Econometric methods for business cycle dating: a practical guide *

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Abstract

Business cycle dating helps in developing economic analysis and is useful for economic agents whether they be policy makers, investors or academics. This paper reviews old and recent research on dating the reference cycle turning points and is intended as a guide to the applied researcher. All these methods provide a statistical alternative to cycle dating committees, although full automatism and researcher's art could be complements rather than substitutes in some dating scenarios. Our survey divides the dating literature into two groups with different approaches to dating the business cycle from a set of coincident economic indicators: average-then-date or date-then average. In both cases, the dating techniques can be divided into non-parametric and parametric. The paper shows the theoretical foundations of both types of techniques and describes in detail the algorithms or estimation methods necessary for their implementation. Finally, the paper describes empirical applications of the different methods with data of different frequencies, trying to show how they work in practice and pointing out their advantages and disadvantages. This empirical illustrations include a compilation of the codes in different languages (R, Matlab or Gauss). In our opinion, future research should focus on developing methods that are robust to changes in volatility or large outliers and on exploring the usefulness of big data sources and the classification ability offered by machine learning methods.

Keywords: Business cycles, dating methods, non-parametric, parametric, classification techniques.

JEL Classification: E32, C22, E27.

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1 Introduction

Dating peaks and troughs helps to shed light on economic analysis in multiple dimensions. Firstly, dating the turning points plays an important role in policy as Governments and Central Banks need to know when and how to implement countercyclical policies to stabilize the fluctuations of the business cycle. Secondly, knowing shifts from one phase of the cycle to the next helps investors to evaluate and adjust their exposure to different types of investments and provide reasonable guidance for management decisions. Thirdly, in academic studies, dating of the business cycle chronology allows researchers to conduct worldwide cyclical comparisons, to analyze the evolution of synchronization between countries and to determine the international transmission of business cycles.

Aware of this requirement, Martin Feldstein established a Business Cycle Dating Committee of National Bureau of Economic Research (NBER) scholars and gave it responsibility for business cycle dating when he became president of the institution in 1978. Inspired by the work of Burns and Mitchell (1946), the committee looked at several coincident economic indicators to make informative judgments on when to set the historical dates of the peaks and troughs of past US business cycles. Following these guidelines, other countries have created similar committees, such as the Euro Area Business Cycle Dating Committee of the Center for Economic Policy Research, founded in 2002, and the Spanish Business Cycle Dating Committee (SBCDC) of the Spanish Economic Association, created in 2014.[1]

While there is considerable interest in establishing and maintaining a historical chronology of the business cycle, the dating methodology of the Committees has received some criticism because, after all, their decisions represent the consensus of individuals, which leads to two drawbacks. First, although the committees measure the business cycle using various econometric methods, they also use qualitative approaches based on the opinions of the experts composing the committees. Thus, their dating methodology is neither transparent nor reproducible.

Second, the committees wait until the existence of a peak or trough is not in doubt. Thus, they date the turning points after a considerable time lag. For example, Table 1 shows that, since 1980, the lag of the NBER in announcing the US peaks ranges from 4 to 12 months, and the lag in announcing the troughs is even longer, ranging from 8 to 21 months. These time lags in announcements reduce the interest in the committees' decisions from the point of view of providing real-time assessments of business cycle changes.

An alternative to business cycle dating committee procedures is to propose statistical frameworks that provide automatic dating of turning points. These are based on algorithms that treat business cycle dating as a formal statistical issue, addressing the two drawbacks of the dating committees described above. Firstly, dating through algorithms does not rely on the judgmental decisions of committee members. By contrast, the algorithms are specified rules that apply to the data directly, so they are transparent and reproducible. Secondly, the algorithms try to automate the dating procedure so that assessments on the business cycle can be updated easily in real time as new data become available. Thus, dating with algorithms tends to reduce the speed with which

[1] To name a few, Brazil, Canada, France and Mexico have also set up dating committees.

the dates of the changes in business cycle phases are provided in real time.

However, timeliness sometimes comes at the price of accuracy, which, as documented by Hamilton (2011), implies a big challenge to academics. This happens because the statistical assessments of turning point dates are typically performed in real time with the first data releases, which are often based on incomplete data and subject to different numbers of revisions in subsequent releases as they incorporate better and more complete data sources. In contrast, the lags in the announcements of dating committees enable them to have not only more data but also more accurate data in the decision process, which mitigates the risk of wrongly announcing or missing a phase change. For these reasons, real-time assessments of dating algorithms may provide somewhat less accurate representations of the reference cycle than those of business cycle dating committees.

The aim of this paper is to survey some of the old and more recent statistical developments provided to automate the dating of business cycle turning points. Our contribution is close in spirit to the works reported by Proietti (2005), Hamilton (2011), Harding and Pagan (2016), and Romer and Romer (2020). However, our approach to dating the reference cycle complements the existing literature in several ways. First, our focus is to provide a practical guide to dating the reference cycle with the aim of filling the typical academic-practitioner gap. Our contribution does not intend to be exhaustive but to conduct a useful research survey on some of the most popular developments in dating the reference cycle. For this purpose, we try to identify the pros and cons and suggest ways of working with some of the existing dating methods.

Second, we develop several illustrative empirical applications and collect the data and codes that replicate the results, which are available from our websites. All of them include readme files that provide some practical guidance and facilitate maximum reuse. The codes are sometimes the direct byproduct of our own research while some others have been adapted from other researchers. In all cases, we have tried to ensure they achieve the five characteristics that Benureau and Rougier (2018) considered a scientific code should possess: they should be re-runnable, repeatable, reproducible, reusable, and replicable. We used three languages to write the codes: R, GAUSS and Matlab, although they can be easily translated to other languages.

Our survey divides the dating literature into two groups with different approaches to dating the business cycle from a set of coincident economic indicators. The first approach is known as the average-then-date method. The approach begins by computing a reference series of the aggregate economy, usually by averaging the indicators across the cross sectional dimension.[2] Then, the global turning points are dated on the aggregate indicator by using one of the business cycle dating models available in the literature.

In this context, there are two broad classifications of these dating methods. The nonparamertic procedures consist of algorithms that try to automatize the dating procedure according to the tradition of the NBER by looking at local maxima and minima in the reference series. The most popular approaches are the monthly algorithm proposed Bry and Boschan (1971) and its quarterly version advocated by Harding and Pagan (2002). On the other hand, the parametric procedures presume that the dynamics of the reference cycle is generated by a known function with unknown parameters, which are estimated from the data. The parametric dating methods transform the

[2] A trivial case is assuming that the reference series is simply one of the economic indicators.

data of the reference series into probabilities of recession, which are used to locate global regime changes by employing threshold rules. Examples are the threshold autoregressive (TAR) model (Tong, 1978), the self exciting threshold autoregressive (SETAR) model (Tong and Lim, 1980), the smooth transition autoregressive (STAR) model (Terasvirta, 1994) and the Markov-switching model (Hamilton, 1989).

The second approach discussed in this survey is the date-then-average procedure. It consists in dating the peaks and troughs in a set of coincident business cycle indicators separately, making assessments on the reference cycle itself in those periods where the individual turning points cohere. Although the literature has focused primarily on average-then-date methods, the date-then-average alternative has recently proved to be very successful in dating the turning points. Examples of date-then-average approaches used to provide assessments of the reference cycle turning point dates are Harding and Pagan (2006, 2016), Chauvet and Piger (2008), Stock and Watson (2010, 2014), and Camacho, Gadea and Loscos (2022).

This survey is organized as follows. Section 2 begins with a preliminary exposition and then describes the notation used throughout the text. Sections 3 and 4 provide a comprehensive survey of average-then-date and date-then-average methods. Section 5 presents empirical applications of the methods described in the survey. It includes a description of the challenges posed by the influential Covid data points. Section 6 concludes and indicates some lines of further research.

2 Preliminaries and notation

2.1 Definition of business cycles

Although it may seem a truism to emphasize that the reference cycle is not observable, the fact is that economists actually observe only tables of figures or charts that show the evolution of a set of selected economic indicators. To complicate the process of dating the reference cycle turning points even further, it is worth recalling that there is no single definition of recession and expansions or when they start and end.

The definition of recession commonly used in the media is two consecutive quarters of negative economic growth as measured by GDP. Despite the popularity of this definition of a so-called technical recession, it does not fully characterize a recession for several reasons. First, this rule does not always coincide with the recessions as determined by the business cycle dating committees. For example, the NBER recession of 2001 does not contain two consecutive falls of US GDP. Second, although this rule may help in dating the beginning of economic recessions, it does not identify their end. Third, GDP could decline by tiny amounts in two consecutive quarters without warranting the determination that a peak had occurred. Fourth, tracking the business cycle sometimes requires focusing on the monthly chronology while GDP is defined on a quarterly basis.

In a more complete definition of cycles, growth cycles are viewed as fluctuations in the deviations of an economic indicator around its generally rising trend. The standard practice, when considering growth cycles, is to discard long term trends, thus keeping only the fluctuations around the trend. This deviation-from-trend or cycle determines the recurrent phases of the cycle and may be used to date the turning points.

In the literature, the phases of the deviation cycle are defined according to two different criteria. The first criterion focuses on the gap between the actual value of the economic indicator and its trend growth. In this context, an expansion occurs when the gap is positive and a recession happens when the gap becomes negative. Thus, peaks and troughs alternate at dates at which the gap is zero. The second criterion, followed by the OECD, considers that the turning points occur when the cycle of the economic indicator reaches a local maximum (peak) or a local minimum (trough). In this case, growth cycle peaks (end of expansions) occur when the economic activity, measured by the economic indicator, is furthest above its trend level, whereas growth cycle troughs (end of recessions) occur when activity is furthest below its trend level.

The main drawback of the growth cycle approach is that the growth cycle characteristics vary widely across detrending methods. In an influential paper, Canova (1998) showed that determining growth cycles involves the controversial issue of detrending because different statistical representations for the trend embed different business cycle properties. In addition, Canova (1999) found that statements concerning the location of turning points of growth cycles are not independent of the statistical assumptions needed to extract trends.

