Monitoring the world business cycle*

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Abstract

We propose a Markov-switching dynamic factor model to construct an index of global business cycle conditions, to perform short-term forecasts of world GDP quarterly growth in real time and to compute real-time business cycle probabilities. To overcome the real-time forecasting challenges, the model accounts for mixed frequencies, for asynchronous data publication and for leading indicators. Our pseudo real-time results show that this approach provides reliable and timely inferences of the world quarterly growth and of the world state of the business cycle on a monthly basis.

Keywords: Real-time forecasting, world economic indicators, business cycles, non-linear dynamic factor models.

JEL Classification: E32, C22, E27.

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1 Introduction

The drastic downturn in the global economy in 2009 led the economic agents to acknowledge the need for new tools to monitor the ongoing world economic developments, which may especially affect small open economies. Although there is currently no global statistical institute in charge of providing official quarterly national accounts at a global level, the IMF releases real GDP annual growth rate figures on an annual basis. However, the IMF only releases its GDP figures twice a year (usually in April and October), although there are two additional updates in January and July, but these provide much less detail.

Therefore, the target of the recent literature has been focusing on several indicators at higher frequencies, which are promptly available and are used to construct early estimates of the world GDP. Rossiter (2010) uses bridge equations to show that PMIs are useful for forecasting developments in the global growth. Within the bridge equation framework, Golinelli and Parigi (2014) detect that short-run indicators from advanced and emerging countries help in predicting the world variables and Drechsel et al. (2014) find that several monthly leading global indicators improve upon the world forecasts of the IMF. Ferrara and Marsilli (2014) develop a linear Dynamic Factor Model (DFM) to summarise the information of a large monthly database into a small numbers of factors and use the MIDAS framework to show that they improve upon the IMF forecasts, at least at the beginning of each year.

These approaches suffer from two limitations. The first limitation is that they focus on GDP on an annual basis while the IMF also releases GDP quarterly growth rates sampled on a quarterly basis. In spite of the advantage in managing data on a quarterly basis, the IMF releases its quarterly figures sporadically, with long publication delays (9 months on average) and no fixed starting date. Hence, to the best of our knowledge, its longest available world GDP quarterly series begins in 2007, which clearly restricts the econometric analysis. To overcome this drawback, we reproduce the IMF releases of the world GDP on a quarterly basis by following the same methodology. Both series are the same from 2007 up to the present, but our series dates back to 1991.

The second limitation of these approaches is that they all rely on linear specifications, which handicaps the models in capturing the nonlinearities that characterise the world business cycle fluctuations and can be used for detecting turning points. To overcome this drawback, we propose an extension of the Markov-switching DFM (MS-DFM) suggested in Camacho et al. (2012). As in their proposal, the MS-DFM advocated by Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998) is enabled to deal with mixing frequencies, publication delays and different starting dates in the economic indicators. In this framework, low-frequency indicators are treated as high-frequency indicators with missing observations and the model is estimated by using a time-varying nonlinear Kalman filter. In addition, we extend the model to handle leading indicators, which are very useful in short-term forecasting (Camacho and Martinez-Martin, 2014) since they usually start to decline before the economy as a whole declines, and start to improve before the general economy begins to recover from a slump. We allow the data to select the number of periods by which the leading indicators lead the broad economic activity, from a minimum of one quarter to one and a half years.

In the empirical application, we use this extension to evaluate the accuracy of the model in computing short-term forecasts of world GDP, and to reveal inferences about the state of the global economy from six economic indicators: the quarterly world GDP and the monthly global industrial production index, the global manufacturing Purchasing Manager Index (PMI), the employment index, the new export orders index and the CBOE volatility index (VIX). For this purpose, we develop a pseudo real-time forecasting exercise, where data vintages are constructed from successive enlargements of the latest available data set by taking into account the real-time data flow (and hence the publication lags). Therefore, the experiment tries to mimic as closely as possible the real-time analysis that would have been performed by a potential user of the models when forecasting, in each period, on the basis of different vintages of data sets. In line with the substantial publication delay of world GDP, in each forecast period we perform backcasts (predict the previous quarters before data for those quarters are released) nowcasts (predict the current period) and forecasts (predict the next quarter).

