. The United Kingdom is identified as the country whose evolution determines the switch from one regime to the other. The data for this country are identified with dots in Figure 1. The overall position of this country over the whole period has evolved from being the richest country at the very beginning of the period to one of the lowest position in the middle the sample, to finally end up at an intermediate level at the end of the period. In other words, this country can be seen as representative of the intensive convergence process among the countries of this first group. This may be an explanation of why it has been picked to form the transition variable. The estimate value of the delay parameter d is 1, so that the transition variable is $g_{UK,t} - g_{UK,t-1}$. As far as the threshold parameter λ is concerned, it is estimated at -1.25. Given the definition of $g_{UK,t}$ and the value of d, it means that regime I corresponds to the years in which the growth rate of the UK per capita income was below the average growth rate of the group by more than 1.25 percentage points. That is, regime I refers to years during which the UK grew somewhat slower than the remaining countries of the group. This regime corresponds to 25.5% observations of the sample. On the opposite, regime II, prevailing during 74.5% of the sample, refers to years that were not so sluggish or more flourishing than the average. The periods corresponding to each regime, as well as the threshold position and the value of the transition variable can be examined in Figure 2. It can be seen that regime II has been dominating during the last twenty years. As far as convergence is concerned, the null of divergence is rejected both in regime I and regime II (see Table 1.b), with a p-value of 0.034 and 0.000, respectively. There are thus some symptoms that convergence is somewhat more intensive under regime II. This regime also exhibits stronger signs of absolute convergence (p-value of 0.675) than regime I (p-value of 0.288). The overall conclusion is, though, absolute convergence under both regimes. We may therefore conclude that these countries have shared a common steady state path over the five last decades. By the same token, the weighted average of their per capita GDP (using the population as a weight) is economically meaningful and can be used as a benchmark in the analysis of the convergence of other countries towards this first group. This is what we do next.

Tables 2.a and 2.b offer the results concerning the study of the convergence process of Spain, Portugal and Greece and the average of the richer countries. Figure 3 reproduces the data of their per capita GDP in logarithms and reflects that these poorer countries have narrowed their distance with respect to the first group, especially in the last years of the sample. However, Table 2.a indicates that the linear model does not detect convergence, since the p-value for the null of divergence is $0, 114$. As a result, the test to discriminate between absolute and conditional convergence does not apply.

In Table 2.b, the results from the TAR model give another image. According to the linearity test, there is no doubt that the TAR model is superior to the linear one (p-values of 0.009 and 0.011 for the unrestricted and restricted bootstrap, respectively). The transition variable corresponds to the relative growth rate of Greece (estimated d is 1) and the estimated value of the threshold parameter is 2.14. This implies that regime I, which takes place in 83.7% of the sample, prevails when the difference between the growth rate of the Greek GDP per capita and the average one is below 2.14 percentage points. Accordingly, regime II takes place when the relative growth rate of Greece is above this level. The results of the convergence tests reveals that divergence cannot be excluded under regime I (p-value of 0.131), whereas convergence takes place under regime II (p-value of 0.016). So there is partial convergence under this last regime. The periods when regime II prevails can be easily located in Figure 4. They broadly coincide with the seventies and the last years of the sample. From Figure 3, it can be seen that the countries grew closer together during these years. On the opposite, regime I completely dominates the decades of the eighties and nineties, which coincide with the period when Greece grew much slower than the other countries and progressively fell down to the worst position. Hence, the TAR model proves to be useful in announcing the results of the convergence analysis. Note finally that the test of absolute against conditional convergence gives a p-value of 0.385 under regime II, which is favorable to absolute convergence.