The definition of business cycle used by dating committees, and that we pursue in this paper, is deliberately less precise.[3] Following the lines suggested by Burns and Mitchell (1946), the business cycle is a recurrent sequence of expansions and recessions, which is marked by the peaks and trough dates. A recession happens when there is a significant decline in economic activity spread across the whole economy and not confined to only one sector. By contrast, an expansion is the normal state of the economy and occurs after a recovery of economic activity following a recession.

Within this view of the business cycle, recessions begin with cascading drops observed in several coincident indicators, typically including output, employment, income, and sales. Usually, a compendium of initial disturbances to some sectors of the economy propagates from industry to industry and region to region, expanding the disturbances across the economy, driving the comovement among the coincident economic indicators and the persistence of the recession. Symmetrically, expansions begin with a recovery in some sectors that reverse the phase of the cycle, increasing output, income, employment and sales. The recovery leads to a sustained period of improving business activity.

2.2 A dual approach to business cycle dating

In this paper, we view the reference cycle as a partition of the time calendar into segments of recurrent expansions and recessions phases, whose break dates are determined by the business cycle turning points. In particular, the reference cycle breaks the time calendar through a sequence of K bivariate turning point dates, $\{\psi_1, \ldots, \psi_K\}$, that produces a partition of the sample calendar time period $\{1, \ldots, \mathbf{T}\}$ into a sequence of recurrent and non-overlapping segments of expansions and recessions.

Without loss of generality, we assume that the reference cycle begins with a peak and ends with a trough. Thus, the reference cycle is determined by the turning points $\psi = \{\psi_1^P, \psi_1^T, \ldots, \psi_K^P, \psi_K^T\}$,

[3] Sometimes, this concept is referred to as the classical approach to the business cycle.

which are unusually stacked in ascending order, where $1 < \psi_1^P$ and $\psi_K^T < \mathbf{T}$.[4] The turning points separate the time span into K periods of recession and $K+1$ periods of expansion. The first expansion covers the period from $t = 1$ to $t = \psi_1^P$, which is followed by the first recession that covers the period from $t = \psi_1^P + 1$ to $t = \psi_1^T$. Following this reasoning, the last recession occurs in the time period from $\psi_K^P + 1$ to ψ_K^T and the last expansion in the period from $\psi_K^T + 1$ to $\mathbf{T}$.

The turning point dates, ψ, are not observable. Thus, the interest in business cycle dating is to draw inference on peaks and troughs using observed economic indicators y_t, which could be univariate or multivariate. Some methods use the series of interest in levels or logs while others require using their stationary transformations, which, in case of unit root in the logarithms, implies using the growth rates.

The segmentation of the time calendar can also be viewed in the context of statistical classification by using a latent binary variable approach. If we label the expansions with 0 and the recessions with 1, the recurrent segments of expansions and recessions can also be labeled through a discrete indicator variable s_t taking values in the set $\{0, 1\}$ in the whole sequence of realizations, which are collected in $S = \{s_1, \ldots, s_{\mathbf{T}}\}$. Therefore, $s_t = 1$ indicates that t belongs to the i-th recession, which occurs whenever $\psi_i^P + 1 < t \leq \psi_i^T$ for some $i = 1, \ldots, K$. In the same vein, $s_t = 0$ indicates that the observation t belongs to an expansion, which occurs whenever $\psi_i^T + 1 < t \leq \psi_i^P$ for some $i = 1, \ldots, K$. The task of recovering the binary indicator $\{s_1, \ldots, s_{\mathbf{T}}\}$ from $\{y_1, \ldots, y_{\mathbf{T}}\}$ is called decoding.

3 Average-then-date

Burns and Mitchell (1946) defined the business cycle as "*expansions occurring at about the same time in many economic activities, followed by similarly general recessions*". Thus, they devoted special attention to the comovements among a set of coincident economic indicators over the business cycle, which implies that dating the reference cycle requires collecting a number of coincident indicators from which to determine the global change-points.

The average-then-date approach consists of summarizing the information of the coincident economic indicators in a single reference series, which captures the general state of the economy and from which the global turning points are determined. Although Burns and Mitchell (1946) were reluctant to use the average-then-date approach because they failed to find a satisfactory aggregate economic activity to set the business cycle reference dates, nowadays this is the most popular approach to dating the business cycle.

The reference series is usually associated to a composite index, which is computed from a set of coincident indicators whose specific cycles are aggregated into a single variable capturing the general state of the economy by using weighted averages. In some cases the weights are based on national accounts methods, such as those used for the calculation of GDP, and in some others the weights are estimated with statistical procedures obtaining a latent factor, which is interpreted as an overall economic activity indicator. Without being exhaustive, examples are Stock and Watson (1991),

[4] To facilitate exposition, we are assuming there is a first expansion ending at $\psi_1^P > 1$ and a last expansion starting at $\psi_K^T + 1 < \mathbf{T}$.

Diebold and Rudebusch (1996), Aruoba, Diebold and Scotti (2009), and Aruoba and Diebold (2010), who constructed the reference cycle as a weighted average of several key macroeconomic time series, where the weights are obtained from a single-index dynamic factor model.

We do not address in this survey the way in which the reference series is obtained from the set of coincident economic indicators. We focus instead on the methods used to date the reference cycle turning points from the reference series once the series is provided to the researcher. The methods, which are described below, are classified into non-parametric and parametric approaches.

3.1 Non-parametric approaches

Much of the non-parametric approaches to dating the reference cycle consist in methods using pattern recognition algorithms capable of detecting the local maxima (peaks) and minima (troughs) in a single series of interest. When this series is a reference series, the algorithms computes the reference cycle or global turning points whereas when the time series refers to a partial indicator, the output of the algorithms are specific turning points.

These approaches were born as an attempt to formalize and give statistical content to the informal and subjective procedure described by Burns and Mitchell (1946) to determine peaks and troughs in a time series.

3.1.1 Bry-Boshcan type algorithms

The first attempt to formalize the Burns-Mitchell approach to dating the reference cycle turning points was the dating algorithm developed by Bry and Boschan (BB, 1971). Accompanying the algorithm, the authors wrote a Fortram code, which was popularized by the conversion to Gauss advocated by Watson (1994), implementing the Bry-Boschan rules. The algorithm was initially developed for monthly data but was extended to quarterly frequency by Harding and Pagan (BBQ, 2002), and has been converted from Gauss to Matlab by several researchers, e.g. Inklaar (2003) who also adapted it to determine turning points in annual series. These codes and several modifications proposed since then have been used extensively by academics and practitioners to date turning points in economic time series.

A detailed discussion of the Bry-Boschan algorithm and its subsequent modifications lies outside the scope of this survey.[5] Essentially, the algorithm isolates local maxima and minima in a reference series (in level or logs), which are identified with the peaks and troughs that separate periods of expansions and recessions. The algorithm consists of several steps ensuring that the final set of turning points simultaneously satisfies the requirements that peaks and troughs alternate and that both phase and cycle have a minimum duration.

The steps involved in BB are summarized in Algorithm 1 in the Appendix. The algorithm starts with an initial filtering of the series before starting the process of pinpointing peaks and troughs. Depending on the nature of the time series, the filters include seasonal adjustment and, if the series is very noisy, extract trend-cycle components of the series, using bandpass filters or Tramo-Seats. Subsequently, the Spencer curve is used to remove outliers, which are defined as

[5] There are countless references in the literature describing the algorithm. Among them, we find Harding and Pagan (2016) one of the most useful.

values whose differences in absolute value from the 15-month Spencer curve are larger than 3 standard deviations.[6]

Then, the algorithm locates the local maximum and minimum in a m-period moving average of the outlier-free series of interest, y_t, where $m = 12, 4, 1$ for monthly, quarterly and yearly data, respectively. For a given symmetric window of size w, a potential peak at time t is identified if $y_t > y_s$ for $t - w < s < t$ and $t + w > s > t$ whereas a potential trough appears whenever $y_t > y_s$ for the same window. The choice of w must be sufficiently large to capture significant drops or increases in economic activity but it cannot be too low in order to avoid a large number of potential turning points. Typically, algorithms designed for monthly data adopt $w = 5$ months, but they are set to $m = 2$ quarters in the case of quarterly frequency and $w = 1$ year for yearly data.

The set of tentative turning points are subjected to censoring rules that ensure turning points alternation and phase and cycle length constraints. For contiguous peaks and troughs, the algorithm iteratively eliminates the lowest peaks and highest troughs that are adjacent to each other.

The following step consists of computing a new set of turning point dates from a 15-month centered Spencer curve, which is compared with the previous set of turning points. After an alternation check, the durations of a peak to peak or a trough to trough (a full cycle) are enforced to be at least 15 months.[7] If the durations are too short, the lower of two peaks or the higher of two troughs are eliminated. Now, a further refinement is conducted with the local maxima and minima of a short-term moving average of 3 to 6 months. These values depend on the Months for Cyclical Dominance (MCD), which is the minimum number of months over which the average rate of change in the Spencer curve exceeds the average change in the irregular component (difference between the original series and the Spencer curve).[8] Again, alternation is checked in the set of local maxima and minima.

The last step of the Bry-Boschan algorithm is the determination of turning points in the unsmoothed series. The local optima are achieved as the highest or lowest values within $\pm$ 4 months or MCD, whichever is larger. Then, the set of turning points is enforced to achieve a final set of constraints: (i) peaks and troughs must alternate; (ii) peaks and troughs within 6 months of the beginning and end of the series are eliminated; (iii) check if the peak-to-peak and the trough-to-trough cycles are less than 15 months ; and (iv) phases (peak to trough or trough to peak) whose duration is less than 5 months are eliminated.