Our main results are as follows. First, the percentage of the variance of world GDP growth that is explained by our MS-DFM is slightly above 70%, indicating the high potential ability of our extension to explain global growth. Second, our pseudo real-time analysis shows that our MS-DFM clearly outperforms univariate forecasts, especially when backcasting and nowcasting. In addition, our MS-DFM also outperforms the forecasts of a linear DFM. Third, our business cycle indicator is in striking accord with the consensus of the history of the world business cycle (Grossman et al. 2014). Fourth, we also compare the performance of the fully non-linear MS-DFM (*one-step* approach) with the "shortcut" of using a linear DFM to obtain a coincident indicator which is then used to compute the Markov-switching probabilities (*two-step* approach). In line with Camacho et al. (2015), our results suggest that the *one-step* approach is preferred to the *two-step* approach to compute inferences on the business cycle phases.

The structure of this paper is organised as follows. Section 2 describes the methodological considerations of the model. Section 3 contains data descriptions and the main empirical results. Section 4 concludes.

2 The econometric model

To account for the peculiar characteristics of the data flow in real time, the comovements across the economic indicators and the business cycles asymmetries, we start from the approach suggested in Camacho et al. (2012). This model is extended to handle economic indicators that lead the broad economic activity.

2.1 Mixing frequencies

The approach deals with the problem of mixing monthly and quarterly frequencies of flow data by treating quarterly series as monthly series with missing observations. Let us assume that the levels of the quarterly flow variable in the quarter that ends in month t, G_t, can be decomposed as the sum of three unobservable monthly values X_t, X_{t-1}, X_{t-2}, where t, $t-1$ and $t-2$ refer to the three months of that quarter

$$G_t = 3\frac{X_t + X_{t-1} + X_{t-2}}{3}. \tag{1}$$

Following the linear framework described in Mariano and Murasawa (2003), let us assume that the arithmetic means can be approximated by geometric means. Hence, the level of the quarterly flow variable becomes

$$G_t = 3(X_t X_{t-1} X_{t-2})^{1/3}. \tag{2}$$

Applying logs and taking the three-period differences for all t

$$\triangle_3 \ln G_t = \frac{1}{3}(\triangle_3 \ln X_t + \triangle_3 \ln X_{t-1} + \triangle_3 \ln X_{t-2}). \tag{3}$$

Calling $\triangle_3 \ln G_t = g_t$, and $\triangle \ln X_t = x_t$, and after a little algebra

$$g_t = \frac{1}{3}x_t + \frac{2}{3}x_{t-1} + x_{t-2} + \frac{2}{3}x_{t-3} + \frac{1}{3}x_{t-4}, \tag{4}$$

which shows that that quarterly growth rates of the quarterly flow variable can be viewed as sums of underlying monthly growth rates of the underlying monthly series.

2.2 Dynamic properties

Let us assume that the n indicators included in the model, y_{it} , $i = 1., ..., n$, admit a dynamic factor representation. In this case, the indicators can be written as the sum of two stochastic components: a common component, f_t, which represents the overall business cycle conditions, and an idiosyncratic component, u_{it}, which refers to the particular dynamics of the series.

To account for the business cycle asymmetries, we assume that the underlying business cycle conditions evolve with $AR(p_f)$ dynamics, which is governed by an unobserved regime-switching state variable, s_t,

$$f_t = c_{s_t} + \theta_1 f_{t-1} + ... + \theta_{p_f} f_{t-p} + \varepsilon_{ft}, \tag{5}$$

where $\varepsilon_{ft}\tilde{\ } iN\left(0, \sigma_f^2\right)$. Within this framework, one can label $s_t = 0$ and $s_t = 1$ as the expansion and recession states at time t. In addition, it is standard to assume that the state variable evolves according to an irreducible 2-state Markov chain, whose transition probabilities are defined by

$$p(s_t = j|s_{t-1} = i, s_{t-2} = h, ..., I_{t-1}) = p(s_t = j|s_{t-1} = i) = p_{ij} \tag{6}$$

where $i, j = 0, 1$ and I_t is the information set up to period t.

Coincident indicators, such as the monthly growth rates of GDP and the monthly growth rates of industrial production index, the global manufacturing Purchasing Manager Index (PMI) index, the employment index and the new export orders index depend contemporaneously on f_t and on their idiosyncratic dynamics, u_{it}, which evolve as an $AR(p_i)$

$$y_{it} = \beta_i f_t + u_{it}, \tag{7}$$

$$u_{it} = \theta_{i1} u_{it-1} + ... + \theta_{ip_i} u_{t-p_i} + \varepsilon_{it}, \tag{8}$$

where $\varepsilon_{it} \tilde{} iN\left(0, \sigma_i^2\right)$, $i = 1, ..., 5$. However, since leading indicators lead the economy movements by h periods, VIX is assumed to depend on the h-period future values of the common factor and its idiosyncratic dynamics, u_{it}, which evolve as an $AR(p_6)$