Now, the previous analysis is extended with the inclusion of Poland, Hungary and Czechoslovakia. The data and the results are presented in Figures 5 and 6 and in Tables 3.a and 3.b. The visual information from Figure 5 is far from favorable to convergence: compared with the fifties and the sixties, during the seventies and the eighties the per capita GDP of these three East-European countries decreased with respect to those of the West-European countries. This tendency changed in the second half of the nineties, although the series still evolve nowadays far away from each other. Both the linear and the TAR model reflect this divergence situation. According to the linear model, divergence cannot be rejected with a p-value equal to 0.091. Within the framework of the TAR model, the most conservative p-value of the linearity test is equal to 0.05, which is a limit value. The transition country is Greece again, although the estimated d turns to be 3 in this case. So the threshold refers to the relative growth rate over three years. It is estimated at 5.85 (which roughly coincides with 2 percentage points in annual terms). So regime I corresponds to the periods in which the relative growth rate of Greece over the last three years has been above the average by somewhat less than 2 points, which is consistent with the results obtained in the previous group. From Figure 6, it can be seen that regime I is the dominating situation, especially in the last three decades. The non-linear convergence tests exhibit the same results in both regimes: divergence cannot be discarded in any of them. Given how the GDP series and the transition variable evolve by the end of the sample period, it is to be expected that these results could change in favour of some type of convergence once more data will be made available.

5 Conclusions

In this paper, we have developed a new method of testing real convergence, in terms of the per capita real GDP, in a panel-data non-linear framework. The non-linear model that we use belongs to the class of TAR models with two regimes. Our model extends the existing literature in two directions at a time. On the one hand, it extends the usual

TAR model to a panel data framework. At the same time, it adds to non-linearity the possibility of non-stationarity due to the presence of a unit root. Allowing for the presence of a unit root is crucial in our study, since this is precisely what happens when one or various countries of the panel diverge. On the opposite, when all the countries converge, no GDP series should exhibit a unit root.

We apply our methodology to study the convergence of several sets of EU countries. The first one concerns a group of nine rich countries for which we obtain that the convergence process has been somewhat more intensive in the last two decades. The second group of countries includes the weighted average of the previous group plus three poorer old EU members, Spain, Portugal and Greece. The linear model does not detect any sign of convergence, whereas the TAR model indicates that these countries have been converging to the richer ones in the years of more intensive growth of one of the poorest country of the group. Finally, two new EU members (Hungary and Poland) together with Czechoslovakia have been added to the analysis. As expected, no form of convergence is detected in any regime. Our empirical results are therefore very reasonable and add interesting nuances to what can be obtained from a linear approach. It indicates that our approach can shed more light on the processes of real convergence.

Finally, it is worth pointing out that our methodology offers an interesting by-product that can be exploited for the analysis of other economic problems. Our TAR methodology to discriminate between divergence and convergence is in fact a way of detecting the presence of a unit root. Therefore, this paper may be applied to detect unit roots in any set of nonlinear multivariate time series.

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Table 1. Nine EU richer countries

<table>
<tr><td colspan="8">1.a. Linear model</td></tr>
<tr><td colspan="3">Divergence vs convergence</td><td colspan="5">Absolute vs conditional convergence</td></tr>
<tr><td colspan="3">0.000
Convergence</td><td colspan="5">0.902
Absolute</td></tr>
<tr><td colspan="8">1.b. TAR model</td></tr>
<tr><td colspan="3">Linearity tests</td><td rowspan="2">Transition country</td><td rowspan="2">d</td><td rowspan="2">λ</td><td rowspan="2" colspan="2">% observations in Regime I</td></tr>
<tr><td colspan="2">Unrestricted</td><td>Restricted</td></tr>
<tr><td colspan="2">0.013</td><td>0.011</td><td>UK</td><td>1</td><td>-1.25</td><td colspan="2">25.5</td></tr>
<tr><td colspan="8">Convergence tests</td></tr>
<tr><td colspan="3">Divergence vs convergence</td><td colspan="5">Absolute vs conditional convergence</td></tr>
<tr><td>Regime I</td><td>Regime II</td><td>Both</td><td>Regime I</td><td>Regime II</td><td colspan="3">Both</td></tr>
<tr><td>0.034</td><td>0.000</td><td>0.000</td><td>0.288</td><td>0.675</td><td colspan="3">0.534</td></tr>
<tr><td colspan="3">Full convergence</td><td colspan="5">Absolute</td></tr>
</table>

Notes. Entries refer to bootstrap *p*-values computed as described in the text. The selected lag length is 2 since it was the lowest value for which the residuals of the linear panel estimated by FGLS were uncorrelated (on the basis of the Ljung-Box statistic) for each country.