Despite the apparent simplicity of the algorithm, some empirical applications could lead to misleading results.[9] The main reason is that the minimum phase and cycle length restrictions are heavily biased towards the characteristics of the US business cycle established by the NBER. Thus, users interested in non US applications need to adapt the hyperparameters that control the censoring rules to their reference series. Examples are Monch and Uhlig (2005), McKay and Reis (2008) and Gadea et al. (2012) who have included additional restrictions in accordance with the specific features of their data sets. In addition, the dating codes do not usually include amplitude

[6] The choice of the type of filtering and the degree of smoothing will depend on the type of time series characteristics of the reference series. For example, monthly series tend to exhibit more noise than quarterly series and, therefore, a higher degree of smoothing will be necessary to isolate the business cycle signals.

[7] In lower frequencies, the duration requirement is 4 quarters and 2 years.

[8] If it is less than 3 months, the MCD is set to 3, while it is set to 6 if its value is more than 6 months.

[9] In some cases, the Bry-Boschan routine is not able to find a set of turning points fulfilling all the constraints.

restrictions but they can be added easily, as in Gadea et al. (2012).

Taking this argument further, Harding and Pagan (2016) contend that the final result of non-parametric dating routines, although they usually constitute a good empirical approximation to the turning point dates determined by dating committees, should be subjected to expert judgment and compared to the known business cycle narrative. Berge and Jorda (2011) suggest that it is not enough to come up with a chronology of turning points determined by a business cycle indicator. They suggest assessing the quality of the chronology and propose tools for comparing the accuracy of various business cycle dating methods. Some statistical measures of classification accuracy, such as the those based on the Receiver Operating Characteristic (ROC) or the Brier score, could be of considerable use in this context.

3.1.2 Other non-parametric approaches

Although BB is, by far, the best known and most widely used dating algorithm, there are other non-parametric methods of business cycle dating. The simplest one consists of accounting for the change in the sign of growth rates of the reference series. If we denote the growth rates in the reference series as Δy_t, we say that there is a peak in t when $\Delta y_t > 0$ and $\Delta y_{t+1} < 0$ whereas there is a trough in t and when $\Delta y_t < 0$ and $\Delta y_{t+1} > 0$. The problem of this method is that it will typically generate many more estimated peaks and troughs than the true reference cycle turning points or the set of turning points dated by a dating committee.

For its simplicity and adaptability, the dating algorithm developed by Dupraz et al. (2021), which is based on the "plucking model" advocated by Friedman (1993), deserves some attention in this context. Although this algorithm was designed for unemployment, we adapt it in this survey to a procyclical reference series. In addition, we describe the algorithm assuming that the first turning point in the time series is a peak, although it can easily be modified if the first turning point is a trough.

The algorithm, which is summarized in Algorithm 2 in the Appendix, begins by assuming that the first observation is a candidate for a peak $cp = 1$. If, in all the subsequent months until the series becomes δ percentage points lower than y_{cp}, the series is actually lower than y_{cp}, then we confirm that $cp = 1$ is a peak. If, instead, the series becomes greater than y_{cp} before it is confirmed as a peak, the month in which this happens, t^*, becomes the new candidate peak, $cp = t^*$ and the process is repeated. Once we have identified a peak at a date p_1, we add this date to the set of peaks and set $ct = p_1 + 1$ as the first candidate to be a trough.

If, in all the subsequent months until the series becomes δ percentage points greater than y_{ct}, then we confirm that ct is a trough. If, instead, the series becomes lower than y_{ct} before it is confirmed as a trough, the month in which this happens, t^{**}, becomes the new candidate trough, $ct = t^{**}$ and the process is repated. Once we have identified a trough at a date t_1, we add this date to the set of troughs and set $cp = t_1 + 1$ as the second candidate to be a trough. The process is iterated until the last observation.

It is obvious that the main advantage of this procedure over BB-type algorithms is its simplicity. Despite its lack of sophistication, we show in the empirical section that the procedure is able to replicate NBER decisions with reasonable accuracy. However, its drawback is that the output of the algorithm is highly dependent on the choice of an appropriate δ.

In a different setting, one notable contribution to the theory of classification is the hierarchical factor segmentation (HFS) algorithm introduced by Fushing et al. (2006), which has been used for dating the reference cycle by, for example, Fushing et al. (2010) and Berge and Jorda (2013). The procedure exploits the recurrence distribution of separating events, but does not provide a joint parametric specification of the stochastic process of the series. In short, the non-parametric decoding algorithm partitions the time series y_t into segments where the intensity of the recurrence distribution in adjacent segments differs significantly.

The resulting business cycle indicator s_t is obtained in several stages, which are summarized in Algorithm 3 in the Appendix. First, the algorithm performs an initial separation of the sample into expansions and recessions using a percentile of the distribution of the stationary transformation of y_t (in case of unit root, this implies using the growth rates). For example, in Fushing et al. (2010) the recessionary indicator $s_t^0 = 1$ when $y_t < h$ and $s_t^0 = 1$ otherwise, where $n_1 = \sum s_t^0$. They suggest using the 30-40th percentile distribution of y_t.

Second, the algorithm finds the dates of the regime switches that meet certain inter-event spacing restrictions. For this purpose, it computes τ^1 as the event-time of the occurrence of the events $y_t < h$, with $\tau_k^1 = \{t|s_t = 1\}$, for $k = 1, \ldots, n_1$, and denote the sequence of recurrence time (inter-event spacing) as $R^1 = \{\tau_k^1 - \tau_{k-1}^1\}_{k=1}^{n_1}$. Now, let q_1 be q_1^{th} -percentile of the empirical distribution of R^1 and choose the segments whose distance is above a certain percentile of this empirical distance distribution. This is achieved by transforming the R^1 sequence into a 0-1 digital string C^1 such that $C_k^1 = 1$ if $R_k^1 > q_1$ and $C_k^1 = 0$ otherwise, for $k = 1, \ldots, n_1$.

Third, upon code sequence C^1, the algorithm takes code word 1 as an event and construct its corresponding event-time τ^2, with $\tau_k^2 = \{t|C_t^1 = 1\}$ and denote the sequence of recurrence time as $R^2 = \{\tau_k^2 - \tau_{k-1}^2\}_{k=1}^{n_2}$, where $n_2 = \sum C_t^1$. Again, let q_2 be q_2^{th}-percentile of the empirical distribution of R^2 and transform the R^2 sequence into a 0-1 digital string C^2 such that $C_k^2 = 1$ if $R_k^2 > q_2$ and $C_k^2 = 0$ otherwise, for $k = 1, \ldots, n_2$.

The resultant code sequence C^2 partitions the time span into clusters where the observed frequency of $y_t < h$ is high and clusters where this frequency is low. The sequence of break dates $\{\hat{\psi}_i^P, \hat{\psi}_i^T\}$ provides the dates of the segments that refer to recessions.

The authors find the optimal threshold parameters q_1 and q_2 as the maximum likelihood estimates of a likelihood function that is based on the observed recurrence times within the two classes of segments. As we will show in the empirical examples, the algorithm provides reasonable partitions of the time span into expansions and recessions. However, the accuracy of the results could depend on the appropriate choice of h.

3.2 Parametric methods

We consider that a dating procedure relies on a parametric approach when the approach imposes an a priori structure on the underlying reference series. This structure relies on a function with a parametric form and an unknown set of parameters that control the reference series dynamics and need to be estimated from the data.

The risk of dating with parametric models is that inference could be based upon an incorrectly specified parametric model. In addition, the reliability of the approaches relies dramatically on the existence of data irregularities such as missing data or influence observations. As we will show in the empirical applications, this issue is a challenge nowadays due to the large figures observed in macroeconomic indicators during the Covid pandemic.

3.2.1 Piecewise autoregressive models

Assume that the interest examining the business cycle dynamics of a time series lies in having some measure of the overall economic development, y_t.[10] It is reasonable to think that the performance of this time series is heterogeneous across and homogeneous within the two business cycle regimes. To handle this nonlinear pattern, piecewise autoregressive models arise as natural extensions of linear autoregressive models. These models allow the autoregressive parameters and the error variances to undergo changes at time points determined by the turning point dates while staying constant between adjacent turning points.

It is reasonable to think that y_t has a different probability distribution in each business cycle regime allowing for the parameters changing in the model according to the integer-valued state variable s_t

$$y_t = c^0_{s_t} + c^1_{s_t} y_{t-1} + \cdots + c^p_{s_t} y_{t-p} + \epsilon_t,\ \epsilon_t \sim N(0, \sigma^2_{s_t}) \tag{1}$$

where the error term is serially independent.[11] In this case, the piecewise autoregressive model becomes

$$y_t = \begin{cases} c^0_0 + c^1_0 y_{t-1} + \cdots + c^p_0 y_{t-p} + \epsilon_t,\ \epsilon_t \sim N(0, \sigma^2_0) & \text{if } s_t = 0 \\ c^0_1 + c^1_1 y_{t-1} + \cdots + c^p_1 y_{t-p} + \epsilon_t,\ \epsilon_t \sim N(0, \sigma^2_1) & \text{if } s_t = 1, \end{cases} \tag{2}$$

and model's parameters undergo occasional shifts according to changes in the value of s_t. If we denote $\varphi_t = (y_1, ..., y_t)$, we can define the conditional density of y_t as $f(y_t|s_t, \varphi_{t-1})$, which is commonly Gaussian.

In practice, the regimes are never explicitly observed, which implies that both the outcome of the state variable, s_t, and the estimates of the model parameters must be inferred from the data. In the context of time-series analysis, the problem of regime identification is a particular case of a classification problem. This approach consists of computing a probability statement that the economy is in an expansion ($s_t = 0$) or a recession ($s_t = 1$) at any point in time through the so-called state probabilities.

3.2.2 TAR and STAR models

The simplest model in business cycle classification from piecewise autoregressive models is the threshold autoregressive (TAR) model, first proposed by Tong (1978). The main idea of the TAR model is to describe a given stochastic process by a piecewise linear autoregressive model, where the determination of whether each of the models is active or not depends on the value of a known

[10] In the context of autoregressive models, the focus is on the stationary version of the time series. Typically, this implies using the rates of growth in the measure of economic activity, such as Gross Domestic Product or Industrial Production.