$$y_{lt} = \beta_l f_{t+h} + u_{lt}, \tag{9}$$

$$u_{lt} = \theta_{l1} u_{lt-1} + ... + \theta_{6p_l} u_{t-p_l} + \varepsilon_{lt}, \tag{10}$$

where $\varepsilon_{lt} \tilde{} iN\left(0, \sigma_l^2\right)$. Finally, we assume that all the shocks ε_{ft} and ε_{it}, $i = 1, ..., 6$, are mutually uncorrelated in cross-section and time-series dimensions.

2.3 State space and missing observations

Assuming that all variables are always observed at a monthly frequency, it is standard to cast the model in state form. The state-space model represents a set of observed time series, Y_t, as linear combinations of a vector of auxiliary variables, which are collected on the state vector, h_t. This relation is modeled by the *measurement equation*, $Y_t = H\xi_t + E_t$, with $E_t \sim i.i.d.N\left(0, R\right)$. The Appendix provides more details on the model structure and the specific forms of these matrices. The dynamics of the state vector are modelled by the *transition equation* $h_t = \Lambda_{s_t} + Fh_{t-1} + V_t$, with $V_t \sim i.i.d.N\left(0, Q\right)$. Maximising the exact log likelihood function of the associated nonlinear Kalman filter is computationally burdensome since at each iteration the filter produces a 2-fold increase in the number of cases to consider. The solution adopted in this paper is based on collapsing some terms of the former filter as proposed by Kim (1994). In particular, our proposal is based on collapsing the posteriors $h_{t|t}^{(i,j)}$ and its covariance matrix $P_{t|t}^{(i,j)}$ at the end of each iteration by using their weighted averages for $s_t = j$, where the weights are given by the probabilities of the Markov state

$$h^j_{t|t} = \frac{\sum_{s_{t-1}=0}^{1} p\left(s_t = j, s_{t-1} = i|I_t\right) h^{(i,j)}_{t|t}}{p\left(s_t = j|I_t\right)} \quad (11)$$

$$P^j_{t|t} = \frac{\sum_{s_{t-1}=0}^{1} p\left(s_t = j, s_{t-1} = i|I_t\right) \left(P^{(i,j)}_{t|t} + \left(h^j_{t|t} - h^{(i,j)}_{t|t}\right)\left(h^j_{t|t} - h^{(i,j)}_{t|t}\right)'\right)}{p\left(s_t = j|I_t\right)}. \quad (12)$$

To handle missing data, we use the method used in Camacho et al. (2012). For this purpose, we substitute missing observations with random draws θ_t from $N(0, \sigma^2_\theta)$. This implies replacing the i-th row of Y_t, H_t, E_t (denoted by Y_{it}, H_{it} and w_{it}) and the i-th element of the main diagonal of R_t (R_{iit}), by Y^*_{it}, H^*_{it}, E^*_{it}, and R^*_{iit}, respectively. The starred expressions are Y_{it}, H_{it}, 0, and 0 if the variable Y_{it} is observable at time t, and θ_t, $0_{1,n}$, θ_t, and σ^2_θ in case of missing data. Accordingly, this transformation converts the model in a time-varying state-space model with no missing observations and the nonlinear version of the Kalman filter can be directly applied to Y^*_t, H^*_t, w^*_t, and R^*_t since missing observations are automatically skipped from the updating recursion.

To conclude this section, let us describe how our model can easily compute world GDP growth forecasts. Let us assume that GDP growth is placed first Y_t and let us call T the last month for which we have observed this indicator and that we are interested in the forecast for $T+1$. Let us call $h^{(j)}_{T+1|T}$ the collapsed version of $h^{(i,j)}_{T+1|T}$, and call $h^{(j)}_{T+1|T}(k)$ the k-th element of $h^{(j)}_{T+1|T}$. Taking into account that h_t contains $f_{t\,+h}$ and its lags, as well as the idiosyncratic components and their lags, the forecasts for month $T+1$ when $s_{T+1} = j$ can be computed from the model as

$$y^{(j)}_{1T+1/T} = \beta_1 \left(\frac{1}{3}h^{(j)}_{T+1|T}(h+1) + \frac{2}{3}h^{(j)}_{T+1|T}(h+2) + h^{(j)}_{T+1|T}(h+3) + \frac{2}{3}h^{(j)}_{T+1|T}(h+4) + \frac{1}{3}h^{(j)}_{T+1|T}(h+5)\right) - \left(\frac{1}{3}h^{(j)}_{T+1|T}(h+6) + \frac{2}{3}h^{(j)}_{T+1|T}(h+7) + h^{(j)}_{T+1|T}(h+8) + \frac{2}{3}h^{(j)}_{T+1|T}(h+9) + \frac{1}{3}h^{(j)}_{T+1|T}(h+10)\right). \quad (13)$$