Table 2. Average of nine EU richer countries and three EU poorer

<table>
<tr><td colspan="7">2.a. Linear model</td></tr>
<tr><td colspan="3">Divergence vs convergence</td><td colspan="4">Absolute vs conditional convergence</td></tr>
<tr><td colspan="3">0.114
Divergence</td><td colspan="4">-
-</td></tr>
<tr><td colspan="7">2.b. TAR model</td></tr>
<tr><td colspan="3">Linearity tests</td><td rowspan="2">Transition country</td><td rowspan="2">d</td><td rowspan="2">λ</td><td rowspan="2">% observations in Regime I</td></tr>
<tr><td colspan="2">Unrestricted</td><td>Restricted</td></tr>
<tr><td colspan="2">0.009</td><td>0.011</td><td>Greece</td><td>1</td><td>2.14</td><td>83.7</td></tr>
<tr><td colspan="7">Convergence tests</td></tr>
<tr><td colspan="3">Divergence vs convergence</td><td colspan="4">Absolute vs conditional convergence</td></tr>
<tr><td>Regime I</td><td>Regime II</td><td>Both</td><td>Regime I</td><td>Regime II</td><td colspan="2">Both</td></tr>
<tr><td>0.131</td><td>0.016</td><td>0.068</td><td>-</td><td>0.385</td><td colspan="2">-</td></tr>
<tr><td colspan="3">Partial convergence in Regime II</td><td colspan="4">Absolute in Regime II</td></tr>
</table>

Notes. The selected lag length is 4. See footnote in Table 1.

Table 3. Average of nine EU richer countries, three EU poorer and three new members

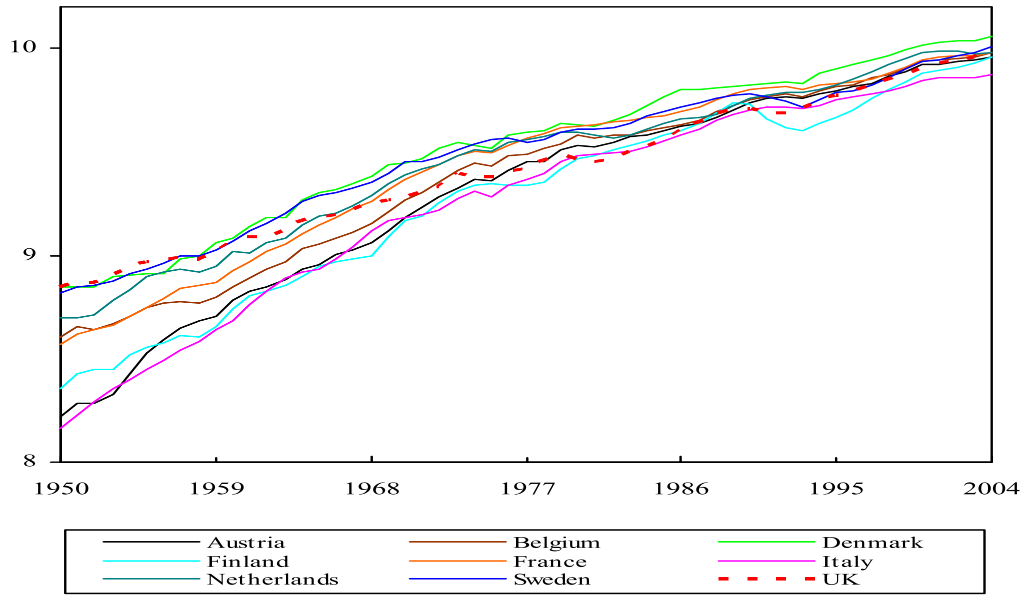
3.a. Linear model	
Divergence vs convergence	Absolute vs conditional convergence
0.091	-
Divergence	-