[11] Conditional on the regime, the related literature typically assumes Gaussian densities.

threshold variable z_t. Thus, if we denote the information available at t as $\varphi_t^* = (y_1, ..., y_t, z_1, ..., z_t)$ and the probability of the state variable conditional on this information as $P(s_t|\varphi_t^*)$, a TAR model considers that there is an expansion at t, $P(s_t = 0|\varphi_t^*) = 1$, when $z_t > \kappa$, whereas there is a recession at t, $P(s_t = 1|\varphi_t^*) = 1$, when $z_t \leq \kappa$.

An important case is the self exciting threshold autoregressive (SETAR) model, introduced by Tong and Lim (1980), where the switches from one regime to another depend on the past values of the dependent variable, $z_t = y_{t-d}$. The scalar d is known as the delay parameter and κ is the length of the threshold. Applied to business cycle analysis from growth rate of the GNP, Potter (1995) proposes a SETAR model with $d = 2$ and $\kappa = 0$, which simplifies the four-regime model specification previously suggested by Tiao and Tsay (1991).

In this context, Chan and Tong (1986) argued that, instead of having an abrupt shift from one regime to the other, one could make the transition smooth. Their extension was approximating the switch by a using a smooth transition function based on the cumulative distribution function of the standard normal variable, G. Thus, the so-called smooth transition autoregressive (STAR) model determines the probability of recession as $P(s_t = 1|\varphi_t) = G(y_{t-d})$. In a popular variant of STAR models, Terasvirta (1994) suggested a logistic function

$$G_L(y_{t-d}) = (1 + exp(-\gamma(y_{t-d} - c)))^{-1}, \tag{3}$$

and called the proposal the Logistic-STAR (LSTAR) model.[12] In this model, $\gamma > 0$ controls the speed of the regime switches and, as γ approaches infinity, G_L converges to the Heaviside function and LSTAR tends to SETAR.

Applied to economic activity data, LSTAR models have a nice business cycle interpretation. In large contractions, the measure of economic activity that acts as the transition variable is sufficiently lower than the threshold for keeping the transition function close to zero. Thus, the probability of recession can be approximated by $P(s_t = 1|\varphi_t) = 1 - G_L(y_{t-d})$. Within a LSTAR framework, Terasvirta and Anderson (1992) examined the nonlinearity of business cycles of 13 OECD countries using their industrial production indices.

3.2.3 The Markov-switching approach

In the Markov-switching model advocated by Hamilton (1989), the changes between regimes do not follow a logistic function, which depends upon observable variables. In contrast, the indicator variable s_t is assumed to evolve according to a latent 2-state first order stationary and ergodic Markov Chain. Thus, the probability that s_t equals some particular value j conditional on the information set available at $t-1$ depends on the past only through the most recent value s_{t-1}

$$p(s_t = j|s_{t-1} = i, s_{t-2} = k, ..., \varphi_{t-1}) = p(s_t = j|s_{t-1} = i), \tag{4}$$

[12] Terasvirta (1994) also suggested exponential functions, but they are not appropriate in the context of business cycles.

which is abbreviated as p_{ij}. The collection of transition probabilities in a transition matrix characterizes the properties of the Markov process.[13]

Hamilton (1989) proposed an iterative algorithm that uses repeated predicting and updating procedures similar in spirit to a Kalman filter in which, given $P(s_t|\varphi_{t-1})$, the so-called filtered probabilities of recession are obtained using Bayes's Law as

$$P(s_t = 1|\varphi_t) = \frac{P(s_t = 1|\varphi_{t-1})f(y_t|s_t = 1, \varphi_{t-1})}{f(y_t|\varphi_{t-1})}, \tag{5}$$

where $f(y_t|\varphi_{t-1}) = P(s_t = 0|\varphi_{t-1})f(y_t|s_t = 0, \varphi_{t-1}) + P(s_t = 1|\varphi_{t-1})f(y_t|s_t = 1, \varphi_{t-1})$. Hamilton (1989) also suggested a backward-recursion filter to compute the smoothing probabilities, $P(s_t = 1|\varphi_{\mathbf{T}})$, which are based on the full-sample information.

There have been numerous applications of the Markov switching model to make inferences about the business cycles at worldwide level (Camacho and Martinez-Marin, 2015), country level (McConnell and Perez-Quiros), state level (Owyang et al, 2005) and city level (Owyang et al, 2008).

3.2.4 Business cycle classification

The parametric models used to handle business cycle dynamics are not in themselves dating rules. In order to establish a chronology of the business cycle dates, one first needs to translate the state probabilities that result from the classification approaches into a zero-one indicator of the state of the economy at any particular time, which would implicitly identify the historical turning points as the phase change dates. Many rules have appeared in the literature to convert recession probabilities into a business cycle dummy.

The simplest decision rule, used by Hamilton (1989), is based on whether the economy is more likely than not to be in a recession. This implies considering t as a recession whenever $P(s_t = 1|\varphi_t) > 0.5$, with the phase changes occurring when the probability of a recession crosses the 0.5 level. Thus, a date τ is designated a peak if $P(s_\tau = 1|\varphi_\tau) < 0.5$ and $P(s_{\tau+1} = 1|\varphi_{\tau+1}) > 0.5$. Likewise, a date τ is designated a peak if $P(s_\tau = 1|\varphi_\tau) > 0.5$ and $P(s_{\tau+1} = 1|\varphi_{\tau+1}) < 0.5$.

In the case of monthly data, Chauvet and Piger (2008) suggested a more conservative two-step approach. The authors identify a new period of recession when, at month τ, the conditions $P(s_{\tau-1} = 1|\varphi_{\mathbf{T}}) < 0.8$ and $P(s_{\tau+k} = 1|\varphi_{\mathbf{T}}) > 0.8$, for $k = 0$ to 2, hold. Then, they find the smallest value of q for which $P(s_{\tau-q-1} = 1|\varphi_{\mathbf{T}}) < 0.5$ and $P(s_{\tau-q} = 1|\varphi_{\mathbf{T}}) \geq 0.5$. Finally, the peak of this recession is the last month of the previous expansion phase, or month $\tau - q - 1$. Analogously, the business cycle troughs are dated using a 20% decision rule.

3.2.5 Other approaches

The parametric approaches described in this section belong to the family of unsupervised classifiers which endogenously determine the alternating periods of expansion and recessions without the need for training the approaches in a labeled subsample to learn where the reference cycle is assumed to be known.

[13] Extensions of the baseline model allow the transition probabilities to depend on exogenous variables (Filardo, 1994), and permit three regimes (Boldin, 1996).

Although not pursued in this survey, there is also a long tradition of using supervised classifiers to make business cycle inferences. Such approaches are considered as supervised classifiers because they require learning from labeled data, which typically assume the business cycle dating committees' chronologies as the reference cycle. Much of the supervised approaches used for business cycle analysis follow the lines suggested by Estrella and Mishkin (1998) and use limited dependent variable models, such as a logit or probit.

4 Date-then-average

In contrast to the average-then-date approach, the date-then-average procedure to date the reference cycle consists of dating the turning points from the set of N coincident indicators separately and then making an assessment on the reference cycle itself.

Burns and Mitchell (1946) were the first in proposing a date-then-average procedure to provide a dating of the US reference cycle. They identify the specific turning points from a list of forty economic indicators as movements of rise and fall, using a combination of a duration rule and a minimum amplitude rule.[14] With the series-specific turning points, they date business cycle reference dates by marking off the zone within which a succession of these series reaches the specific turning points and choosing the date of their central tendency as the reference date.

It is worth emphasizing that the date-then-average procedure of Burns and Mitchell (1946) is full of judgmental decisions. In an attempt to automate the process, some algorithms have recently been proposed in the literature. This section describes some of these methods and lists some of their pros and cons.

Before using the aggregating algorithms described in this section, it is worth mentioning that one needs to first extract the K turning point dates for each of the individual time series, $\{\hat{\psi}^P_{1j}, \hat{\psi}^T_{1j}, \ldots, \hat{\psi}^P_{Kj}, \hat{\psi}^T_{Kj}\}_{j=1}^N$. We suggest using the non-parametric dating algorithms described above.

4.1 Harding and Pagan (2006, 2016)

An interesting proposal is put forward by Harding and Pagan (2006, 2016) who obtain the reference cycle by combining specific turning points. These authors consider individual turning points as sample realizations of a (relatively small) number of economic indicators and propose a nonparametric algorithm to codify the procedures used to aggregate the specific turning points to obtain economy-wide turning points.

Let us focus on peaks and consider t as a point in time that is a candidate for the k-th economy-wide turning point. Thus, we should find around t a set of specific peaks of the jth series that will lie close to it, $\hat{\psi}^P_{kj}$. Let $d_{tj} = |\hat{\psi}^P_{kj-t}|$ be the distance between the specific peak and the candidate for of reference peak. This produces a collection of distances, which is summarized by the median, d_t as the central tendency measure. Then, we compute the median distances for a set of $2k+1$ candidate peak dates, which produces a collection of median distances $d_{t-k}, \ldots, d_{t+k}$.[15] If the series

[14] If there are multiple peaks or troughs, the authors date the turning point at the latest extreme.

[15] Chauvet and Piger (2008) considered a 31-month window centered at time t, that is, from $t-15$ to $t+15$.

of distances has a local minimum at $\hat{\psi}_k^P$ then that is the peak for the reference cycle.

Despite the simplicity of the method, Harding and Pagan (2006) and Chauvet and Piger (2008) showed that it produces turning point dates that are in high concordance with those of the NBER dating committee. However, it requires some specific restrictions because the minimum value of the sequence of distances does not need to be unique and may occur at a range of values between $t-k$ and $t+k$. To overcome this drawback, Harding and Pagan (2006) propose using higher percentiles than the median until a unique local minimum is found, whereas Harding and Pagan (2016) propose choosing the value of t with the smallest value of dispersion, measured as $\sum_{j=1}^{N}(d_{tj})^2$.