Using the matrix of transition probabilities, one can easily obtain $p\left(s_{T+1} = j, s_T = i|I_t\right)$, which can be used to compute

$$p\left(s_{T+1}=j|I_T\right)=\sum_{i=1}^{2}p\left(s_{T+1}=j,s_T=i|I_T\right) \tag{14}$$

Then, the unconditional forecast of GDP is

$$y_{1T+1/T}=\sum_{j=1}^{2}p\left(s_{T+1}=j|\chi_t\right)y_{T+1/T}^{(j)}. \tag{15}$$

It is worth noting that these forecasts are easily computed in practice by including missing observations of GDP growth in the dataset, since the model will automatically replace at missing observation with a dynamic forecast. Following the same reasoning, forecasts for longer horizons and forecasts for other indicators can be computed in this way.

3 Empirical results

3.1 Preliminary analysis of the data

In this section, we identify potential indicators that reflect the economic dynamics of the world, and might therefore be well suited for the prediction of the GDP aggregates and the business cycle conditions. Indicators should lead or coincide with the macroeconomic dynamics of the particular aggregate, and should have a wide coverage of the economy as a whole. In addition, they should have a high frequency, should be released before the GDP figure for the respective quarter becomes available and must be available in at least one third of the sample. For the world indicators, we have selected the following five coincident indicators: the quarterly world GDP and the monthly global industrial production index, the global manufacturing Purchasing Manager Index (PMI), the employment index and the new export orders index. In addition, we include the CBOE volatility index (VIX), which is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices.

The data set managed in this paper was collected on 15 December, 2014 and the maximum effective sample spans the period from January 1991 to October 2014. The indicators used in the empirical analysis, their respective release lag-time, their sources and the data transformations applied to achieve stationarity are listed in Table 1. GDP enters the model as its quarterly growth rate, industrial production and the VIX enters in monthly growth rates while other indicators enter with no transformation. All the variables are seasonally adjusted. Before estimating the model, the variables are standardised to have a zero mean and a variance equal to one. Therefore, the final forecasts are computed by multiplying the initial forecasts of the model by the sample standard deviation, and then adding the sample mean.[1]

[Insert Table 1 about here]

The IMF releases the quarterly data sporadically, with a long publication delay of 9 months on average. However, the most important limitation is that the historically available series starts in 2007, which clearly restricts the econometric analysis. To overcome this drawback, we aim to re-construct the IMF releases of the world GDP on a quarterly basis by following the same methodology. Our proxy is based on the aggregation of national quarterly growth rates (official Quarterly National Accounts) of 69 countries, which are weighted by their share of GDP ppp in the world.[2] Both series are the same from 2007 up to the present but our world GDP quarterly growth is dated back to 1991.

3.2 In-sample analysis

We examine in this section the in-sample estimates of the nonlinear dynamic factor model outlined in Section 2. For the contribution of this paper, the selection of the lead time profile of the VIX becomes of particular interest, since it is allowed to lead the business cycle dynamics in h months, with $h = 0, 1, ..., 6$. To select the optimal number of leads, we computed the log likelihood values associated with these lead times and we observed that the maximum of the likelihood function was achieved when the VIX led the common factor by $h = 3$ months.

The loading factors, whose estimates appear in Table 2 (standard errors in parentheses), allow us to evaluate the correlation between the common factor and each of the

[1] In the simulated real-time analysis, the sample means and standard deviations are also computed using only the observations available up to the forecast jump-off point.

[2] It covers about 92% of total world GDP ppp.

indicators used in the model.[3] Apart from GDP (loading factor of 0.22), the economic indicators with the largest loading factors are industrial production (loading factor of 0.47) and the VIX (loading factor of -0.22). As expected, the loading factors for all of the indicators except the VIX are positive and statistically significant, indicating that these series are procyclical, i.e., positively correlated with the common factor that represents the world overall economic activity. The significant negative sign of the VIX's loading factor agrees with the view that it becomes a leading proxy of global financial risk aversion.[4] For the sake of comparison, the table also shows the factor loadings of the linear version of MS-DFM.

