Non-symmetric polarization

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Let P be an m-homogeneous polynomial in n-complex variables $x_1, \dots, x_n$. Clearly, P has a unique representation in the form

$$P(x) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{(j_1,\ldots,j_m)} x_{j_1} \cdots x_{j_m},$$

and the m-form

$$L_P(x^{(1)}, \dots, x^{(m)}) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{(j_1,\ldots,j_m)} x_{j_1}^{(1)} \cdots x_{j_m}^{(m)}$$

satisfies $L_P(x, \dots, x) = P(x)$ for every $x \in \mathbb{C}^n$. We show that, although L_P in general is non-symmetric, for a large class of reasonable norms $\|\cdot\|$ on $\mathbb{C}^n$ the norm of L_P on $(\mathbb{C}^n, \|\cdot\|)^m$ up to a logarithmic term $(c \log n)^{m^2}$ can be estimated by the norm of P on $(\mathbb{C}^n, \|\cdot\|)$; here $c \ge 1$ denotes a universal constant. Moreover, for the ℓ_p-norms $\|\cdot\|_p$, $1 \le p < 2$ the logarithmic term in the number n of variables is even superfluous.