4.2 Stock and Watson (2010,2014)

Stock and Watson (2010) propose a new dating method to date the reference cycle from a set of specific turning points obtained from N disaggregated time series. As the authors treat peaks and troughs in a similar way, we will focus on a set of specific peaks $\{\hat{\psi}_{1i}^P, \ldots, \hat{\psi}_{Ki}^P\}_{i=1}^N$, which are aggregated to obtain estimates of the reference cycle peaks $(\hat{\psi}_1^P, \ldots, \hat{\psi}_K^P)$.

Conditioning on the knowledge of the K non-overlapping episodes of the reference cycle and assuming that each indicator has a mean lead/lag relative to the reference cycle of L_i^P, estimating the dates of the reference turning points can be stated as a standard panel data structure. In particular, the panel data model is defined as

$$\hat{\psi}_{ik}^P = \psi_k^P + L_i^P + \eta_{ik}^P, \tag{6}$$

where ψ_k^P is the k-th reference cycle turning point, η_{ik} is the deviation of the specific cycle from the reference cycle, $i = 1, \ldots, N$ and $k = 1, \ldots, K$. To define the individual dimension k, the authors rely on the business cycle phases proposed by the NBER.

This fixed-effect panel data model requires handling unbalanced panels because each business cycle episode contains a different number of specific turning points and missing observations because some series might not have turning points in a given reference peak or trough. The authors propose estimating the panel data model by ordinary least squares to obtain estimates of the reference cycle peaks $\{\hat{\psi}_1^P, \ldots, \hat{\psi}_K^P\}$. The analysis of troughs is developed in the same way.

In a separate proposal, Stock and Watson (2014) propose an alternative nonparametric approach to dating the reference cycle from a set of coincident indicators. These authors assume that each turning point of the reference cycle is a local measure of the central tendency of the population distribution of the set of disaggregated turning points, conditional on the turning point having occurred.

In practice, they recommend using the mode of the kernel density of the sample of individual turning point dates, conditional on the occurrence of a single phase shift in a given episode k covering a known time interval, $\{\hat{\psi}_{1k}^P, \ldots, \hat{\psi}_{n_k K}^P\}$, where n_k is the number of specific turning points in this interval. In their application to dating the US turning points from monthly indicators, the authors define the intervals of the different turning points as the NBER turning point dates $\pm$ 12 months.

One significant contribution of these two procedures is that, along with the estimates of the reference cycle turning point dates, they also produce standard errors and confidence intervals for the estimates of the reference cycle turning points. In addition, the methods are easily implemented for large data sets. However, one important limitation of both approaches is that they require knowing the turning point dates that they are trying to estimate.

4.3 Camacho, Gadea, Loscos (2022)

Camacho, Gadea and Loscos (2022) contribute to this literature by estimating the reference cycle turning points from a multiple change-point model with an unknown number of K breaks. In this approach the estimates of the reference cycle are not conditional on the known occurrence of the phase shifts. Thus, the number of business cycle phases and the dates of the turning points are estimated from the data in a single step.

If the reference cycle phases were known as in Stock and Watson (2010, 2014) the binary business cycle indicator, S, would be known. In this case, Camacho, Gadea and Loscos (2022) assume that each individual pair of peak and trough dates, ψ_i, is a realization of a bivariate Gaussian density. The mean of this distribution, $\psi_k = (\psi_k^P, \psi_k^T)$, is the reference cycle peak-trough vector and Σ_k is its covariance matrix

$$\psi_i | s_i = k \sim N(\psi_k, \Sigma_k). \tag{7}$$

However, in most empirical applications, the state is unknown. To overcome this issue, the individual pair of peak and trough dates is viewed as a realization of a mixture of K of separate bivariate Gaussians and the transition between the mixture components is governed by an unobservable first-order K-state Markov chain. Let $\Psi_{k-1} = \{\psi_1^P, \psi_1^T, \ldots, \psi_{k-1}^P, \psi_{k-1}^T\}$ be the past turning point dates and $P(s_t = k|\Psi_{k-1})$ the probability of state k conditioned on these dates. The mixture becomes

$$\psi_i \sim \sum_{k=1}^{K} P(s_t = k|\Psi_{k-1}) N(\psi_k, \Sigma_k). \tag{8}$$

The transition probabilities are constrained to reflect the one-step ahead dynamics of a multiple change-point specification. In particular, the transition probabilities are restricted as follows

$$p(s_i = k|s_{i-1} = l) = \begin{cases} p_{ll} & \text{if } k = l \neq M \\ 1 - p_{ll} & \text{if } k = l + 1 \\ 1 & \text{if } l = K \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

This parameterization implies that when the process reaches one regime, for example regime l, it remains in this regime with probability p_{ll} or moves to regime $l+1$ with a probability $1 - p_{ll}$. The process starts at regime 1 and moves forward to the next regime until it reaches regime K in which the process stays permanently.

The authors estimate the model parameters and compute inference about unobserved state variable using a Markov Chain Monte Carlo (MCMC) method. To select the number of components in the mixture, they follow two approaches. First, they apply AIC, BIC and Bayes factor sequential comparisons as in Kass and Raftery (1995). Second, they take the number of regimes as random whose duration follows a Poisson distribution as in Koop and Potter (2007). As in Stock and Watson (2010, 2014), standard errors and confidence bands of the estimated turning point dates are obtained from the model.

5 Empirical examples

The accuracy of the methods used to estimate the reference cycle turning points will ultimately be a matter of practice. Thus, the interest of this section is to provide empirical applications that try to investigate how well the methods that we describe in this survey are able to replicate the decisions of the NBER Business Cycle Dating Committee, with special attention paid to comparing their strengths and weaknesses.

5.1 Univariate non-parametric methods

In this study, we use the nonparametric dating method proposed by Bry and Boschan (1971), along with its quarterly (Harding and Pagan, 2002) and annual (Inklaar, 2003) versions. To simplify notation, the results of these three alternatives are labeled as BB. In addition, we also date the US turning points with the method proposed by Dupraz et al. (DNS, 2021) and with the hierarchical factor segmentation (HFS) advocated by Fushing et al. (2006).

Depending on the frequency we are considering, we use three different measures of the US aggregate economic activity. For monthly frequency, we use the seasonally adjusted total index of Industrial Production (IP) for the sample 1919.01 to 2022.06. In the case of quarterly frequency, the aggregate measure of economic activity is the seasonally adjusted Real Gross Domestic Product (billions of chained 2012 dollars) from 1947.1 to 2022.2. For annual data, we use Real Gross Domestic Product (billions of chained 2012 dollars) from 1929 to 2021.

Figure 1 provides a comparative assessment of the business cycle dating performance for the three alternative non-parametric dating methods. In columns, the figure shows the monthly, quarterly and annual measures of economic activity. In rows, the figure plots the recession periods identified by each of the the three dating methods, which are represented by shaded areas.

A glance at the figures leads to the conclusion that all methods reproduce the US business cycle periods quite reliably. As expected, the recessionary periods identified by the algorithms coincide with reductions in the measures of economic activity. To investigate the extent to which the measures of economic activity and the states of the economy identified by the dating algorithms cohere, we propose using the Area Under the Receiver Operating Characteristic curve (AUROC).

For each threshold value c we can define a binary prediction recession whenever $y_t < c$ and expansion whenever $y_t \geq c$. Let S_j represents the classification of states generated by method j, with $j = BB, DNS, HFS$. Thus, we can define true positives $TP(c)$ sensitivity) and false positives

$FP(c)$ (1-specificity) using the following conditional probabilities

$$TP(c) = P[y_t < c | s_{jt} = 1] \quad (10)$$

$$FP(c) = P[y_t \geq c | s_{j,t} = 0] \quad (11)$$

where s_{jt} defines the cyclical state of the economy at t, as a result of S_j.

The ROC curve plots the entire set of possible combinations of $TP(c)$ and $FP(c)$ $c \in (-\infty, \infty)$. When $c \to \infty$, $TP(c) = FP(c) = 0$ and, conversely, when when $c \to -\infty$, $TP(c) = FP(c) = 1$. Therefore, the ROC curve is an increasing function in $[0,1]x[0,1]$ space. If y_t is an uninformative classifier of the underlying state of the economy s_{jt}, then $TP(c) = FP(c)$ $\forall c$, and the ROC curve would be the 45-degree line, a natural benchmark with which to compare classifiers. On the other hand, if y_t is a perfect classifier of the business cycle s_{jt}, then the ROC curve will hug the north-west border of the positive unit quadrant.

From this curve it is possible to define a scalar measure, known as AUROC, which represents the area under the curve. This quantity takes values between 0.5 for a random classifier and 1 for a perfect classifier. Thus, AUROC serves as a basis from which to compare the ability of the aggregate economic activity measures to perform accurate classifications of the three business cycles S_j.

Table 2 shows that, regardless of the frequency and the dating method, the values of AUROC are very high, which implies that the measures of the aggregate economy are good classifiers of the business cycle dated by the three non-parametric approaches. For the monthly frequency, industrial production is a slightly better classifier of the business cycles identified by DNS and HFS. For the quarterly frequency BB and DNS tend to outperform HFS.

However, one important question remains: how accurate is the chronology of recessions dated by the nonparametric algorithms? One way to assess such relative accuracy is to compare the outcome of the three methods with that of the NBER-referenced business cycle chronology. To this end, we use the the concordance index proposed by Harding and Pagan (2002). The index measures the proportion of time that the estimated reference cycles and the NBER-referenced cycle are in the same phase, with a value of 1 implying that the two dating methods are in the same phase 100% of the time. In particular, if S_{NBER} is the official chronology, this expression becomes:

$$IC_{j,NBER} = T^{-1}\{\sum_{t=1}^{T} S_{j,t}S_{NBER,t} + (1 - S_{j,t})(1 - S_{NBER,t})\}, \quad (12)$$

where $j = BB, DNS, HFS$.