[Insert Table 2 about here]

Although GDP is generally regarded as the most appropriate indicator of economic activity, global GDP is only available on an annual basis or, sporadically, a quarterly basis. As such, many important questions cannot be addressed in a satisfactory manner, especially those related to the analysis of business cycles. One partial solution is found by constructing aggregate indexes at monthly frequency, which are computed as linear combinations of meaningful economic indicators, as in Aruoba et al. (2011). However, these indexes are not related to the particular variable of interest, what makes it difficult to find an economic interpretation of their movements or their reactions to shocks. One significant contribution of our methodology is that it allows to construct quarterly GDP growth rates for the world economy on a monthly basis, from both available quarterly GDP data and global monthly indicators. This time series is plotted in Figure 1.

[Insert Figure 1 about here]

Our model is based on the notion that co-movements among the macroeconomic variables have a common element, the common factor, that moves in accordance with the dynamics of the world business cycle. To check whether the business cycle information

[3] The lag lengths used in the empirical exercise were always set to 2. However, we performed several exercises to check that our results were robust to other reasonable choices of the lag lengths.

[4] Under the framework of financial publications and business news' shows on CNBC, Bloomberg TV and CNN/Money, the VIX is often referred to as the "fear index".

that can be extracted from the common factor agrees with the world business cycle, the coincident indicator is also plotted in Figure 1.[5] According to this figure, the evolution of the factor is in clear concordance with GDP growth and contains relevant information of its expansions and recessions. In particular, the common factor is consistent with the recession of the early 1990s, the expansionary period of the late 1990s and the downturn of 2001. The evolution of the factor also picks up the short-lived effects of the Asian financial crisis of 1997 and the Russian default of 1998 as an emerging markets slowdown. In addition, it also captures the magnitude of the global recession of 2008, which was unprecedented in scale, and the subsequent (albeit fragile) recovery.

3.3 Pseudo real-time analysis

To begin with, it is worth mentioning that we could not perform the forecast evaluation in pure real-time, i.e., by using only the information that would have been available at the time of each forecast. The reason is that the historical records of the time series used in the analysis were not available to construct all the real-time vintages for all the indicators in the panel. One feasible alternative, employed frequently in the forecasting literature, is to develop a pseudo real-time analysis. The method consists of computing forecasts from successive enlargements of a partition of the latest available data set by taking into account the typical real-time data flow when constructing the data vintages.[6] Therefore, the method tries to mimic as closely as possible the real-time analysis that would have been performed by a potential user of the models when forecasting, at each time period.

Our pseudo real-time analysis begins with data of all the time series from the beginning of the sample until October 2000. Since the average reporting lag of world GDP is three quarters, we assume that the latest available figure of GDP refers to the first quarter of 2000. Using this sample, the model is estimated and 15-months-ahead forecasts of

[5] To facilitate graphing, the coincident indicator has been transformed to exhibit the same mean and variance as world GDP growth.

[6] Hence, each vintage used at the forecast periods contains missing data at the end of the sample reflecting the calendar of data releases. We assume that the pattern of data availability is unchanged throughout the evaluation sample since the timing of data releases varies only slightly from month to month.

quarterly GDP growth are computed. This implies performing backcasts for the first three quarters of 2000, nowcasts for the last quarter of 2000 and forecasts for the first quarter of 2001. In addition, we collect the probabilities of recession for the first month of the quarter for which GDP is unavailable. In November 2000, the sample is updated by one month of each indicator, the model is reestimated and 15-months-ahead forecasts of quarterly GDP growth and the inferences on the business cycle state for the second month of the quarter for which GDP is unavailable are computed again. Then, the sample is again updated by one month and the forecasting exercise is developed similarly in December 2000.

In January 2001, GDP for the first quarter of 2000 is assumed to be released so the forecast moves forward one quarter. The forecasting procedure continues iteratively until the final forecast with the last vintage of data that refers to July 2014, with the 15-months-ahead forecasts moved forward one quarter in accordance with the publication date of GDP. This procedure ends up with 165 blocks of forecasts. To examine the forecasting accuracy, we compute the mean-squared forecast errors (MSE), which are the average of the deviations of the predictions from the final releases of GDP available in the data set.

