A characterization of the Bishop-Phelps-Bollobás property for operators for the pair (ℓ_∞^4, Y)

Maria D Acosta *University of Granada*

Bishop-Phelps Theorem states that the set of norm attaining functionals is dense in the (topological) dual of a Banach space. Bollobás showed a "quantitative" version of that result called nowadays the Bishop-Phelps-Bollobás Theorem. He proved that every pair of elements (x_0, x_0^*) in $S_X \times S_{X^*}$ such that $x_0^*(x_0) \sim 1$ can be approximated by another pair (x, x^*) in $S_X \times S_{X^*}$ such that $x^*(x) = 1$. In 2008 it was initiated the study of versions of such result for operators. A pair of Banach spaces (X, Y) has the Bishop-Phelps-Bollobás property for operators (BPBp for short) whenever every pair (x_0, S_0) in $S_X \times S_{L(X,Y)}$ such that $\|S_0(x_0)\| \sim 1$ can be approximated by another pair (x, T) in $S_X \times S_{L(X,Y)}$ such that $\|T(x)\| = 1$. Here we denote by $L(X, Y)$ the space of bounded and linear operators from X to Y. It is known that the previous property is non trivial. It is an open problem whether or not the pair (c_0, ℓ_1) has the BPBp in the real case. It is known that the pair (ℓ_∞^3, ℓ_1) has that property. Here we provide a characterization of the Banach spaces Y such that (ℓ_∞^4, Y) has the BPBp. As a consequence, we provide examples of spaces Y satisfying the previous condition.
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