The results presented in the Table 2 show that all methods have a high classification ability, although BB has a slight advantage. In order to check the impact of the Covid data on the models performance, Table 2 displays the concordance index in shorter samples that start in 2010. According to the figures in the table, the pandemic data had little effect on the dating processes, although BB and DNS date the pandemic peak at a monthly frequency by the end of 2019 because industrial production started to fall before the pandemic restrictions took place.

Summing up, the non-parametric methods work well to produce adequate classifications of the

US economy into periods of expansion and recession. As they focus on picking local maxima and minima, they are robust to breaks in volatility or large values such as those of the past financial crisis or those caused by the restrictions occurred during the Covid pandemic. On the downside, however, these methods tend to depend on the appropriate choice of certain parameters, especially DNS.

5.2 Univariate parametric methods

In this section, we focus on the most popular parametric method used to compute business cycle inferences: the Markov Switching (MS) approach developed by Hamilton (1989). The method requires a stationary transformation of the series of aggregate activity, so we rely on the rates of growth of IP and GDP.

Specifically, we propose a Markov-switching specification that follows the lines suggested by Camacho and Perez-Quiros (2007), who show that a model that decomposes the growth rates of economic activity into a state-dependent mean and a stationary process captures the US business cycle dynamics with high precision. In this case, the model becomes

$$y_t = c_{s_t} + \epsilon_t \tag{13}$$

where the errors are serially independent and $\epsilon_t \sim N(0, \sigma)$.

Let us focus on GDP growth rates as the measure of economic activity. The maximum likelihood estimates, which are displayed in Table 3, show that if we do not include the Covid data, the expected growth rates are -0.43% in recessions and 0.96% in expansions. According to the estimated transition probabilities, expansions are highly permanent because the estimate of p_{00} is above 0.9, while recessions are less permanent because the estimats of p_{11} falls to about 0.7. Conditional on being in a given state, the expected duration of a typical US recession is 3.14 quarters, while the expected duration of an expansion is 20 quarters, which consistent with the historical average duration of the NBER recessions and expansions.[16]

Figure 2 displays the filtered probability of being in the negative growth state that comes from the Markov-switching model together with the NBER chronology. The probabilities provide a clear classification of the time span as they become high at about the NBER peaks and remain high until the NBER troughs, where the probabilities fall drastically. Thus, the figure shows that the probabilities of recession are in close agreement with the NBER-referenced cycles, although they are not able to identify the 2001 recession clearly. If we convert the probability series into a dichotomous variable with $s_t = 1$ when the probability of recession is higher than 0.5, the concordance index would be of 0.86.

However, this nice picture changes dramatically when the sample is enlarged to include the Covid data. In this case, although the within-expansion growth is almost unaltered (0.8%), the recessions are characterized by an unprecedented negative growth of -9.4%. This change is due to the very large drop in GDP growth documented in 2020.2. Figure 2 reveals that the 2020.2 figure is so influential that the model relegates all the previous recessions to the high-growth state.

[16] Conditional on being in state j, the expected duration is $(1 - p_{jj})^{-1}$.

Regarding monthly IP, the restricted sample shows that the within-expansion growth rates are 0.48% and the within-recession growth rates are -4.2%. Again, expansions are more permanent than recessions, with their expected duration of 77 and 4 months, respectively. This implies that the expected duration of recessions is much shorter than the historical average duration of the NBER recessions.

Figure 3 shows that, although the MS model is able to produce high filtered probabilities of the negative growth regime in some of the NBER-referenced recessions, the model misses much of the most recent recessions. This happens because the model is considerably influenced by the large drops in IP at the beginning of the sample. For this reason, neither the estimates of Table 3 nor the probabilities of recession of Figure 3 are greatly altered when the sample is enlarged to include the pandemic data.

To summarize, in the absence of outliers or breaks, parametric dating methods are able to produce business cycle inferences that are in close agreement which the NBER-referenced business cycles. However, structural changes or large outliers affect parametric models more than non-parametric models and tend to distort their dating outcomes.

5.3 Multivariate approach

In this section, we show a multivariate dating procedure updating the average-then-date method proposed by Camacho et al. (2022), whose empirical exercise ended in 2010.

The set of coincident economic indicators that these authors used to obtain the specific turning points followed the lines suggested by the NBER memorandum explaining the June 2009 trough. In particular, the dating committee paid attention to ten monthly indicators.[17] These indicators are a measure of monthly GDP that has been developed by the private forecasting firm Macroeconomic Advisers, three measures of monthly GDP and GDI that have been developed by some of the members of the committee, real manufacturing and trade sales, industrial production, real personal income excluding transfers, the payroll and household measures of total employment, and an aggregate of hours of work in the total economy. The largest sample spans the period from January 1959 to August 2010, during which there were eight complete NBER-referenced business cycles. The collection of specific turning points for this sample is obtained by using BB in each time series.

Unfortunately, the members of the committee have not updated the measures of monthly GDP and GDI. For this reason, we updated only the monthly series and extended the list of monthly economic indicators with real personal consumption expenditures.[18] In line with the results that we obtained in the application of the non-parametric dating approach, we complete the collection of specific turning points up to July 2022 by using HFS.

[17] An Excel file with the ten time series is available from the NBER website at

mirror.nber.org/cycles/BCDCFiguresData100920 ver5.xls

[18] The latest list of monthly indicators that the NBER consult is available at

https://www.nber.org/business-cycle-dating-procedure-frequently-asked-questions

Figure 4 provides a preliminary view of the individual chronologies of turning points. The figure displays the kernel density of the turning points, which exhibits several modes that cluster the turning points around the periods of NBER-referenced peaks and troughs. The method assumes that each of the specific points is generated by a mixture of K Gaussian densities, whose means are the reference cycle turning points. The transition between the components of the mixture is restricted to force a left-to-right transition dynamic.

As a first approximation to the determination of the number of clusters, Figure 4 suggests that the tentative number of components of the mixture, which refer to the distinct local maxima of the kernel distribution, could be between seven and ten. To formally determine the number of clusters, Table 4 shows that the local minima of AIC and BIC are achieved for nine clusters. The sequence of Bayes factors (twice their logs) also points to 9 different clusters of turning points given that the Bayes factor that establishes the comparison of the model with 8 and the model with 9 clusters favors the extra cluster while the comparison of 9 versus 10 clusters does not suggest adding an extra cluster. Although this result does not require prior knowledge of the number of clusters, it is worth noting that the NBER also establishes a set of 9 pairs of peak and trough dates.

Table 5 shows the results of evaluating the multivariate date-then-average proposal in terms of its ability to capture the turning point dates established by the NBER business cycle dating committee. The columns labeled NBER show the official turning points. The following two columns display the means of the components of the mixtures, which are estimated using the posterior distributions obtained with the Gibbs sampler algorithm. The last two columns show the deviation in months between the NBER and the estimated turning points.

The table reveals that the method provides very precise estimates of the NBER-referenced dates and supports the view that the economic indicators tend to provide accurate signals of the business cycle turning points. In particular, the estimated turning point dates deviate from the NBER dates only by a maximum of 3 months in the case of peaks and 4 months in the case of troughs.

6 Concluding remarks

Awareness of the benefits of dating the reference cycle and the costs of the delays in detecting turning point changes has recently led to the publication of an increasing number of theoretical and empirical studies proposing various alternatives for establishing a set of reference dates that mark the phases or states of the economy. The aim of this survey is to link together theoretical and computational problems of some of these business cycle dating procedures.

Our results suggest that none of the methods can be applied by researchers with their eyes closed. Researchers must actively participate in the selection of relevant parameters, be aware of circumstances that may distort the computation and critically analyze the dating results in terms of finding agreement with the widely recognized historical business cycle phases.

The economic shock caused by Covid has been a challenge for all macroeconomic and econometric models, including business cycle dating methods. As a result of the mobility restrictions during the pandemic, the typical indicators of economic activity recorded one of their largest drops and higher rises . These highly influential observations may distort the results of the dating methods, especially those of the parametric approaches because the outliers have changed the empirical distribution and the time series dynamics.

Future work on dating the reference cycle should focus on establishing dating methods robust to the presence of influential data or other disturbances. Focusing on non-parametric techniques (Camacho et al. 2022) or Bayesian methods (Leiva-Leon et al. 2020) would be a good starting point.

Although not considered in this survey, dating in data-rich environments is also a promising research line. Recently, Piger (2020) provided a survey of supervised machine learning classification techniques applied to dating the reference cycle. Artificial neural networks, k-nearest neighbors, boosting, naïve bayes, classification trees and learning vector quantization are some of the methods that have only recently begun to be widely used in dating the business cycle.

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Table 1: NBER dates

Turning point	Date	Announcement	Lag
Trough	2020.04	July 19, 2021	15
Peak	2020.02	June 8, 2020	4
Trough	2009.06	September 20, 2010	15
Peak	2007.12	December 1, 2008	12
Trough	2001.11	July 17, 2003	20
Peak	2001.03	November 26, 2001	9
Trough	1991.03	December 22, 1992	21
Peak	1990.07	April 25, 1991	9
Trough	1982.11	July 8, 1983	8
Peak	1981.06	January 6, 1982	6
Trough	1980.06	July 8, 1981	13
Peak	1980.01	June 3, 1980	5

Note: The table shows the NBER dates of peaks and troughs, the announcement release times, and the time lag in months between them.

Table 2: Classification ability

	BB	DNS	HFS
		quarterly	
AUROC	0.97	0.98	0.91
CI 1947.1-2022.2	0.94	0.93	0.92
CI 2010.1-2022.2	1.00	1.00	1.00
		monthly	
AUROC	087	0.94	0.96
CI 1919.01-2022.06	0.89	0.85	0.88
CI 2010.01-2022.06	0.77	0.77	1
		annual	
AUROC	094	1.00	0.79

Note: The table checks the classification ability of three dating procedures - Bry-Boschan (BB), Dupraz at al. (DNS, 2021) and hierarchical factor segmentation (HFS) - as compared with measures of economic activity (AUROC) and with the NBER-referenced chronology (Concordance Index, CI).

