Stability properties of the differential process generated by complex interpolation

Jesús M. F. Castillo *Universidad de Extremadura*

The stability of the differential process associated to an interpolation scale has been a central topic in the theory since its inception. We will present here several advances obtained in collaboration with Willian Corrêa, Valentin Ferenczi and Manuel González. At the ground zero level the problem can be described as the study of properties P so that whenever the interpolation space X_θ has P (or an interpolated operator T has P on X_θ) there is an open neighborhood of θ so that all spaces X_t have P (or all interpolated operators T have P on X_t) for all t in that neighborhood. At level 1, which is the one we will consider in this talk, the problem is the study of properties P so that whenever the derived space dX_θ has P there is an open neighborhood of θ so that all derived spaces dX_t have P. We will focus on triviality and singularity properties.

In other words (or, better, in other drawings): Let $\mathbb{D}$ be the complex unit disk and let $(X_\xi)_{\xi\in\partial\mathbb{D}}$ be an interpolation family. Pick $\theta \in \mathbb{D}$ and assume that the induced exact sequence

$$0 \longrightarrow X_\theta \longrightarrow dX_\theta \longrightarrow X_\theta \longrightarrow 0$$

is trivial (resp. strictly singular). Does there exist $\varepsilon > 0$ so that the induced exact sequences $0 \to X_z \to dX_z \to X_z \to 0$ are trivial (resp. singular) for all $|z-\theta| < \varepsilon$?

References

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