Coordinatewise summability, inclusion theorems and p-Sidon sets

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Let $m \geq 1$, $X_1, \ldots, X_m$, Y Banach spaces and $T : X_1 \times \cdots \times X_m \to Y$ m-linear. For $r \geq 1$ and $\mathbf{p} = (p_1, \ldots, p_m) \in [1, +\infty)^m$, we say that T is multiple $(r, \mathbf{p})$-summing if there exists a constant $C > 0$ such that for all sequences $x(j) \subset X_j^{\mathbb{N}}$, $1 \leq j \leq m$,

$$\left(\sum_{\mathbf{i} \in \mathbb{N}^m} \| T(x_{\mathbf{i}}) \|^r \right)^{\frac{1}{r}} \leq C w_{p_1}(x(1)) \cdots w_{p_m}(x(m))$$

In this talk, we discuss the multiple summability of T when we have information on the summability of the maps it induces on each coordinate, extending important results of Defant, Popa and Schwarting for $(r, 1)$-multiple summability. Our methods have applications to inclusion theorems for multiple summing multilinear mappings and to the product of p-Sidon sets.