Table 3: Markov-switching estimates

	Quarterly		Monthly	
	1947.1-2022.2	1947.1-2019.4	1919.01-2022.06	1919.01-2020.02
c_0	0.788 (0.058)	0.960 (0.0642)	0.450 (0.051)	0.479 (0.049)
c_1	-9.362 (1.000)	-0.431 (0.260)	-5.152 (0.541)	-4.23 (0.302)
σ^2	1.000 (0.085)	0.0632 (0.063)	2.632 (0.115)	2.548 (0.112)
p_{00}	0.997 (0.010)	0.950 (0.010)	0.987 (0.001)	0.987 (0.001)
p_{11}	0.000 (0.997)	0.682 (0.105)	0.657 (0.087)	0.747 (0.062)

Note: The figures show model estimates (standard deviations in brackets) of two means (c_1 and c_2), variance (σ), and transition probabilities (p_{11} and p_{22}).

Table 4: Number of cycles

K	LogLik	AIC	BIC	Bayes factor
1	-597.87	1205.74	1216.29	-
2	-256.41	534.83	558.05	658.24
3	-226.02	486.04	521.93	36.12
4	-188.75	423.49	472.04	49.89
5	-157.68	373.35	434.57	37.47
6	-129.37	328.74	402.62	31.95
7	-112.29	306.57	393.12	9.50
8	-87.16	268.32	367.53	25.59
9	-60.69	227.39	339.26	28.27
10	-66.70	251.39	375.93	-36.67

Note: The (log) likelihoods appear in the second column. AIC and BIC model selection criteria are in the third and fourth columns. The Bayes factors for models of K versus K+1 clusters appear in the last column.

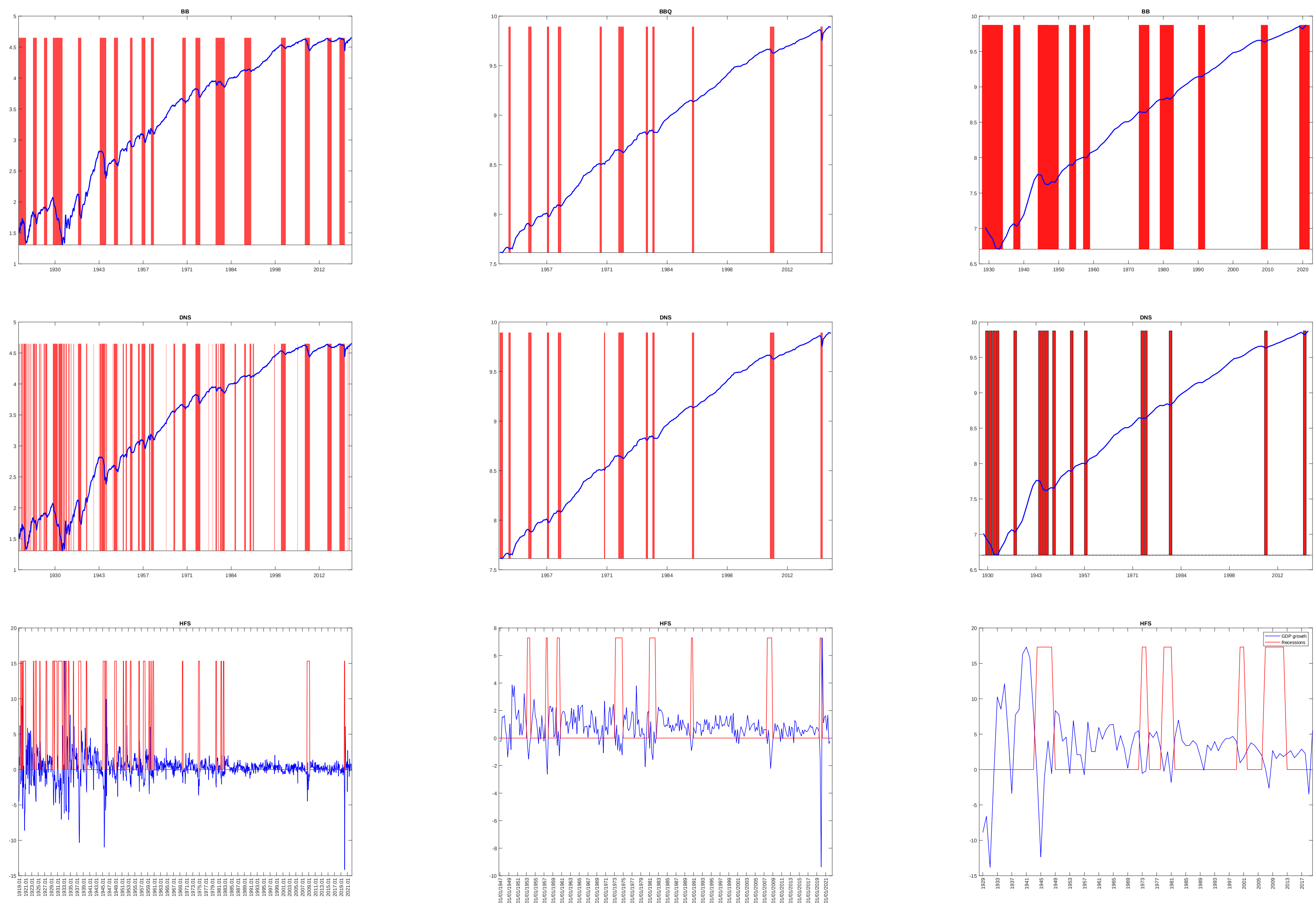
Table 5: Multivariate dating procedure

NBER		Estimates		Deviation	
Peaks	Troughs	Peaks	Troughs	Peaks	Troughs
1960.04	1961.02	1960.03	1961.02	1.00	0.00
1969.12	1970.11	1969.12	1970.11	0.00	0.00
1973.11	1975.03	1974.02	1975.03	-3.00	0.00
1980.01	1980.07	1979.11	1980.06	2.00	1.00
1981.07	1982.11	1981.07	1982.10	0.00	1.00
1990.07	1991.03	1990.04	1991.01	3.00	2.00
2001.03	2001.11	2000.12	2002.03	3.00	-4.00
2007.12	2009.06	2007.09	2009.08	3.00	-2.00
2020.02	2020.04	2020.03	2020.07	-1.00	-3.00

Note: The NBER-established dates appear in the first two columns. The peaks and troughs estimated with the mixture model are in Columns 3 and 4. The last two columns show the months of difference between the NBER turning points and those obtained from the mixture model.

Figure 1: Business cycle inferences

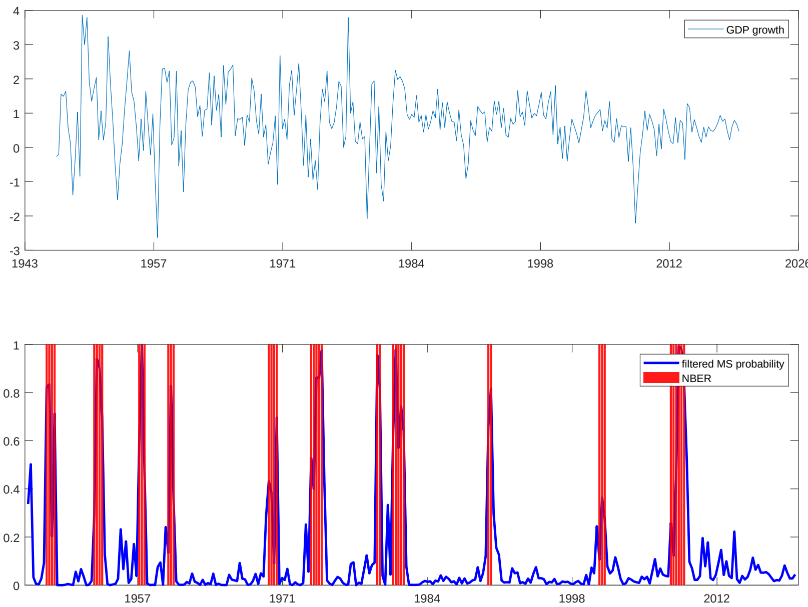
Industrial Production, monthly (1919.1-2022.6) | GDP, quarterly (1947.1-2022.2) | GDP, annual (1929-2021)



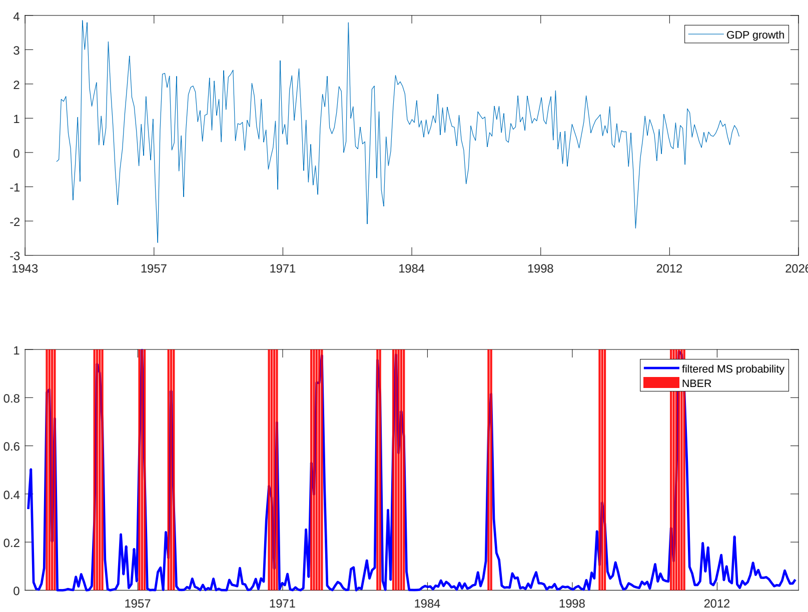
Note: In blue is the real GDP for the BB and DNS methods and its growth rate for the HFS. The red bars represent the recessions detected by each method at the different frequencies of the data.

