Orbits of homogeneous polynomials on Banach spaces

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Let X be a Banach space. A function $F : X \to X$ is said to be hypercyclic if there exists $x \in X$ whose orbit $Orb_F(x) = \{F^n(x) : n \in \mathbb{N}\}$ is dense in X. It is a known fact that there are linear hypercyclic operators in arbitrary separable infinite dimensional Banach spaces. The dynamical system induced by a homogeneous polynomial is quite different. Associated to each (non linear) homogeneous polynomial there is a ball, centered at zero, that is invariant under the action of the polynomial. Moreover, the behavior of the orbits that meet this limit ball is clear: they simply tend to zero. Therefore homogeneous polynomials on Banach spaces are far from being hypercyclic.
However, the behavior of the orbits that never meet the limit ball can be highly non trivial. Indeed, in [1] Bernardes showed the existence of orbits oscillating between infinity and the limit ball. He also proved that there are supercyclic homogeneous polynomials in arbitrary separable infinite dimensional Banach spaces.
In this talk we will show a simple and natural 2-homogeneous polynomial that is at the same time d-hypercyclic (the orbit meets every ball of radius d), weakly hypercyclic (the orbit is dense with to respect the weak topology) and Γ-supercyclic ($\overline{\Gamma Orb_P(x)} = X$) for each subset $\Gamma \subseteq \mathbb{C}$ unbounded or not bounded away from zero. We will also generalize the construction to arbitrary infinite dimensional Fréchet spaces.

The talk is based on joint work with Santiago Muro.

References

[1] N.C. Bernardes. On orbits of polynomial maps in Banach spaces. *Quaest. Math.*, 21(3-4):318,1998.