3.b. TAR model						
Linearity tests		Transition country	d	λ	% observations in Regime I	
Unrestricted	Restricted					
0.050	0.033	Greece	3	5.85	77.1	

Convergence tests					
Divergence vs convergence			Absolute vs conditional convergence		
Regime I	Regime II	Both	Regime I	Regime II	Both
0.429	0.476	0.569	-	-	-
Divergence			-		

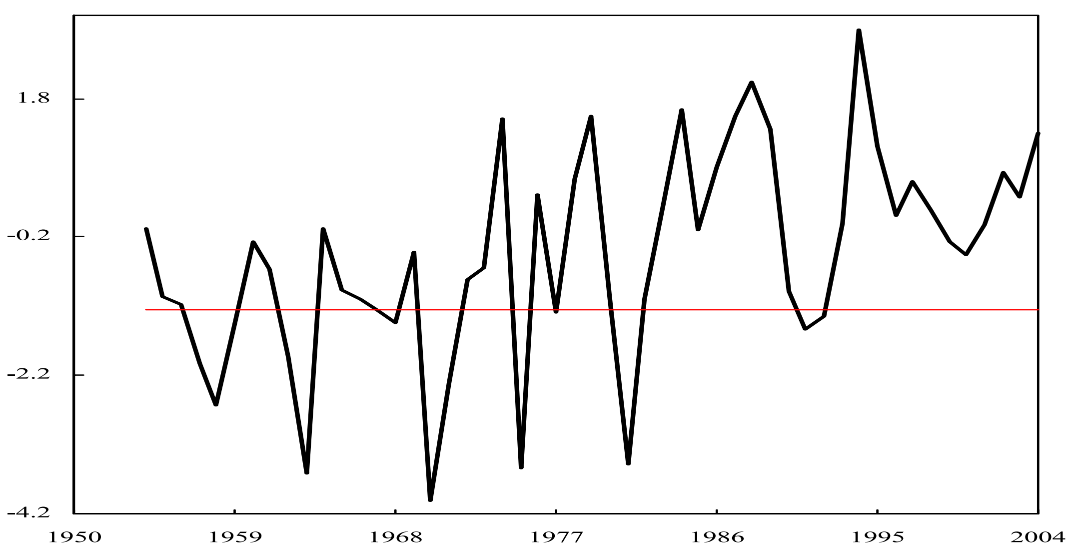
Notes. The selected lag length is 4. See footnote in Table 1.

Figure 1: Output evolution of nine richer



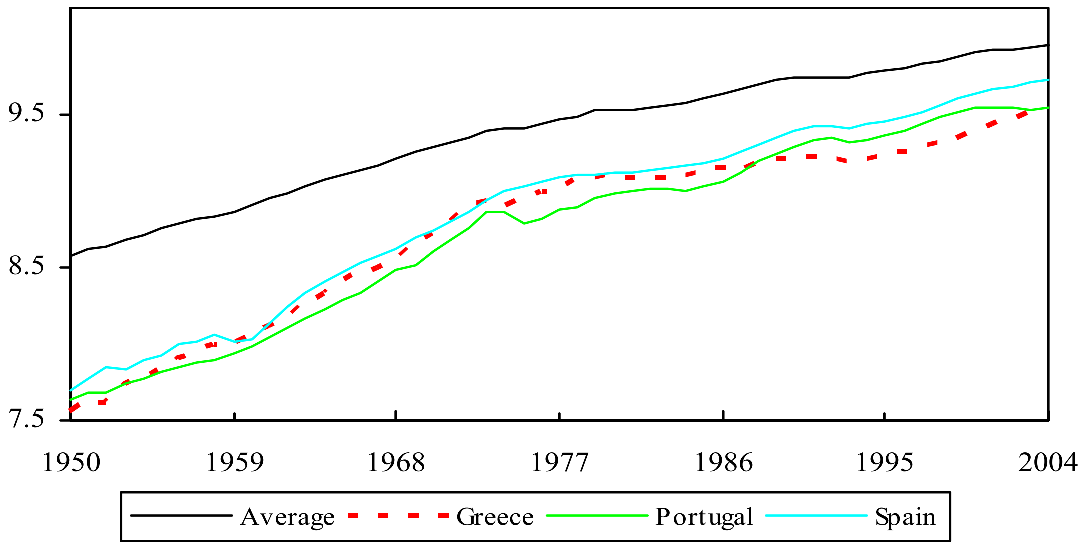
Notes: Logs of annual per capita GDP for Austria, Belgium, Denmark, Finland, France, Italy, Netherlands, Sweden and UK. Sample: 1950-2004.

