Multilinear Marcinkiewicz-Zygmund inequalities

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Given $1 \leq p, q, r \leq \infty$, the triple (p, q, r) is said to satisfy a Marcinkiewicz-Zygmund inequality if there is a constant $C \geq 1$ such that for each continuous linear operator $T : L^q(\mu) \to L^p(\nu)$, each $n \in \mathbb{N}$ and functions $f_1, \ldots, f_n \in L^q(\mu)$,

$$\left\| \left(\sum_{k=1}^{n} |T(f_k)|^r \right)^{1/r} \right\|_{L^p(\nu)} \leq C \|T\| \left\| \left(\sum_{k=1}^{n} |f_k|^r \right)^{1/r} \right\|_{L^q(\mu)}. \tag{1}$$

Particular cases of (1) include well-known inequalities of Marcinkiewicz, Zygmund, Paley, Grothendieck and Herz among others. A systematic study of these vector-valued inequalities was addressed by Defant-Junge and Gasch-Maligranda, who determined the set of triples (p, q, r) and, in almost all the cases, the best constants $C \geq 1$ satisfying (1). The aim of this talk is to discuss the extension of these classical inequalities to the multilinear setting. As an application, we will obtain vector-valued estimates for multilinear singular integrals. We will also study the connection between Marcinkiewicz-Zygmund inequalities and weighted inequalities.