Figure 2: Markov-switching model for GDP growth

Sample 1947.1-2019.4



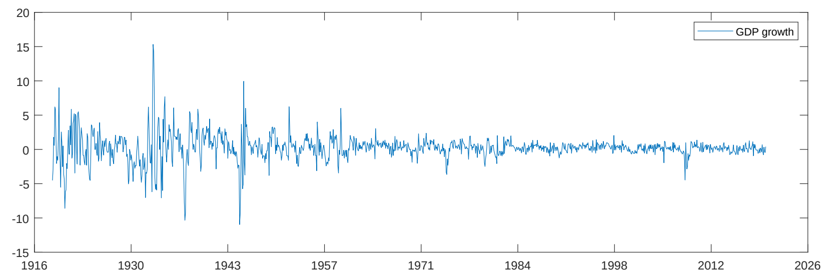
Sample 1947.1-2022.2

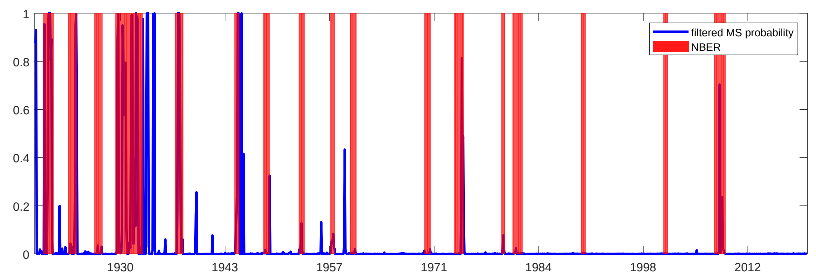


Note: The graph above shows the evolution of the real GDP growth rate. In the graph below, the blue line represents the filtered probability estimated with the MS model; the red bars indicate the recessions reported by the NBER.

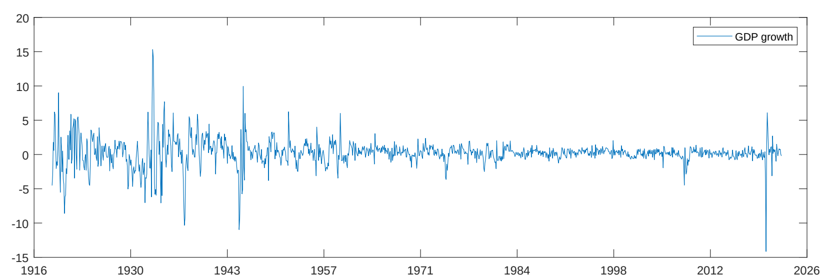
Figure 3: Markov-switching model for GDP growth

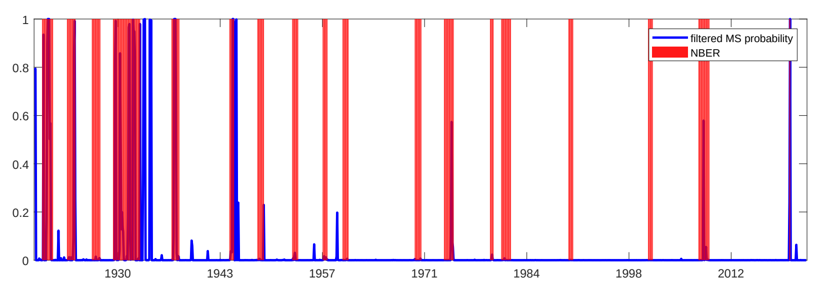
Sample 1919.01-2020.02





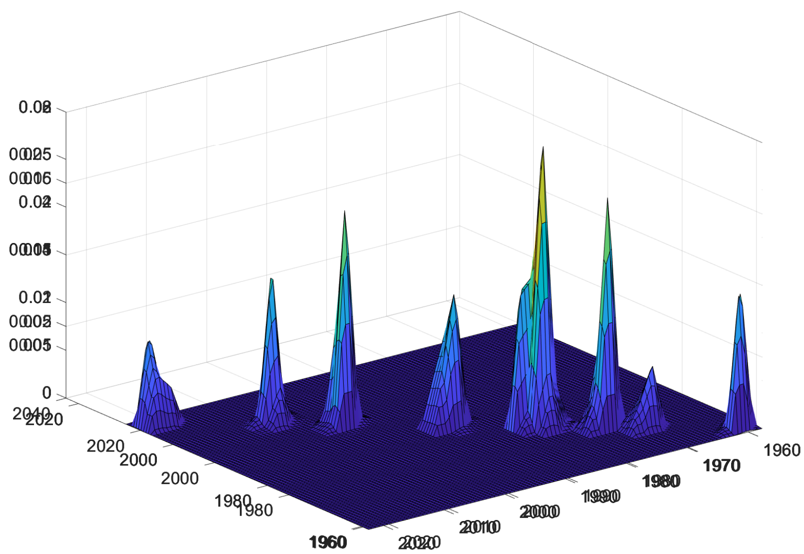
Sample 1919.01-2022.06





Note: The graph above shows the evolution of the real GDP growth rate. In the graph below, the blue line represents the filtered probability estimated with the MS model; the red bars indicate the recessions reported by the NBER.

Figure 4: Bivariate distribution of specific turning point dates



Note: The figure plots the bivariate kernel density of the specific pairs of peak-trough dates.

7 Appendix: Algorithms

Algorithm 1 Bry and Boschan (1971) dating algorithm

Step 0 Adjust seasonally and remove noisy components in the series of interest y_t

Step I Remove outliers (absolute difference with Spencer curve is larger than three standard deviations)

Step II Dating with moving-average.

1. Compute m centered moving average y_t^m, where $m = 12, 4, 1$ for monthly, quarterly and yearly data, respectively
2. Date a first set of turning points $\widehat{\psi}_2 = \{\widehat{\psi}^P_{1,2}, \widehat{\psi}^T_{1,2}, ..., \widehat{\psi}^P_{K,2}, \widehat{\psi}^T_{K,2}\}$ where the value of y_t is higher (or lower) than those of the w periods on either side; usually, $w = 5, 2, 1$ for monthly, quarterly and yearly data
3. Ensure alternating peaks and troughs: select from the adjacent peaks the one with the larger value and from the adjacent troughs the one with the smaller value

Step III Refine peaks and troughs with Spencer curve

1. Compute the Spencer curve y_t^{sp}
2. Date a new set of turning points $\widehat{\psi}_{3a} = \{\widehat{\psi}^P_{1,3a}, \widehat{\psi}^T_{1,3a}, ..., \widehat{\psi}^P_{K,3a}, \widehat{\psi}^T_{K,3a}\}$
3. Compare $\widehat{\psi}_2$ with $\widehat{\psi}_{3a}$ and ensuring alternating peaks and troughs
4. Ensure a minimum distance between cycles P-P or T-T of 15 months (4 quarters or 2 years). The resulting dates $\widehat{\psi}_{3b} = \{\widehat{\psi}^P_{1,3b}, \widehat{\psi}^T_{1,3b}, ..., \widehat{\psi}^P_{K,3b}, \widehat{\psi}^T_{K,3b}\}$ meet the constraints

Step IV Refine peaks and troughs with short-term moving average

1. Compute short-term moving average of 3 to 6 months
2. Identify a new set of turning points $\widehat{\psi}_4 = \{\widehat{\psi}^P_{1,4}, \widehat{\psi}^T_{1,4}, ..., \widehat{\psi}^P_{K,4}, \widehat{\psi}^T_{K,4}\}$
3. Ensure alternating peaks and troughs.

Step V Refine peaks and troughs with the unsmoothed series

1. Identify a new set of turning points within 4 or MCD periods
2. Ensure alternating peaks and troughs
3. Eliminate turns within 6 months of beginning and end of series
4. Enforcement minimum cycle duration: 15 months, 4-6 quarters, 2 years.
5. Enforcement minimum phase duration: 5 months, 2 quarters, 1 year.

The output is a set of turning points $\widehat{\psi}_5 = \{\widehat{\psi}^P_{1,5}, \widehat{\psi}^T_{1,5}, ..., \widehat{\psi}^P_{K,5}, \widehat{\psi}^T_{K,5}\}$ that fulfill all the constraints

Algorithm 2 Plucking algorithm, Dupraz et al. (2021)

while $t \leq T$ **do**

Step 1 set $cp = t$ and $t = t + 1$

if $y_t \geq y_{cp}$ **then**

Step 2 go back step 1

end if

if $y_{cp} \geq y_t \geq y_{cp} - \delta$ **then**

Step 3 set $t = t + 1$ go back step 2

end if

if $y_t < y_{cp} - \delta$ **then**

Step 4 add cp to the set of peaks

end if

Step 5 set $ct = t$ and $t = t + 1$

if $y_t \leq y_{ct}$ **then**

Step 6 go back step 5

end if

if $y_{ct} \leq y_t \leq y_{ct} + \delta$ **then**

Step 7 set $t = t + 1$ bo back step 6

end if

if $y_t > y_{ct} + \delta$ **then**

Step 8 add ct to the set of troughs

end if

end while

Algorithm 3 Hierarchical Factor Segmentation, Fushing et al. (2010a, 2010b)

Initialization Select two initial threshold values for distance between recurrent events, q_1, q_2, by maximizing the likelihood function proposed in Fushing et al. (2010a).

Step 1 Choose h and perform an initial decoding S^0. For example, set $S^0_t = 1$ when $y_t < h$ and $S^0_t = 0$ otherwise

Step 2 Compute the event-time of the occurrence $S^0_t = 1$, and denote the sequence of inter-event spacing as R^1. Transform R^1 into sequence into a 0-1 digital string denoted by C^1, where $C^1_t = 1$ if $R^1_t > q_1$

Step 4 Compute the event-time of the occurrence $C^1_t = 1$, and denote the sequence of inter-event spacing as R^2. Transform R^2 into sequence into a 0-1 digital string denoted by C^2, where $C^2_t = 1$ if $R^2_t > q_2$

Step 6 The resultant code sequence is mapped back into y_t. This produces a partition the time span, $\{\hat{\psi}^P_i, \hat{\psi}^T_i\}$, that refers to recessions