Figure 2: Threshold variable for nine richer



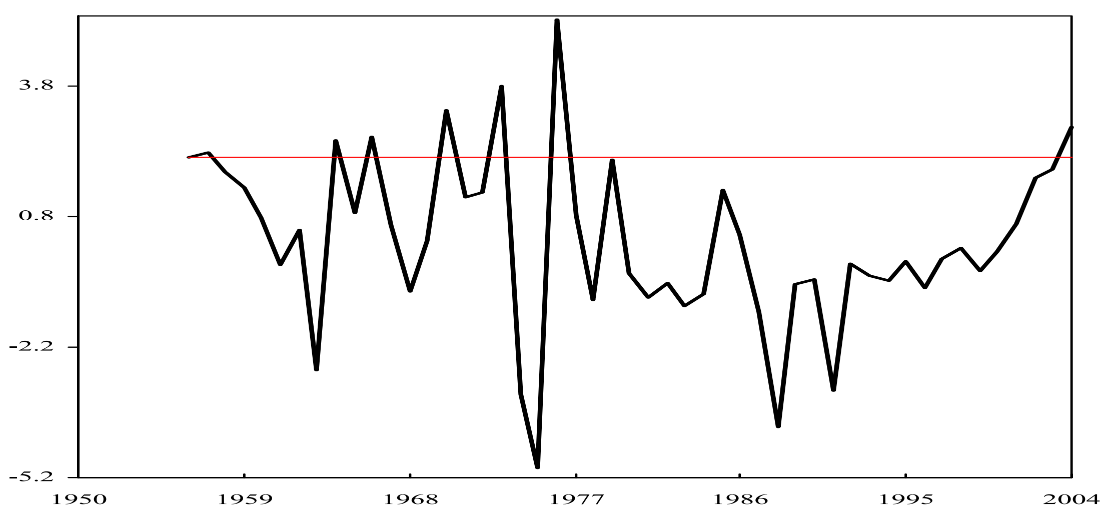
Notes: Threshold variable refers to UK data (*d*=1). Horizontal line refers to the threshold (-1.25). Sample: 1950-2004.

