Systems of seminorms on spaces of operators and weak bounded approximation properties

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A classical result, essentially due to Grothendieck, says that the approximation property (AP) of a dual Banach space X^* is metric whenever X^* or X^{**} have the Radon–Nikodým property (RNP). A version of its proof by Oja deals with the *weak metric* AP.
The weak metric AP and its generalization, a *bounded AP for a Banach operator ideal*, were introduced by Lima, Lima, and Oja in order to investigate the gap between the AP of a dual Banach space and its metric version.
We observe that in many cases the equivalence of such bounded APs boils down to a certain equivalence of systems of seminorms defined on all spaces of bounded linear operators. Such a language seems to help in highlighting the cornerstones of the theory. In particular, it enables one to simplify the aforementioned Oja's proof, eliminating the need to enter the original space X, and shows that the only essential part is Oja's "RNP impact lemma".