This paper establishes two naïve benchmark models against which to compare the results of the MS-DFM model. The first benchmark is a random-walk model, where the forecasts for the global output growth are equal to the last observed value. The second benchmark model is an autoregressive model, in which GDP is regressed on its lags and the forecasts are computed recursively from the in-sample estimates. Since these types of models traditionally perform reasonably well with macro data, the key challenge then is to determine whether the addition of global indicator variables can improve the forecasting performance of the benchmark models. In addition, the paper also establishes as a benchmark the linear dynamic factor model underlying the nonlinear specification. In this case, the challenge is to determine whether accounting for the potential nonlinearities help in improving the forecasting accuracy.

The predictive accuracy of the models is under examination in Table 3. Results for backcasts, nowcasts and forecasts are summarised in the second, third and fourth columns, respectively. Remarkably, the multivariate models perform better than the univariate benchmarks, with largest improvements for shortest forecast horizons. This result reinforces the view that the monthly global indicators are useful for forecasting the developments in the global economy. When comparing MS-DFM and the linear DFM, the former performs better than the latter. This suggests that to obtain accurate forecasts of world GDP, the nonlinear information on the total economy matters.

[Insert Table 3 about here]

In spite of the good performance in forecasting world GDP, our nonlinear MS-DFM exhibits a clear advantage with respect to the linear proposals. The model automatically converts the economic information contained in the global indicators into inferences of the world business cycle. In particular, the model computes probabilities of global recessions, which are transparent, objective and free of units of measurement, facilitating business cycle comparisons with national developments. Figure 2 plots the real-time probabilities whenever the coincident global indicator is under a recessionary state. To facilitate comparisons, the figure also shows shaded areas that refer to the chronology of global recessions suggested in Grossman et al. (2014).[7] Notably, they are in striking accord: the probabilities of recession jump quickly around the peaks, remain at high values during recessions and fall to almost zero after the troughs.

[Insert Figure 2 about here]

As a final check, we compare the performance of our fully nonlinear multivariate specification (*one-step* approach) with the 'shortcut' of using a linear factor model to obtain a coincident indicator, which is then used to compute the Markov switching probabilities (*two-step* approach). We quantify the ability of these procedures to detect the actual state of the business by computing the Forecasting Quadratic Probability Score (FQPS), i.e., the mean squared deviation of the probabilities of recessions from a recessionary indicator that takes on the value of one in the periods classified by Grossman et al. (2014) as recessions and zero elsewhere. We obtain a FQPS of 0.09 for the *two-step* approach, which falls to 0.06 in the case of the *one-step* approach. Therefore, our results suggest that, in line

[7] These authors date the turning points by using univariate dating algorithms to global aggregates of a sample of 84 countries.

with Camacho et al. (2015), the *one-step* approach is preferred to the *two-step* approach to compute inferences on the business cycle phases.

4 Concluding remarks

Albeit that several approaches have been employed, it has been a recent challenge to construct practical and satisfactory tools to monitor global business cycles. The methodology we use in this paper has several advantages over existing approaches. First, our measure of global economic activity captures common movements among a wide range of indicators that can exhibit different frequencies, different sample start dates and different release lags. In particular, we include GDP, industrial production, manufacturing PMI, employment, new export orders and VIX. Second, our framework is useful for computing short-term forecasts of world GDP in real time. In addition, it combines the information provided by monthly and quarterly data to obtain a monthly measure of quarterly growth rates of world GDP. Managing data on a monthly basis has enormous advantages in several macroeconomic applications, especially those related to business cycle analyses. Third, our framework is also useful for monitoring global economic activity in real time. The nonlinear nature of the model proposed in this paper helps it to capture the asymmetric dynamics of the recurrent sequence of expansions and recessions that characterise the world business cycle.

5 Appendix

Without loss of generalisation, we assume that our model contains only world GDP, one coincident indicator and one leading indicator, which are collected in the vector $Y_t = (y_{1t}, y_{2t}, y_{lt})'$. For simplicity's sake, we also assume that the autoregressive processes for the idiosyncratic components are of order one, that the common factor is just a switching mean ($p_f = 0$) and that the lead for the leading indicator is $h = 1$. Let 0_{ab} be $a \times b$ matrix of zeroes.

In this case, the observation equation, $Y_t = Hh_t + E_t$, where $E_t\tilde{}iN(0, R)$, can be stated by using $E_t = 0_{3\times1}$, $R = 0_{3\times3}$ and

$$
\begin{aligned}
H &= \begin{pmatrix} 0 & \frac{2\beta_1}{3} & \frac{\beta_1}{3} & \beta_1 & \frac{\beta_1}{3} & \frac{2\beta_1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \beta_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (16) \\
h_t &= (f_{t+1}, f_t, ..., f_{t-4}, u_{1t}, ..., u_{1t-4}, u_{2t}, u_{lt})'. \qquad (17)
\end{aligned}
$$

In this expression, the factor loadings, β_1, β_2 and β_l, measure the sensitivity of each series to movements in the latent factor. The quarterly variable is expressed as a moving average of the underlying monthly non-observable variables that are related to the factor and the monthly shocks.