Figure 3: Output evolution of average and poorer



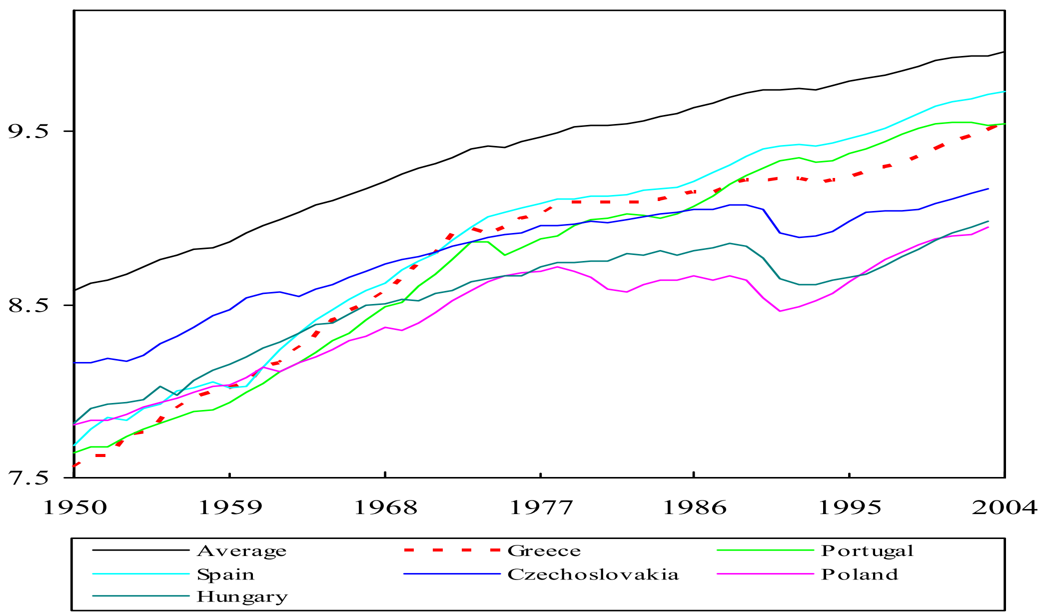
Notes: Logs of weighted average of annual per capita GDP for countries of Figure 1 and logs of GDP for Greece, Portugal and Spain. Sample: 1950-2004.

Figure 4: Threshold variable for average and poorer



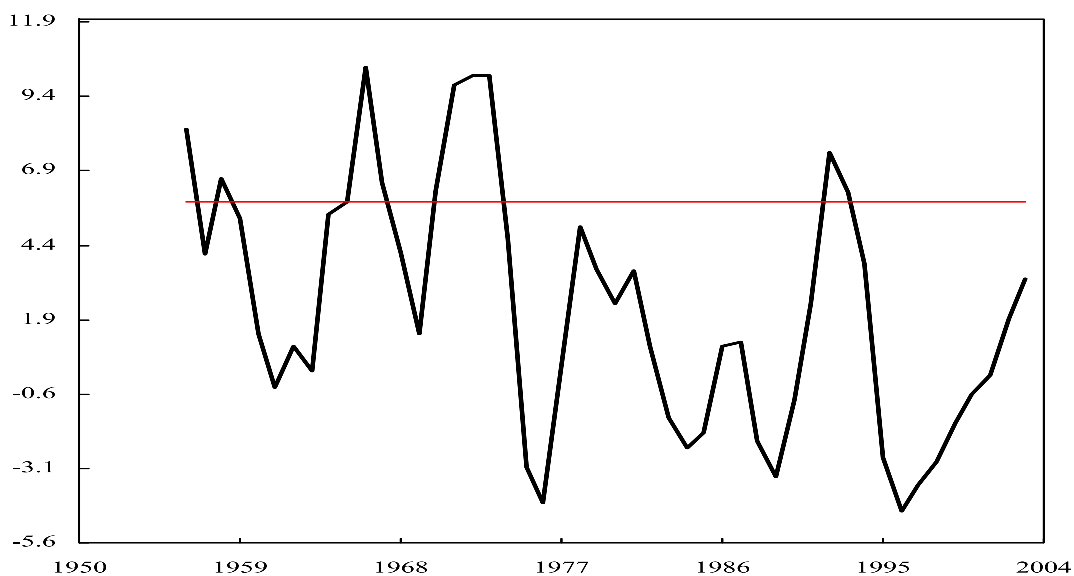
Notes: Threshold variable refers to Greece data (*d*=1). Horizontal line refers to the threshold (2.14). Sample: 1950-2004.

Figure 5: Output evolution of average, poorest and newly acceded



Notes: Logs of annual per capita GDP for countries of Figure 2 along with Czechoslovakia, Poland and Hungary. Sample: 1950-2003.

Figure 6: Threshold variable for average, poorest and newly acceded



Notes: Threshold variable refers to Greece data (d=3). Horizontal line refers to the threshold (5.85). Sample: 1950-2003.