Lifting of nest approximation properties and related principles of local reflexivity

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Recall that a Banach space X has the approximation property if there exists a net of bounded linear finite-rank operators on X converging uniformly on compact subsets of X to the identity operator. Nest approximation properties are defined by the requirement that the finite-rank approximating operators leave invariant all subspaces in a given nest of closed subspaces of X.

Nest approximation properties were launched by T. Figiel and W. B. Johnson in *J. Funct. Anal.* 271 (2016), 566–576. These properties refine the concept of approximation properties for pairs due to T. Figiel, W. B. Johnson, and A. Pełczyński, *Israel J. Math.* 183 (2011), 199–231. The latter concept requires the finite-rank approximating operators to leave invariant a given closed subspace of X and it is, in turn, a refinement of the classical approximation properties.

In this lecture, we review some old and new results and open problems related to the lifting of various approximation properties between the dual space X^* and X. An emphasis is given to nest approximation properties, where lifting theorems rely on some new forms of the principle of local reflexivity which respect given nests of subspaces (cf. E. Oja, *Adv. Math.* 258 (2014), 1–12, and E. Oja, S. Veidenberg, *J. Funct. Anal.* 273 (2017), 2916–2938).