The transition equation, $h_t = \Lambda_{s_t} + Fh_{t-1} + V_t$, where $V_t\tilde{}iN(0, Q)$, is

$$\Lambda_{s_t} = (c_{s_t}, 0_{1\times 12})', \tag{18}$$

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{l1} \end{pmatrix}, \tag{19}$$

$$V_t = (\varepsilon_{ft}, 0_{1\times 5}, \varepsilon_{1t}, 0_{1\times 5}, \varepsilon_{2t}, \varepsilon_{lt})', \tag{20}$$

$$Q = diag\left(\sigma_f^2, 0_{1\times 5}, \sigma_1^2, 0_{1\times 5}, \sigma_2^2, \sigma_l^2\right). \tag{21}$$

The identifying assumption implies that the variance of the common factor, σ_f^2, is normalised to a value of one.

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Table 1: Variables included in the model

Series	Sample	Source	Publication delay	Data transformation
Real Gross Domestic Product (GDP, SAAR, Mill. 1993 ARS)	1991.1 2014.1	National accounts	2 to 3 months	QGR
Industrial Production Index (IPI) – Global (SA, 2000=100)	1992.01 2014.10	CPB	2.5 months	MGR
JP Morgan Global PMI – (+50: Expansion)	1998.01 2014.10	Markit Economics	0 months	Level
Employment Index – Global Manufacturing (+50: Expansion)	1998.06 2014.10	Markit Economics	0 months	Level
New Export Orders Index – Global Manufacturing (+50: Expansion)	1998.06 2014.10	Markit Economics	0 months	Level
VIX – CBOE Market -Volatility Index	1991.01 2014.10	CBOE - Bloomberg	0 months	MGR

Notes. SA means seasonally adjusted. MGR and QGR mean monthly growth rates and quarterly growth rates, respectively. CPB: Netherlands Bureau of Economic Analysis. CBOE: Chicago Board Options Exchange.

Table 2: Loading factors

	GDP	IP	PMI	EMPL	NExO	VIX
MS-DFM	0.22 (0.03)	0.47 (0.04)	0.10 (0.01)	0.06 (0.01)	0.10 (0.02)	-0.22 (0.09)
DFM	0.06 (0.00)	0.14 (0.01)	0.22 (0.01)	0.16 (0.02)	0.22 (0.02)	-0.04 (0.01)

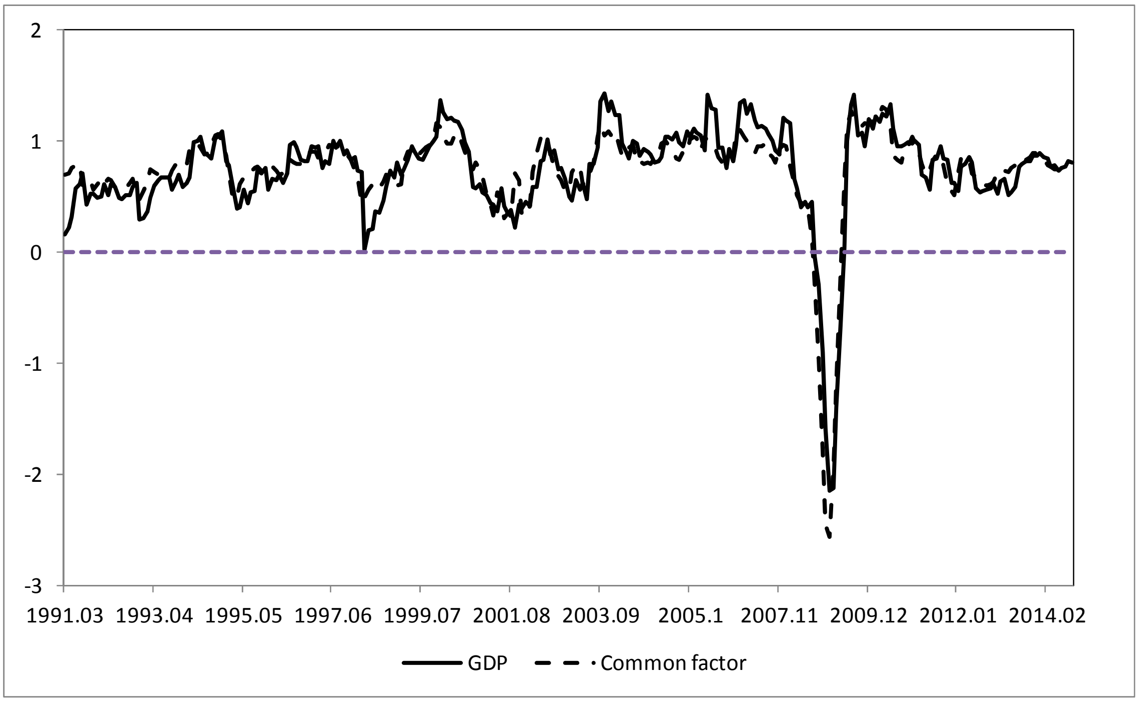
Notes. The loading factors (standard errors are in brackets) measure the correlation between the common factor and each of the indicators. See notes of Table 1.

Table 3: Predictive accuracy

	Backcasts	Nowcasts	Forecasts
	Mean Squared Errors		
RW	0.31	0.32	0.33
AR	0.21	0.24	0.25
Linear DFM	0.14	0.20	0.25
MS-DFM	0.12	0.19	0.24

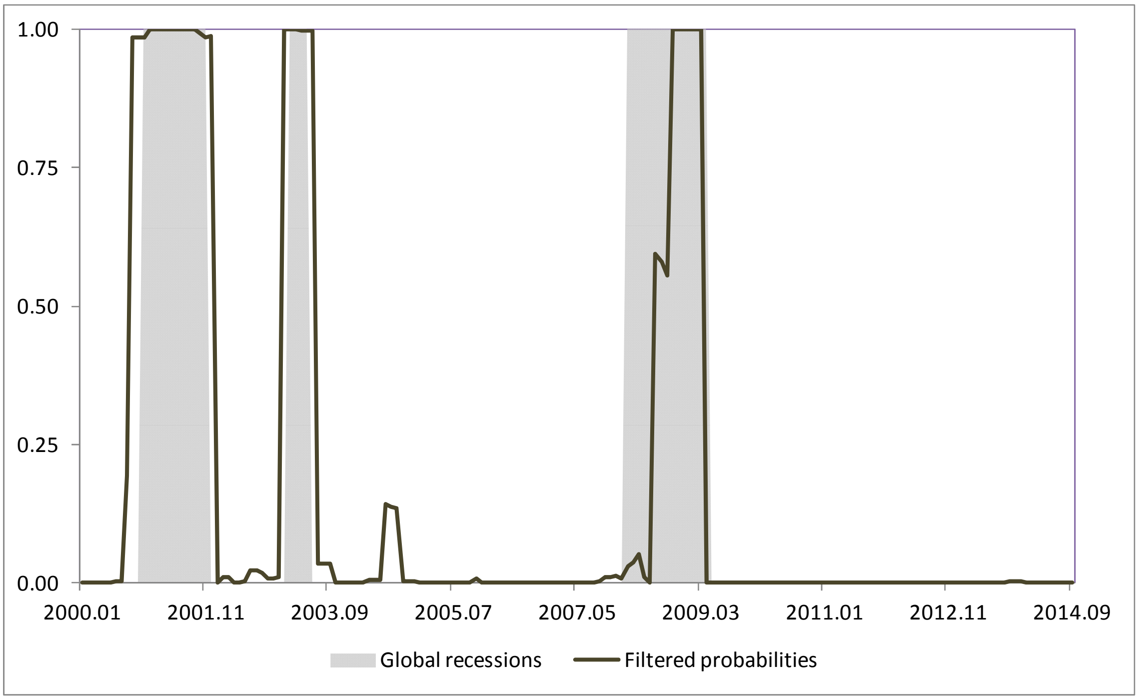
Notes. The forecasting sample is 2000.1-2013.3. The top panel shows the Mean Squared Errors of a random walk (RW), an autoregressive model (AR), a linear dynamic factor model and our Markov-snitching dynamic factor model.

Figure 1: Common factor and monthly GDP



Notes. Quarterly growth of global GDP at monthly frequency and common factor estimated from 1991.03 to 2014.09.

Figure 2: Probabilities of global recessions



Notes. The figure plots the probabilities of recession in real time from 200.01 to 2014.09. Shaded areas correspond to global recessions as documented by Grossman et al. (2014).